15 Dicembre Buongiorno!

Titolo nota 15/12/2020

14/01/2020

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 4 & 2 & -4 \\ 0 & 0 & 0 \end{bmatrix}$$

Trovare autovalori e autovettori di A

$$p_A(\lambda) = \det(A - \lambda I_3) = \ldots = \lambda^2 \cdot (2-\lambda)^1 ; \quad \lambda_1 = 0 \quad \lambda_2 = 2$$

autovettori relativi a $\lambda_1 = 0$

$$\begin{bmatrix} 0 & 0 & 0 \\ 4 & 2 & -4 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{cases} 0 = 0 \\ 4x + 2y - 4z = 0 \\ 0 = 0 \end{cases} \qquad 2x + y - 2z = 0$$

$$y = 2z - 2x$$

$$(x, y, z) = (x, 2z - 2x, z) = x(1, -2, 0) + z(0, 2, 1)$$

autovettori $\forall (x, z) \in \mathbb{R}^2 - \{(0,0)\}$

$\lambda_1 = 0$ base $E(\lambda_1) = ((1, -2, 0), (0, 2, 1))$

$$m_g(\lambda_1) = \dim(E(\lambda_1)) = 2$$

$$\lambda_2 = 2 \qquad \begin{bmatrix} -2 & 0 & 0 \\ 4 & 0 & -4 \\ 0 & 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{cases} -2x = 0 & \Rightarrow x = 0 \\ 4x - 4z = 0 \\ -2z = 0 & \Rightarrow z = 0 \end{cases}$$

nessuna

$4\cdot 0 - 4\cdot 0 = 0$ identità

informazione su y che quindi è LIBERA

$$(x,y,z) = (0,y,0) = y\,(0,1,0)$$

$$\forall y \in \mathbb{R} - \{0\}$$

$$\lambda_2 = 2 \qquad \text{base } E(\lambda_2) = (0,1,0)$$

$$m_g(\lambda_2) = \dim E(\lambda_2) = 1$$

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(2) π' piano passante per $A(9, \sqrt{5}, 23)$ parallelo alla retta $r: \underbrace{x+3y-7}_{} = \underbrace{2y+z}_{} + 9 = 0$ e perpendicolare al piano $\pi: 3x + 2z + 13 = 0$. Trovare il punto B d'intersezione tra π' e $\underbrace{\text{asse } z}_{x=y=0}$.

$$\pi': ax + by + cz + d = 0$$

$$\vec{u}_{\pi'} = (a,b,c) \neq \vec{0} \qquad \vec{u}_{\pi'} \perp \pi'$$

$$\pi' /\!/ r \iff \det\begin{bmatrix} a & b & c \\ 1 & 3 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{matrix} a & b \\ 1 & 3 \\ 0 & 2 \end{matrix} = 0$$

$$\boxed{3a + 2c - b = 0} \Rightarrow b = 3a + 2c$$

$$\pi' \perp \pi \iff \vec{u}_{\pi'} \cdot \vec{u}_{\pi} = 0 \iff 3\cdot a + 0\cdot b + 2\cdot c = 0$$

$3x+2z+13=0$

$\boxed{3a+2c=0}$ $\qquad b=3a+2c \Rightarrow \boxed{b=0}$ fisso

↑ scelgo, a piacere, $a=2$ e $c=-3$

$$\pi': \ 2\cdot x + 0\cdot y + (-3)\cdot z + d = 0$$

$$A(9,\sqrt{5},23) \in \pi' \Rightarrow 2\cdot 9 - 3\cdot 23 + d = 0 \Rightarrow$$

$$\Rightarrow 18 - 69 + d = 0 \Rightarrow d = +51$$

$$\pi': \ 2x - 3z + 51 = 0$$

$$\pi' \cap \underbrace{\text{asse } z}_{x=y=0} \Rightarrow B(0,0,17)$$

$-3z+51=0$

$51=3z$

$z=\frac{51}{3}=17$

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(3) $r: x+3y-27 = y-2z = 0$ $\qquad$ sghembe

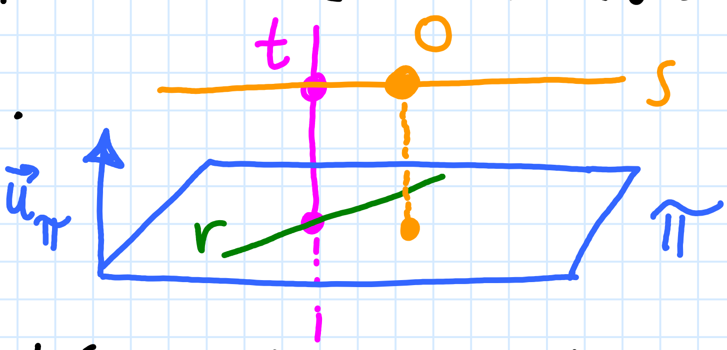
$\boxed{s: 3x+5z = x+2y+2z=0}$

Sia t la retta di minima distanza

Sia D la distanza tra le 2 rette

Trovare vettore avente la direzione di t e lunghezza D.

$O(0,0,0) \in s$



$$D = d(r,s) = d(O,\pi) \leftarrow \text{formula}$$

$\pi : \pi \in F(r)$ et $\pi // s$

$\pi \in F(r) : \lambda(x+3y-27) + \mu(y-2z) = 0$

$$\pi : \underbrace{\lambda}_{a} \cdot x + \underbrace{(3\lambda+\mu)}_{b} \cdot y + \underbrace{(-2\mu)}_{c} \cdot z \underbrace{(-27 \cdot \lambda}_{d} = 0$$

$s : \underline{3x+5z} = \underline{x+2y+2z} = 0$

$$\pi // s \iff \det \begin{bmatrix} \lambda & (3\lambda+\mu) & (-2\mu) \\ 3 & 0 & 5 \\ 1 & 2 & 2 \end{bmatrix} \begin{matrix} \lambda & (3\lambda+\mu) \\ 3 & 0 \\ 1 & 2 \end{matrix} = 0$$

$$\underline{5(3\lambda+\mu)} - 12\mu - 10\lambda \underline{-6(3\lambda+\mu)} = 0$$

$$-12\mu - 10\lambda - (3\lambda+\mu) = 0$$

$$-13\mu - 13\lambda = 0 \qquad \lambda + \mu = 0$$

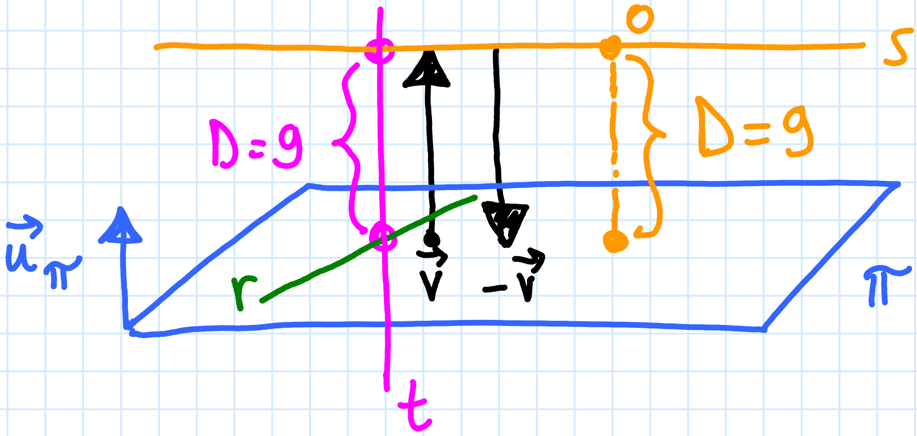
scelgo $\lambda = 1$ e $\mu = -1$

$$\pi : 1 \cdot x + 2 \cdot y + 2 \cdot z - 27 = 0$$

$\vec{u}_\pi = (1,2,2) \perp \pi$ ($\vec{u}_\pi //$ retta minima distanza)

$$d(O, \pi) = \frac{|1 \cdot 0 + 2 \cdot 0 + 2 \cdot 0 - 27|}{\sqrt{1^2+2^2+2^2}} = \frac{|-27|}{\sqrt{9}} = 9$$

$O \rightarrow (0,0,0)$



$\|\vec{v}\| = 9$

$\vec{v} \,//\, t$

$\vec{u}_\pi = (1,2,2) \Rightarrow \|\vec{u}_\pi\| = ? = 3$

$\vec{v} = 3\,\vec{u}_\pi = 3\cdot(1,2,2) = (3,6,6)$

$-\vec{v} = -(3,6,6) = (-3,-6,-6)$

uno dei due

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(4) r retta parallela asse z e passante per $A(2\sqrt{7}, 2\sqrt{7}, \sqrt{3})$. Trovare i piani α_1 α_2 che contengono r e formano un angolo di $\frac{\pi}{4}$ radianti con il piano XZ. $\leftarrow \beta$

Problema principale: angolo tra 2 piani

RIPASSO

$\alpha: ax+by+cz+d=0 \qquad \beta: a'x+b'y+c'z+d'=0$

$$\cos(\widehat{\alpha,\beta}) = \pm\frac{aa'+bb'+cc'}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{(a')^2+(b')^2+(c')^2}}$$

$\frac{\pi}{4}$ rad

β = piano XZ : $y=0$

$\beta: \underbrace{0}_{a'}\cdot x + \underbrace{1}_{b'}\cdot y + \underbrace{0}_{c'}\cdot z + 0 = 0$

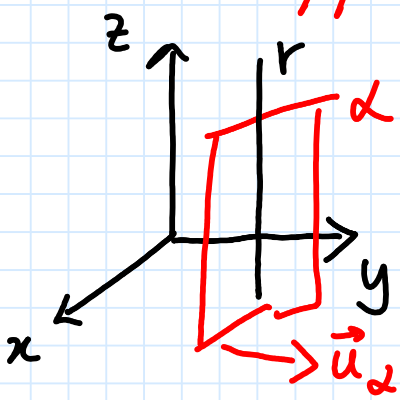
$\alpha : ax+by+cz+d=0$ piano(i) incognito

contiene r // asse z

$\vec{u}_\alpha = (a,b,c) \perp \alpha$

$\uparrow$

0



$r // (0,0,1)$

$(a,b,0) = \vec{u}_\alpha$

$(a',b',c') = (0,1,0) = \vec{u}_\beta$

$\widehat{\alpha,\beta} = \frac{\pi}{4}$ rad

$$\frac{\sqrt{2}}{2} = \pm \frac{a\cdot 0 + b\cdot 1 + 0\cdot 0}{\sqrt{a^2+b^2+0^2}\cdot\sqrt{0^2+1^2+0^2}} = \pm \frac{b}{\sqrt{a^2+b^2}}$$

$$\frac{1}{2} = \frac{b^2}{a^2+b^2} \Rightarrow a^2+b^2 = 2b^2 \Rightarrow$$

$$\Rightarrow a^2-b^2=0 \Rightarrow (a-b)\cdot(a+b)=0 \Rightarrow$$

$$\Rightarrow a-b=0 \quad \vee \quad a+b=0$$

scelgo $a=1$ e $b=1$

scelgo $a=1$ e $b=-1$

$c=0$ fisso per tutti e due

$\alpha_1 : x+y+d_1=0$

$\alpha_2 : x-y+d_2=0$

$A(2\sqrt{7}, 2\sqrt{7}, \sqrt{3}) \in \alpha_1 \Rightarrow 2\sqrt{7} + 2\sqrt{7} + d_1 = 0 \Rightarrow$

$\Rightarrow d_1 = -4\sqrt{7}$

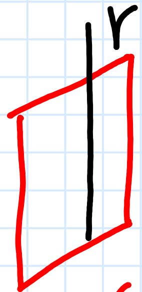
$$\boxed{\alpha_1 : x + y - 4\sqrt{7} = 0}$$

$A(2\sqrt{7}, 2\sqrt{7}, \sqrt{3}) \in \alpha_2 \Rightarrow \cancel{2\sqrt{7}} \cancel{-2\sqrt{7}} + d_2 = 0 \Rightarrow$

$\Rightarrow d_2 = 0$

$$\boxed{\alpha_2 : x - y = 0}$$

$\vec{k} = (0,0,1)$ $\uparrow$ asse z $r \parallel$ asse z (dato del problema)



$r \parallel \vec{k} = (0, 0, 1)$

α (dato del problema) contiene r

$$r \subseteq \alpha \Rightarrow r \parallel \alpha \Rightarrow \underbrace{(0,0,1)}_{\vec{k}} \parallel \alpha$$

$\vec{u}_\alpha$

$\longrightarrow k = (0,0,1) \parallel \alpha$

α

$\vec{u}_\alpha \perp \vec{k}$

$\vec{u}_\alpha \bullet \vec{k} = 0$ (zero)

$$\underbrace{(a, b, c)}_{\vec{u}_\alpha} \bullet \underbrace{(0, 0, 1)}_{k} = 0$$

$$a \cdot \cancel{0} + b \cdot \cancel{0} + c \cdot 1 = 0 \implies \boxed{c = 0}$$

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(5) $3x^2 + 10\sqrt{3}xy - 7y^2 + 4\sqrt{3}x - 44y - 52 = 0$

Trovare equazione CANONICA. Poi classificarla.

$$A = \begin{bmatrix} 3 & 5\sqrt{3} \\ 5\sqrt{3} & -7 \end{bmatrix}; \quad p_A(\lambda) = \ldots = \lambda^2 - \underbrace{(-4)}_{\mathrm{Tr}A}\lambda + \underbrace{(-96)}_{\det A}$$

$\mathrm{Tr}A = 3 + (-7) = -4$ $\quad\quad$ $\det A = -21 - 75 = -96$

$$p_A(\lambda) = \lambda^2 + 4\lambda - 96 = (\lambda + 12) \cdot (\lambda - 8);$$

$\lambda_1 = +8$ $\quad\quad$ $\lambda_2 = -12$

autovettori relativi a $\lambda_1 = +8$

$$(A - 8I_2) = \begin{bmatrix} -5 & 5\sqrt{3} \\ 5\sqrt{3} & -15 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \boxed{-x + \sqrt{3}y = 0}$$

$x = \sqrt{3}y$ $\quad\quad$ scelgo $y = 1 \implies x = \sqrt{3}$

auto <u>vettore</u> $\vec{v}_1 = (\sqrt{3}, 1)$ $\quad\quad$ $\|\vec{v}_1\| = ? = 2$

auto <u>VERSORE</u> (lunghezza 1) $\quad \vec{u}_1 = \frac{1}{2}\vec{v}_1 = \frac{1}{2}(\sqrt{3}, 1) = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$$\vec{u}_1 = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$$

$$\Lambda = \begin{bmatrix} 8 & 0 \\ 0 & -12 \end{bmatrix} \; ; \quad C = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

$\vec{u}_1$ $\quad$ $\vec{u}_2$

$\boxed{\theta = 30^\circ \text{ anti}}$

$\vec{u}_2 \perp \vec{u}_1$ e $\|\vec{u}_i\|$ e $\det C = +1$

$$\underbrace{[4\sqrt{3} \quad -44]}_{d \qquad e} \cdot \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \underbrace{[-16 \quad -24\sqrt{3}]}_{d' \qquad e'}$$

dopo la rotazione C di angolo θ

$$\boxed{8 \cdot (x')^2} + \boxed{(-12) \cdot (y')^2} + \boxed{\underbrace{(-16)}_{d'} \cdot x'} + \boxed{\underbrace{(-24\sqrt{3})}_{e'} \cdot y'} - 52 = 0$$

$$\boxed{2 \cdot (x')^2} \boxed{- 3 \cdot (y')^2} \boxed{- 4x'} \boxed{- 6\sqrt{3} \cdot y} - 13 = 0$$

$$\boxed{2 \cdot \left[(1x' - 1)^2 - 1 \right]} \boxed{- 3 \left[(1y' + \sqrt{3})^2 - 3 \right]} - 13 = 0$$

traslazione $\begin{cases} x'' = x' - 1 \\ y'' = y' + \sqrt{3} \end{cases}$

$$2 \cdot (x'')^2 - 2 - 3(y'')^2 + 9 - 13 = 0$$

$$2\cdot(x'')^2 - 3\cdot(y'')^2 = +6$$

$$\frac{(x'')^2}{3} - \frac{(y'')^2}{2} = +1$$

→ equazione canonica → IPERBOLE

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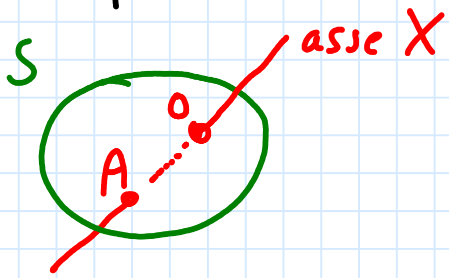
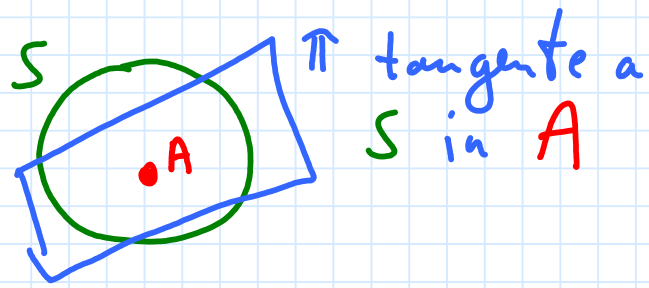
(6) L'asse X incontra la sfera

$$S: x^2+y^2+z^2-5x+2y-z=0$$

nell'origine $O(0,0,0)$ e in un punto A.
Scrivere l'equazione del piano tangente alla sfera nel punto A.





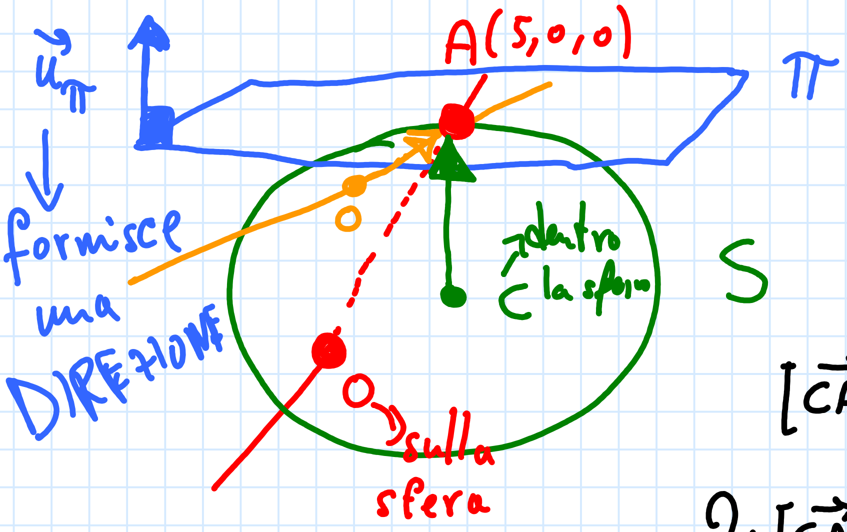
$A = ?$ asse X: $y=z=0$

$$S \cap \text{asse } X : \begin{cases} \text{———} \leftarrow S \\ y=z=0 \end{cases}$$

$$x^2+0^2+0^2-5x+2\cdot 0-0=0$$

$$x^2-5x=0 \ ; \quad x\cdot(x-5)=0 \begin{cases} \nearrow x_1=0 \to O \\ \searrow x_2=5 \to A(5,0,0) \end{cases}$$

$$A(5,0,0)$$



$[\vec{CA}] \perp \pi$

$C\left(\frac{5}{2}, -1, \frac{1}{2}\right)$

$[\vec{CA}] = \left(\frac{5}{2}, +1, -\frac{1}{2}\right)$

$2 \cdot [\vec{CA}] = (5, 2, -1) \parallel [\vec{CA}]$

$$\underset{a\ \ \ b\ \ \ c}{(5, 2, -1)} \perp \pi$$

$$\pi : 5 \cdot x + 2 \cdot y + (-1) \cdot z + d = 0$$

?

$$A(5,0,0) \in \pi \Rightarrow 5 \cdot 5 + 2 \cdot 0 + (-1) \cdot 0 + d = 0 \Rightarrow$$

$$\Rightarrow 25 + d = 0 \Rightarrow d = -25$$

$$\pi : 5 \cdot x + 2y - z - 25 = 0$$

11/06/2014

(1) $t \in \mathbb{R}$: $A = \begin{bmatrix} (-7t) & (-4t) \\ (8t^2) & (t) \end{bmatrix}$ sia

diagonalizzabile

$$p_A(\lambda) = \det \begin{bmatrix} [(-7t)-\lambda] & (-4t) \\ (8t^2) & [t-\lambda] \end{bmatrix} =$$

$$= (-7t-\lambda)(t-\lambda) - 8t^2(-4t) =$$

$$= \underbrace{1}_{a} \cdot \lambda^2 + \underbrace{(6t)}_{b} \cdot \lambda \underbrace{-7t^2 + 32 \cdot t^3}_{c} ;$$

$$\Delta = b^2 - 4 \cdot a \cdot c = (6t)^2 - 4 \cdot 1 \cdot (-7t^2 + 32t^3) =$$

$$= 36t^2 + 28t^2 - 128 \cdot t^3 =$$

$$= 64t^2 - 128t^3 = 64 \cdot t^2 \cdot (1 - 2 \cdot t)$$

ricordare che, se A non è già diagonale, allora è diagonalizzabile $\Leftrightarrow \Delta > 0$

$$\Delta = 64 \cdot t^2 \cdot (1-2t) > 0$$

$$\frac{\Delta}{4} = 16 \cdot t^2 \cdot (1-2t) > 0$$

se $\boxed{t=0}$, allora $\Delta = 0$. PERÒ

per $t=0$ si ha che $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ che è già diagonale (caso particolarissimo)

essendo già diagonale considero $t \neq 0$

$t \neq 0 \Rightarrow t^2 > 0$

$$\Delta = \underbrace{64}_{>0} \cdot \underbrace{t^2}_{>0} \cdot (1-2t) > 0 \iff 1-2t > 0 \iff$$

$$\iff 2t < 1 \iff t < \frac{1}{2}$$

$$\boxed{t < \frac{1}{2}}$$