

$C=(e_1, e_2)$ $e_1=(1,0)$ $e_2=(0,1)$ base canonica di \mathbb{R}^2

$B=(u_1, u_2)$ $u_1=(1,-2)$ $u_2=(0,3)$ base di \mathbb{R}^2

$\underline{B}=(v_1, v_2)$ $v_1=(0,5)$ $v_2=(1,2)$ altra base di \mathbb{R}^2

$$A_{B \rightarrow C} = \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix}$$

$$A_{\underline{B} \rightarrow C} = \begin{bmatrix} 0 & 1 \\ 5 & 2 \end{bmatrix}$$

$$A_{C \rightarrow \underline{B}} = [A_{\underline{B} \rightarrow C}]^{-1} = -\frac{1}{5} \begin{bmatrix} 2 & -1 \\ -5 & 0 \end{bmatrix}$$

$$A_{B \rightarrow \underline{B}} = A_{C \rightarrow \underline{B}} \bullet A_{B \rightarrow C} = -\frac{1}{5} \begin{bmatrix} 2 & -1 \\ -5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 3 \end{bmatrix} = -\frac{1}{5} \begin{bmatrix} 4 & -3 \\ -5 & 0 \end{bmatrix} = \begin{bmatrix} -\frac{4}{5} & \frac{3}{5} \\ 1 & 0 \end{bmatrix}$$

$$A_{B \rightarrow \underline{B}} = \begin{bmatrix} -\frac{4}{5} & \frac{3}{5} \\ 1 & 0 \end{bmatrix}$$

$$t = 3u_1 + 2u_2 = 3(1, -2) + 2(0, 3) = (3, 0) \quad t_B = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

$$t = xv_1 + yv_2 \quad t_{\underline{B}} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$t_{\underline{B}} = A_{B \rightarrow \underline{B}} \bullet t_B \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\frac{4}{5} & \frac{3}{5} \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \end{bmatrix} = \begin{bmatrix} -\frac{6}{5} \\ 3 \end{bmatrix}$$

$$t = -\frac{6}{5}v_1 + 3v_2 = -\frac{6}{5}(0, 5) + 3(1, 2) = (3, 0)$$