Esercizio 1

$$\lim_{x\to\pm\infty}\frac{x-1}{\sqrt{2x^2-1}}=\lim_{x\to\pm\infty}\frac{x\left(1-\frac{1}{x}\right)}{|x|\sqrt{2-\frac{1}{x^2}}}=\lim_{x\to\pm\infty} segnx\,\frac{\left(1-\frac{1}{x}\right)}{\sqrt{2-\frac{1}{x^2}}}=\begin{cases}-\frac{1}{\sqrt{2}}\ se\ x\to-\infty\\ \frac{1}{\sqrt{2}}\ se\ x\to+\infty\end{cases}$$

Esercizio 2

$$\lim_{x\to 2}\frac{2x^3-5x^2-4x+12}{x^4-4x^3+5x^2-4x+4}$$

$$\frac{2x^3-5x^2-4x+12}{x^4-4x^3+5x^2-4x+4}=\frac{n(x)}{d(x)}$$

$$n(2)=0$$

$$d(2)=0$$

$$(x-2)\ divide\ 2x^3-5x^2-4x+12$$

	2	-5	-4	12
2		4	-2	-12
	2	-1	-6	0

$$2x^3-5x^2-4x+12=(x-2)(2x^2-x-6)$$
$$2x^3-5x^2-4x+12=(x-2)^2(2x+3)$$

$$(x-2)\ divide\ x^4-4x^3+5x^2-4x+4$$

	1	-4	5	-4	4
2		2	-4	2	-4
	1	-2	1	-2	0

$$x^4-4x^3+5x^2-4x+4=(x-2)(x^3-2x^2+x-2)$$
$$x^4-4x^3+5x^2-4x+4=(x-2)^2(x^2+1)$$

$$\frac{2x^3-5x^2-4x+12}{x^4-4x^3+5x^2-4x+4}=\frac{(x-2)^2(2x+3)}{(x-2)^2(x^2+1)}=\frac{(2x+3)}{(x^2+1)}$$

$$\lim_{x\to 2}\frac{2x^3-5x^2-4x+12}{x^4-4x^3+5x^2-4x+4}=\lim_{x\to 2}\frac{(2x+3)}{(x^2+1)}=\frac{7}{5}$$

Esercizio 3

$$\lim_{x\to+\infty}\sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)=\lim_{x\to+\infty}\frac{\sqrt{x}\left(\sqrt{x+1}-\sqrt{x}\right)\left(\sqrt{1+x}+\sqrt{x}\right)}{\sqrt{1+x}+\sqrt{x}}=$$

$$=\lim_{x\to+\infty}\frac{\sqrt{x}(x+1-x)}{\sqrt{1+x}+\sqrt{x}}=\lim_{x\to+\infty}\frac{\sqrt{x}}{\sqrt{1+x}+\sqrt{x}}=\lim_{x\to+\infty}\frac{\sqrt{x}}{\sqrt{x\left(\frac{1}{x}+1\right)}+\sqrt{x}}=$$

$$=\lim_{x\to+\infty}\frac{\sqrt{x}}{\sqrt{x}\sqrt{\frac{1}{x}+1}+\sqrt{x}}=\lim_{x\to+\infty}\frac{\sqrt{x}}{\sqrt{x}\left(\sqrt{\frac{1}{x}+1}+1\right)}=\lim_{x\to+\infty}\frac{1}{\sqrt{\frac{1}{x}+1}+1}=\frac{1}{2}$$

Esercizio 4

$$\lim_{x\to0}\frac{e^{(\sin x)^3}-1}{\log(1-x)-\log(1+x)}\sim\lim_{x\to0}\frac{(\sin x)^3}{-x-x}\sim\lim_{x\to0}\frac{x^3}{-2x}=\lim_{x\to0}\frac{x^2}{-2}=0$$

$$\log(1-x)-\log(1+x)=\log\frac{1-x}{1+x}=\log\frac{1+x-x-x}{1+x}=$$

$$\log\left(1+\frac{-2x}{1+x}\right)\sim\frac{-2x}{1+x}\ per\ x\to0$$

$$\lim_{x\to0}\frac{e^{(\sin x)^3}-1}{\log(1-x)-\log(1+x)}\sim\lim_{x\to0}\frac{(\sin x)^3}{\frac{-2x}{1+x}}\sim\lim_{x\to0}\frac{x^3(1+x)}{-2x}=\lim_{x\to0}\frac{x^2(1+x)}{-2}=0$$

Esercizio 5

$$\lim_{x\to+\infty}\sqrt{x}\left(\sqrt[3]{x+1}-\sqrt[3]{x-1}\right)=\lim_{x\to+\infty}\sqrt{x}\sqrt[3]{x-1}\left(\frac{\sqrt[3]{x+1}}{\sqrt[3]{x-1}}-1\right)=$$

$$=\lim_{x\to+\infty}\sqrt{x}\sqrt[3]{x-1}\left(\sqrt[3]{\frac{x+1}{x-1}}-1\right)=\lim_{x\to+\infty}\sqrt{x}\sqrt[3]{x-1}\left(\sqrt[3]{\frac{x-1+1+1}{x-1}}-1\right)=$$

$$=\lim_{x\to+\infty}\sqrt{x}\sqrt[3]{x-1}\left(\sqrt[3]{\frac{x-1+2}{x-1}}-1\right)=\lim_{x\to+\infty}\sqrt{x}\sqrt[3]{x-1}\left(\sqrt[3]{1+\frac{2}{x-1}}-1\right)=$$

$$=\lim_{x\to+\infty}\sqrt{x}\sqrt[3]{x-1}\left(\left(1+\frac{2}{x-1}\right)^{\frac{1}{3}}-1\right)\sim\lim_{x\to+\infty}\sqrt{x}\sqrt[3]{x-1}\left(\frac{1}{3}\left(\frac{2}{x-1}\right)\right)\sim$$

$$\sim\lim_{x\to+\infty}\sqrt{x}\sqrt[3]{x}\left(\frac{1}{3}\left(\frac{2}{x-1}\right)\right)=\lim_{x\to+\infty}x^{\frac{1}{2}}x^{\frac{1}{3}}\left(\frac{1}{3}\left(\frac{2}{x-1}\right)\right)=$$

$$\sim\lim_{x\to+\infty}x^{\frac{5}{6}}\left(\frac{1}{3}\left(\frac{2}{x}\right)\right)\sim\frac{2}{3}\lim_{x\to+\infty}\frac{1}{x^{1-\frac{5}{6}}}=\frac{2}{3}\lim_{x\to+\infty}\frac{1}{x^{\frac{1}{6}}}=\frac{2}{3}\lim_{x\to+\infty}\frac{1}{\sqrt[6]{x}}=0$$