

$$\mathbf{u}=(u_1, u_2, u_3)=u_1\mathbf{i}+u_2\mathbf{j}+u_3\mathbf{k}$$

$$\mathbf{v}=(v_1, v_2, v_3)=v_1\mathbf{i}+v_2\mathbf{j}+v_3\mathbf{k}$$

$$\mathbf{u}\cdot\mathbf{v}=(u_1, u_2, u_3)\cdot(v_1, v_2, v_3)=(u_1\mathbf{i}+u_2\mathbf{j}+u_3\mathbf{k})\cdot(v_1\mathbf{i}+v_2\mathbf{j}+v_3\mathbf{k})=$$

$$=(u_1\mathbf{i})\cdot(v_1\mathbf{i})+(u_1\mathbf{i})\cdot(v_2\mathbf{j})+(u_1\mathbf{i})\cdot(v_3\mathbf{k})+(u_2\mathbf{j})\cdot(v_1\mathbf{i})+(u_2\mathbf{j})\cdot(v_2\mathbf{j})+(u_2\mathbf{j})\cdot(v_3\mathbf{k})+(u_3\mathbf{k})\cdot(v_1\mathbf{i})+(u_3\mathbf{k})\cdot(v_2\mathbf{j})+(u_3\mathbf{k})\cdot(v_3\mathbf{k})=$$

$$=u_1v_1\mathbf{i}\cdot\mathbf{i}+u_1v_2\mathbf{i}\cdot\mathbf{j}+u_1v_3\mathbf{i}\cdot\mathbf{k}+u_2v_1\mathbf{j}\cdot\mathbf{i}+u_2v_2\mathbf{j}\cdot\mathbf{j}+u_2v_3\mathbf{j}\cdot\mathbf{k}+u_3v_1\mathbf{k}\cdot\mathbf{i}+u_3v_2\mathbf{k}\cdot\mathbf{j}+u_3v_3\mathbf{k}\cdot\mathbf{k}=$$

$$=u_1v_1\cdot 1+u_1v_2\cdot 0+u_1v_3\cdot 0+u_2v_1\cdot 0+u_2v_2\cdot 1+u_2v_3\cdot 0+u_3v_1\cdot 0+u_3v_2\cdot 0+u_3v_3\cdot 1=$$

$$=u_1v_1\cdot 1+0+0+0+u_2v_2\cdot 1+0+0+0+u_3v_3\cdot 1=$$

$$=u_1v_1\cdot 1+u_2v_2\cdot 1+u_3v_3\cdot 1=$$

$$=u_1v_1+u_2v_2+u_3v_3$$

$$\mathbf{u}\cdot\mathbf{v}=u_1v_1+u_2v_2+u_3v_3$$

$$\frac{1}{\|\mathbf{v}\|}\mathbf{v}$$

$$\mathbf{v}=(x,y,z)$$

$$\|\mathbf{v}\| = \sqrt{x^2 + y^2 + z^2}$$

$$\frac{1}{\|\mathbf{v}\|}\mathbf{v} = \frac{1}{\sqrt{x^2 + y^2 + z^2}}(x, y, z) = \left(\frac{x}{\sqrt{x^2 + y^2 + z^2}}, \frac{y}{\sqrt{x^2 + y^2 + z^2}}, \frac{z}{\sqrt{x^2 + y^2 + z^2}} \right)$$

$$\left(\frac{1}{\|\mathbf{v}\|}\mathbf{v}\right)\cdot\mathbf{i}=\left(\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}\right)\cdot(1,0,0)=\frac{x}{\sqrt{x^2+y^2+z^2}} = \cos\widehat{\frac{1}{\|\mathbf{v}\|}\mathbf{v}\mathbf{i}}$$

$$\left(\frac{1}{\|\mathbf{v}\|}\mathbf{v}\right)\cdot\mathbf{j}=\left(\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}\right)\cdot(0,1,0)=\frac{y}{\sqrt{x^2+y^2+z^2}} = \cos\widehat{\frac{1}{\|\mathbf{v}\|}\mathbf{v}\mathbf{j}}$$

$$\left(\frac{1}{\|\mathbf{v}\|}\mathbf{v}\right)\cdot\mathbf{k}=\left(\frac{x}{\sqrt{x^2+y^2+z^2}}, \frac{y}{\sqrt{x^2+y^2+z^2}}, \frac{z}{\sqrt{x^2+y^2+z^2}}\right)\cdot(0,0,1)=\frac{z}{\sqrt{x^2+y^2+z^2}} = \cos\widehat{\frac{1}{\|\mathbf{v}\|}\mathbf{v}\mathbf{k}}$$