16 Dicembre Buongiorno!

Titolo nota 16/12/2020

11/02/2020

(1) $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 0 & 7 \\ -2 & 0 & 0 \end{bmatrix}$ Trovare autovalori di A e una base per ogni autospazio.

$$P_A(\lambda) = \det \begin{bmatrix} \boxed{(1-\lambda) \quad 0 \quad 0} \\ 3 \quad -\lambda \quad 7 \\ -2 \quad 0 \quad -\lambda \end{bmatrix} = (1-\lambda)\cdot \det \begin{bmatrix} -\lambda & 7 \\ 0 & -\lambda \end{bmatrix} =$$

$$= (1-\lambda)^1 \cdot \lambda^2 \quad \begin{matrix} \nearrow \lambda_1 = 1 \quad m_a(1) = 1 \Rightarrow m_g(1) = 1 \\ \searrow \lambda_2 = 0 \quad m_a(0) = 2 \Rightarrow m_g(0) \leq 2 \end{matrix}$$

$\boxed{\lambda_1 = 1}$ $(A - 1\cdot I_3) = \begin{bmatrix} 0 & 0 & 0 \\ 3 & -1 & 7 \\ -2 & 0 & -1 \end{bmatrix}$

$$\begin{cases} 0 = 0 \\ 3x - y + 7z = 0 \\ -2x \quad -z = 0 \end{cases} \quad \begin{cases} z = -2x \\ 3x - y - 14x = 0 \end{cases} \quad \begin{cases} z = -2x \\ y = -11x \end{cases}$$

$$(x,y,z) = (x, -11x, -2x) = x(1, -11, -2) \quad \forall x \in \mathbb{R} - \{0\}$$

$\boxed{\lambda_1 = 1}$ base di $E(\lambda_1) = E(1) = (1, 11, -2)$

$\boxed{\lambda_2 = 0}$ $(A - 0\cdot I_3) = A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 0 & 7 \\ -2 & 0 & 0 \end{bmatrix}$ $\begin{cases} x = 0 \\ 3x + 7z = 0 \\ -2x = 0 \end{cases}$

$x=0$ fisso ; $z=0$ fisso ; quindi y è LIBERA

$(x,y,z)=(0,y,0)=y(0,1,0) \quad \forall y\in\mathbb{R}-\{0\}$

$$\boxed{\boxed{\lambda_2=0} \quad \text{base di } E(\lambda_2)=E(0)=(0,1,0)}$$

$m_g(\lambda_2)=m_g(0)=\dim E(0)=1$

$m_g(0)=1<2=m_a(0)$

Quindi, <u>NON</u> esiste base di $\mathbb{R}^3$ formata da autovettori di A ovvero A <u>NON</u> è diagonalizzabile

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(2) $3x-y+10z-22=1x-y+8z-8=2x+y-5z-13=0$

Trovare la soluzione generale oppure una particolare X_p e una base spazio soluzioni sistema <u>omogeneo</u> associato.

$$C=\left[\begin{array}{ccc|c} 1 & -1 & 8 & -8 \\ 3 & -1 & 10 & -22 \\ 2 & 1 & -5 & -13 \end{array}\right] \rightsquigarrow \left[\begin{array}{ccc|c} 1 & -1 & 8 & -8 \\ 0 & \overset{1}{\cancel{2}} & \overset{-7}{\cancel{-14}} & \overset{1}{\cancel{2}} \\ 0 & \underset{1}{\cancel{3}} & \underset{-7}{\cancel{-21}} & \underset{1}{\cancel{3}} \end{array}\right] \rightsquigarrow$$

$$\rightsquigarrow \left[\begin{array}{ccc|c} 1 & -1 & 8 & -8 \\ 0 & 1 & -7 & 1 \\ 0 & 1 & -7 & 1 \end{array}\right] \rightsquigarrow \begin{array}{c} \left[\begin{array}{ccc|c} 1 & -1 & 8 & -8 \\ 0 & 1 & -7 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \\ \begin{array}{cccc} x & y & z & \uparrow \text{NOTI} \end{array} \end{array} = C'$$

$$\begin{cases} x - y + 8z - 8 = 0 \\ \quad\; y - 7z + 1 = 0 \end{cases} \uparrow \qquad \begin{cases} y = 7z - 1 \\ x - 7z + 1 + 8z - 8 = 0 \end{cases}$$

$$\begin{cases} y = 7z - 1 \\ x = -z + 7 \end{cases}$$

$$(x,y,z) = \boxed{(-z+7,\ 7z-1,\ z)} = \quad \forall z \in \mathbb{R}$$

soluzione generale

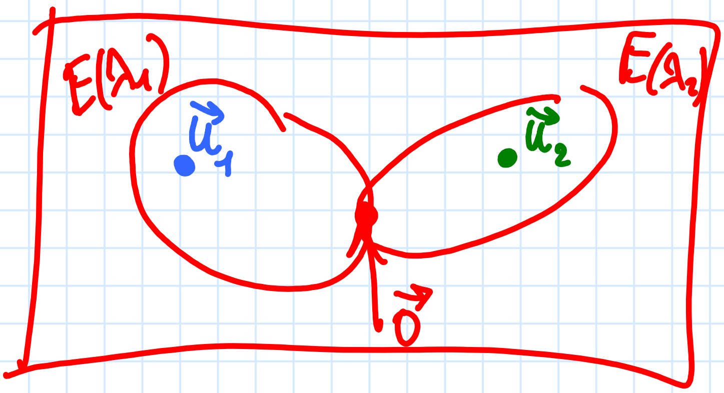
$$= \underbrace{\boxed{z(-1,7,1)}}_{\forall z \in \mathbb{R}} + \boxed{(7,-1,0)} \rightarrow X_p$$

soluzione omogeneo associato

soluzione PARTICOLARE del sistema INIZIALE

$X_p = (7,-1,0)$

$B = (-1,7,1)$ base spazio soluzioni sistema omogeneo associato.



$\mathbb{R}^3$

base $E(\lambda_1) = (\vec{u}_1)$

base $E(\lambda_2) = (\vec{u}_2)$

MA $\nexists$ base di $\mathbb{R}^3$ formata da autovettori di A

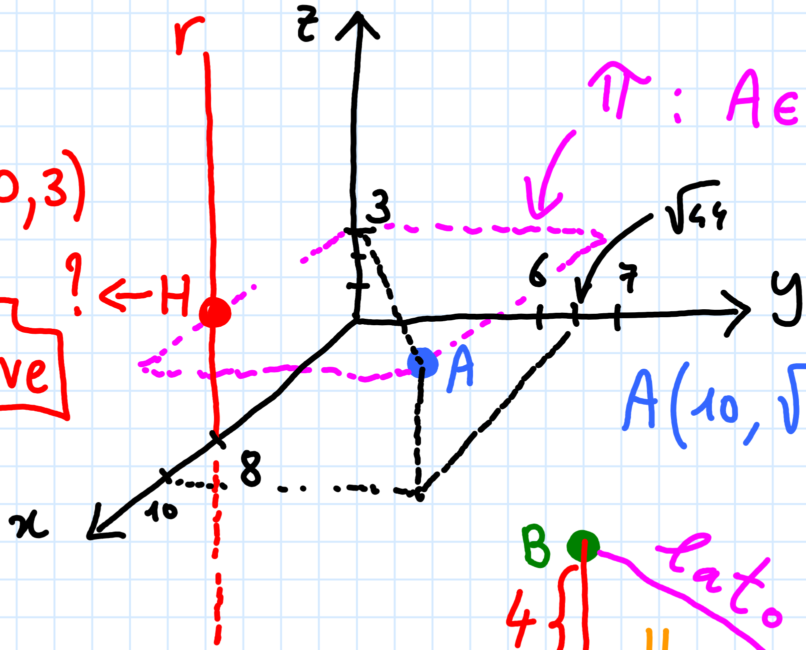
(3) $A(10, \sqrt{44}, 3)$; $r: y = x - 8 = 0$;

Trovare due punti B e C sulla retta r tali che $\hat{ABC}$ (triangolo) sia EQUILATERO.

$y_A = \sqrt{44} \Rightarrow A \notin r$

H(8,0,3)

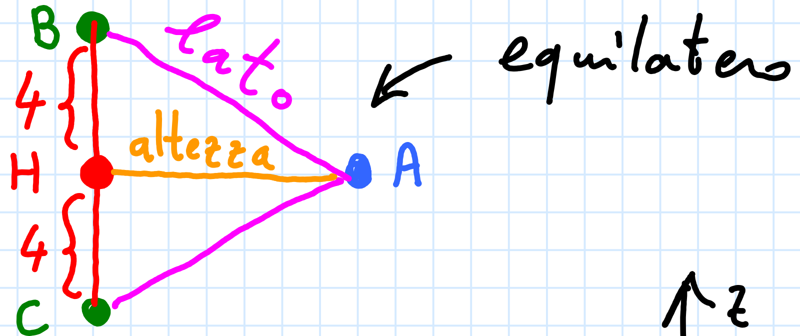
$\boxed{\text{chiave}}$



$\pi : A \in \pi$ et $\pi \perp r$

$\pi : z = 3$

$A(10, \sqrt{44}, 3)$



$A(10, \sqrt{44}, 3)$

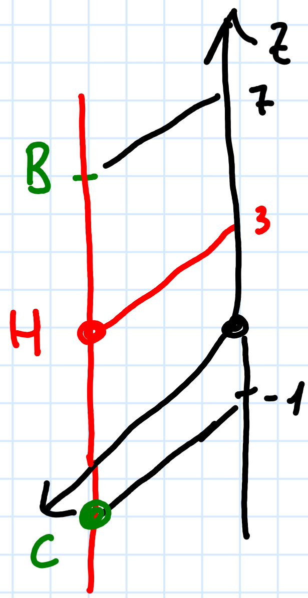
$H(8, 0, 3)$

$d(A,H) = \text{altezza} = \sqrt{48} = 4\sqrt{3}$

$\text{lato} = 8$

$B(8, 0, 7)$

$C(8, 0, -1)$



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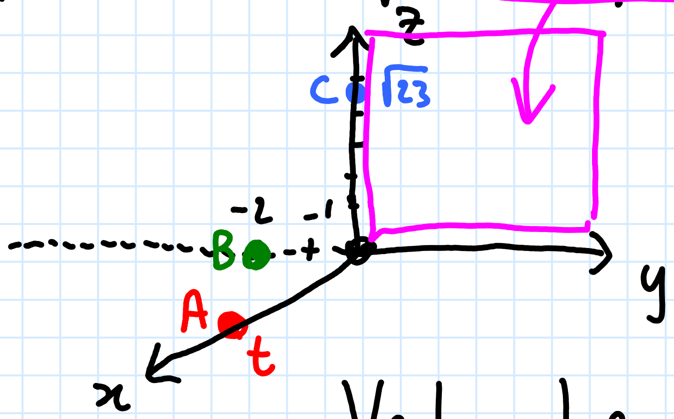
(4) $\boxed{A(t,0,0)}$; $B(0,-2,0)$; $C(0,0,\sqrt{23})$

Trovare i valori di $t \in \mathbb{R}$ per i quali il $\boxed{\text{piano } \alpha}$ passante per A, B e C forma col $\boxed{\text{piano } YZ}$ un angolo di $\frac{2}{3}\pi$ rad.



$A(t,0,0) \in$ asse X

↳ si sposta sull'asse X

$B(0,-2,0) \in$ asse Y

$C(0,0,\sqrt{23})$

Vedo che $\forall t \in \mathbb{R}$ quei tre punti A, B e C NON sono allineati e quindi $\forall t \in \mathbb{R}$ $\exists! \alpha$ (dipendente da t) che li contiene.

Piano YZ : $x = 0 \Leftarrow$ piano β

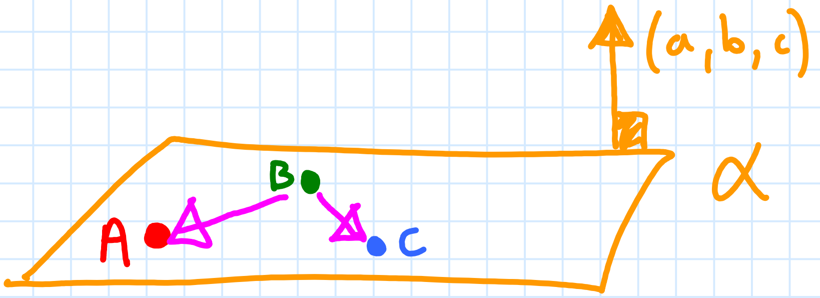
PROBLEMA: angolo fra 2 piani

$$\cos(\widehat{\alpha,\beta}) = \pm \frac{aa' + bb' + cc'}{\sqrt{a^2+b^2+c^2} \cdot \sqrt{(a')^2+(b')^2+(c')^2}}$$

piano β = piano YZ : $x=0$ $\quad \underbrace{1}_{a'} \cdot x + \underbrace{0}_{b'} \cdot y + \underbrace{0}_{c'} \cdot z + \underbrace{0}_{d'} = 0$

$\widehat{\alpha,\beta} = \frac{2}{3}\pi$ radianti $\Rightarrow \cos(\widehat{\alpha,\beta}) = ? = -\frac{1}{2}$

$(a,b,c) = (?,?,?)$ $\quad (a,b,c) \perp \alpha$ passante per A, B e C



$A(t, 0, 0)$

$B(0, -2, 0)$

$C(0, 0, \sqrt{23})$

$[\overrightarrow{BA}] = (t, 2, 0)$

$[\overrightarrow{BC}] = (0, 2, \sqrt{23})$

$[\overrightarrow{BA}] \wedge [\overrightarrow{BC}] \perp \alpha$

$$\det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ t & 2 & 0 \\ 0 & 2 & \sqrt{23} \end{bmatrix} = \underbrace{(2\sqrt{23})}_{a}\vec{i} \underbrace{-(t\sqrt{23})}_{b}\vec{j} + \underbrace{(2t)}_{c}\vec{k}$$

$a = 2\sqrt{23}$; $b = -t\sqrt{23}$; $c = 2 \cdot t$;

$a' = 1 \qquad b' = 0 \qquad c' = 0$

$\cos(\theta) = -\frac{1}{2}$

$$-\frac{1}{2} = \pm \frac{2 \cdot \sqrt{23} \cdot 1 + 0 + 0}{\sqrt{92 + 23t^2 + 4t^2} \cdot \sqrt{1^2 + 0^2 + 0^2}}$$

$$-\frac{1}{2} = \pm \frac{2\sqrt{23}}{\sqrt{92 + 27t^2}}$$

$92 + 27t^2 = 16 \cdot 23$; $27t^2 = 16 \cdot 23 - 92$

$t^2 = \frac{16 \cdot 23 - 92}{27}$; $t^2 = \frac{276}{27} = \frac{92}{9}$

$$t = \pm \frac{\sqrt{92}}{3} = \pm \frac{2\sqrt{23}}{3}$$

$$\boxed{t = \pm \frac{2\sqrt{23}}{3}}$$

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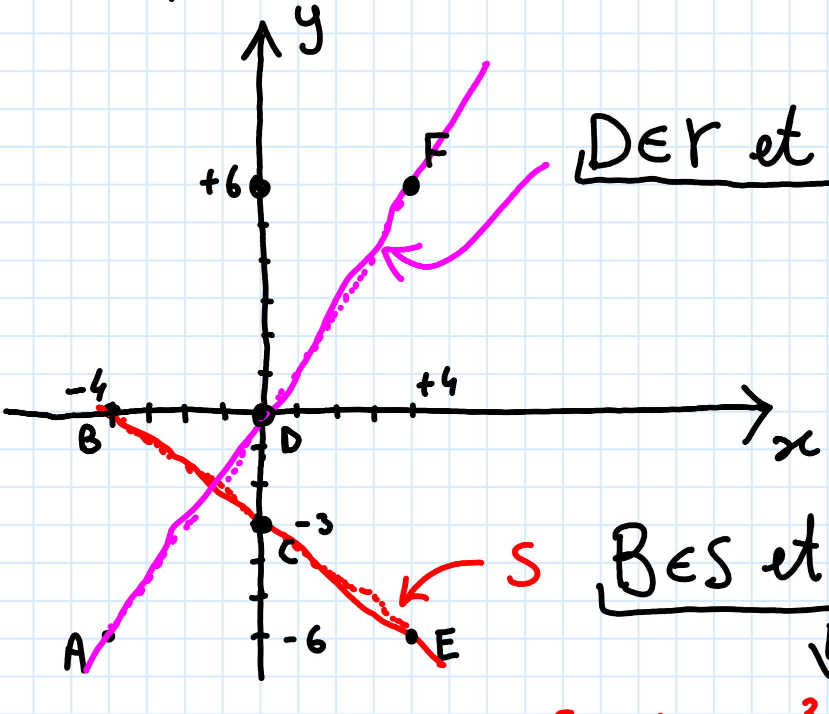
(5) $A(-4,-6)$; $B(-4,0)$; $C(0,-3)$; $D(0,0)$; $E(4,-6)$; $F(4,6)$

Se esiste, una conica passante per quei (TUTTI) punti, allora scrivere la sua equazione.
Altrimenti, spiegare brevemente il motivo.



$D \in r$ et $F \in r$

$$r: y = \frac{3}{2}x$$

$A \in r$? Sì

$B \in s$ et $C \in s$

$$s: y = -\frac{3}{4}x + (-3)$$

$E \in s$? Sì

$$r: y = \frac{3}{2}x$$

$$s: y = -\frac{3}{4} \cdot x - 3$$

$\mathscr{C}$ è DEGENERE (unione di 2 rette)

$$\mathscr{C} = r \cup s$$

$r: y - \frac{3}{2}x = 0 \qquad r: 2y - 3x = 0 \qquad \boxed{r: 3x - 2y = 0}$

$s: y = -\frac{3}{4}x - 3 \qquad s: 3x + 4y + 12 = 0 \qquad \boxed{s: 3x + 4y + 12 = 0}$

$$\boxed{\mathscr{C} = r \cup s : (3x - 2y) \cdot (3x + 4y + 12) = 0}$$

non è necessario togliere le parentesi!!!

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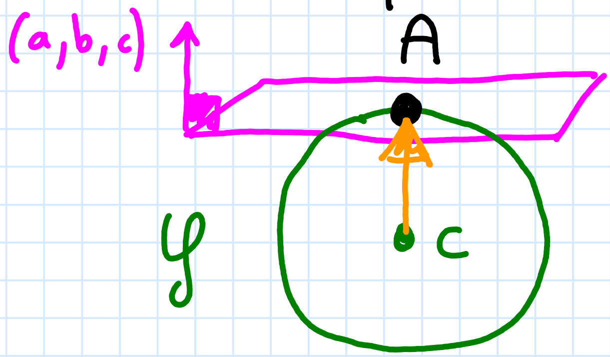
(6) Sia $\mathscr{S}$ la sfera di centro $C(-10, 3, 15)$ e passante per il punto $A(-14, 3, 8)$.
Scrivere l'equazione del piano π tangente a $\mathscr{S}$ in A,



π tangente a $\mathscr{S}$ in A

$\pi: \underset{?}{a}(x - x_A) + \underset{?}{b} \cdot (y - y_A) + \underset{?}{c} \cdot (z - z_A) = 0$

$\pi: \underset{?}{a} \cdot (x + 14) + \underset{?}{b} \cdot (y - 3) + \underset{?}{c} \cdot (z - 8) = 0$

$[\overrightarrow{CA}] \perp \pi \quad ; \quad [\overrightarrow{CA}] = (-4, 0, -7)$

$-[\overrightarrow{CA}] = [\overrightarrow{AC}] = (\underset{a}{4}, \underset{b}{0}, \underset{c}{7})$

$\pi: 4 \cdot (x + 14) + 0 \cdot (y - 3) + 7 \cdot (z - 8) = 0$

$$\boxed{\pi: 4 \cdot x + 7 \cdot z = 0}$$ piano richiesto.

11/06/2014 (5)

$$3x^2 + 2\sqrt{3}\,xy + 1y^2 - 4\sqrt{3}\,x - 4y = 0$$

$$A = \begin{bmatrix} 3 & \sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}; \quad p_A(\lambda) = \dots = \lambda^2 - 4\lambda + 0 = \lambda\cdot(\lambda-4)$$

$$\lambda_1 = 0$$
$$\lambda_2 = 4$$

$\boxed{\lambda_1=0}$ cerco autovettori $\begin{bmatrix} \cancel{3 \quad \sqrt{3}} \\ \sqrt{3} \quad 1 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$$\sqrt{3}\,x + y = 0$$
$$y = -\sqrt{3}\,x \qquad (x,y) = (x, -\sqrt{3}\,x) = x\,(1, -\sqrt{3})$$
$$\forall x \in \mathbb{R} - \{0\}$$

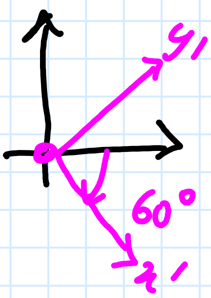
$\boxed{\lambda_1=0}$ $\quad \vec{V}_1 = (1, -\sqrt{3})$; $\quad \|\vec{V}_1\| = 2$

auto<u>VERSORE</u> $\quad \vec{u}_1 = \frac{1}{2}\vec{V}_1 = \frac{1}{2}\cdot(1, -\sqrt{3}) = \left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

$$\Lambda = \begin{bmatrix} 0 & 0 \\ 0 & 4 \end{bmatrix}; \quad C = \begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix}$$

rotazione
$\theta = \frac{\pi}{3}$ rad (orario)

$\vec{u}_1$ $\quad \vec{u}_2 = (?, ?)$



$$\vec{u}_1 \bullet \vec{u}_2 = 0 \text{ (zero)} \qquad \det C = +1$$

$$\begin{bmatrix} \underset{d}{-4\sqrt{3}} & \underset{e}{-4} \end{bmatrix}\cdot\begin{bmatrix} \frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \underset{d'}{0} & \underset{e'}{-8} \end{bmatrix}$$

Dopo la rotazione di 60° orario

$$0\cdot(x')^2+4\cdot(y')^2+0x'-8\cdot y'=0$$

$$(y')^2-2y'=0$$

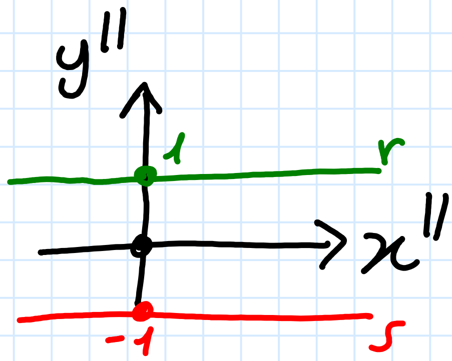
$$(y'-1)^2-1=0$$

traslazione : $\begin{cases} \cancel{x''=x'} \\ y''=y'-1 \end{cases}$ potete anche non scriverla.

Dopo la traslazione

$$(y'')^2-1=0$$

$$\underbrace{(y''-1)}_{r}\cdot\underbrace{(y''+1)}_{s}=0$$



CONICA è UNIONE di 2 rette REALI parallele in senso STRETTO

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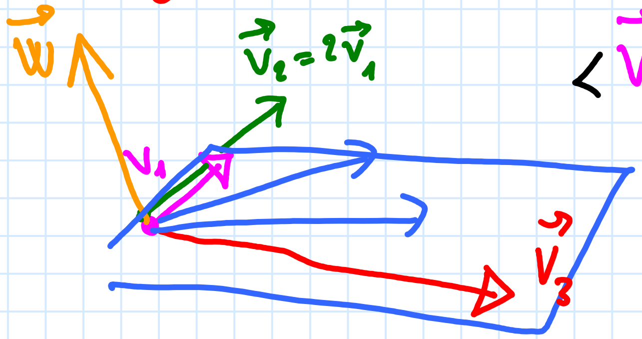
(4) $\boxed{3}$ generatori di $\mathbb{R}^3$

$\vec{v}_1=(1,0,2)$; $\vec{v}_2=(2,0,4)$; $\vec{v}_2=2\vec{v}_1$

se possibile, altrimenti motivare perché?

$\vec{v}_3 = (\quad , \quad , \quad)$



$\langle \vec{v}_1, \vec{v}_3 \rangle =$ piano $= S$

$\vec{v}_2 \in S$

$\langle \vec{v}_1, \vec{v}_2, \vec{v}_3 \rangle = S$ piano

$\vec{w}$ NON su S, non riesco a generarlo quindi la risposta è NON è possibile

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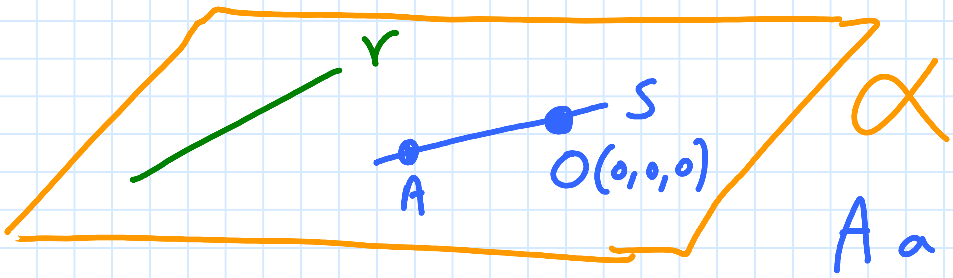
04/02/2010 (2)

piano contenente le 2 rette

$r: 2x + 5y - z - 1 = x - 1 = 0$

$s: 5y - z = 3x + 10y - 2z = 0 \leftarrow O \in S$



A a piacere su S

$\alpha \in F(r)$ et $O \in \alpha$

$\alpha: \lambda(2x + 5y - z - 1) + \mu(x - 1) = 0$

$O(0,0,0) \in \alpha \Rightarrow -\lambda - \mu = 0 \Rightarrow \lambda + \mu = 0$

scelgo $\lambda = 1$ et $\mu = -1$

$\boxed{\alpha: x + 5y - z = 0}$ STOP

poi controllate che $A \in \alpha$

28/01/2020

(1) $A = \begin{bmatrix} 0 & 0 & 0 \\ -8 & 2 & 8 \\ 0 & 0 & 0 \end{bmatrix}$

Trovare autovalori e autovettori di A

$$p_A(\lambda) = \det(A - \lambda I_3) = \det \begin{bmatrix} -\lambda & 0 & 0 \\ -8 & (2-\lambda) & 8 \\ 0 & 0 & -\lambda \end{bmatrix} =$$

$$= (-\lambda)(-\lambda)(2-\lambda) = \lambda^2 \cdot (2-\lambda) \quad \begin{matrix} \nearrow \lambda_1 = 0 \\ \searrow \lambda_2 = +2 \end{matrix}$$

$\boxed{\lambda_1 = 0}$ → autovettori? $\begin{bmatrix} 0 & 0 & 0 \\ -8 & 2 & 8 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightsquigarrow -4x + y + 4z = 0$

$\boxed{y = 4x - 4z}$ $\quad x$ e z libere $\quad \forall (x,z) \in \mathbb{R}^2$

$(x,y,z) = (x, 4x-4z, z) = x(1,4,0) + z(0,-4,1)$

Autovettori di $\lambda_1 = 0$ $\quad x(1,4,0) + z(0,-4,1)$

$\forall (x,z) \in \mathbb{R}^2 - \{(0,0)\}$

NO

$\boxed{\lambda_2 = 2}$ → autovettori?

$$\begin{bmatrix} -2 & 0 & 0 \\ -8 & 0 & 8 \\ 0 & 0 & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -2x=0 \\ -8x+8z=0 \\ -2z=0 \end{cases} \quad \begin{cases} x=0 & \text{fisso} \\ z=0 & \text{fisso} \\ 0=0 & \text{identità} \end{cases} \quad \Rightarrow y \text{ è libera}$$

$$(x,y,z)=(0,y,0)=y(0,1,0) \quad \forall y \in \mathbb{R}$$

Auto VETTORI $\quad y(0,1,0) \quad \forall y \in \mathbb{R}-\{0\}$

No

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(2) piano α: $\quad A(-57, 7, \sqrt{13}) \in \alpha$

$\alpha // r \; : \underbrace{y+z-3}=\underbrace{x-4z+19}=0$

$\alpha \perp \pi : \; 4x-y+5=0$

$\alpha : \; ax+by+cz+d=0$

$\vec{u}_\alpha=(a,b,c)\neq(0,0,0) \quad ; \quad \vec{u}_\alpha \perp \alpha$

$$\alpha // r \iff \det\begin{bmatrix} a & b & c \\ 0 & 1 & 1 \\ 1 & 0 & -4 \end{bmatrix} \begin{matrix} a & b \\ 0 & 1 \\ 1 & 0 \end{matrix} = 0$$

$-4a+b+0-c-0-0 = -4a+b-c=0$

(1) $\boxed{4a-b+c=0}$

$\alpha \perp \pi : \; 4x-y\,\cancel{+}\overset{(+0z)}{}\,5=0 \iff \vec{u}_\alpha \cdot \vec{u}_\pi = 0$ (zero)

$4\cdot a-1\cdot b+0\cdot c=0$

② $\boxed{4a-b=0}$

$$\alpha /\!/ r \ \underline{et}\ \alpha \perp \pi \iff \begin{cases} 4a-b+c=0 \\ 4a-b=0 \end{cases} \iff \begin{cases} \boxed{c=0} \text{ fisso} \\ b=4a \end{cases}$$

scelgo $a=1 \Rightarrow b=4$

$$(a,b,c)=(1,4,0)$$

$$\alpha : 1\cdot x+4\cdot y+0\cdot z+d=0$$

$$\alpha : x+4y+d=0$$

$$A(-57,7,\sqrt{13}) \in \alpha \Rightarrow -57+28+d=0$$

$$\Rightarrow d=29$$

$$\boxed{\alpha : x+4y+29=0}$$

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(3) $r: x-5y-90=3y+2z=0$

$s: 5x+z=x-2y+2z=0 \leftarrow O(0,0,0) \in S$

$$d(r,s)=?$$

($1^{\underline{o}}$ step) r et s sono parallele o no?

(ℓ_r, m_r, n_r) et (ℓ_s, m_s, n_s)

$$r \rightarrow \begin{bmatrix} 1 & -5 & 0 \\ 0 & 3 & 2 \end{bmatrix} \quad \begin{array}{l} -10 \leftarrow l \\ -(2) = -2 \leftarrow m \\ +3 \leftarrow n \end{array}$$

$$\begin{array}{ccc} \uparrow & \uparrow & \uparrow \\ l & m & n \end{array}$$

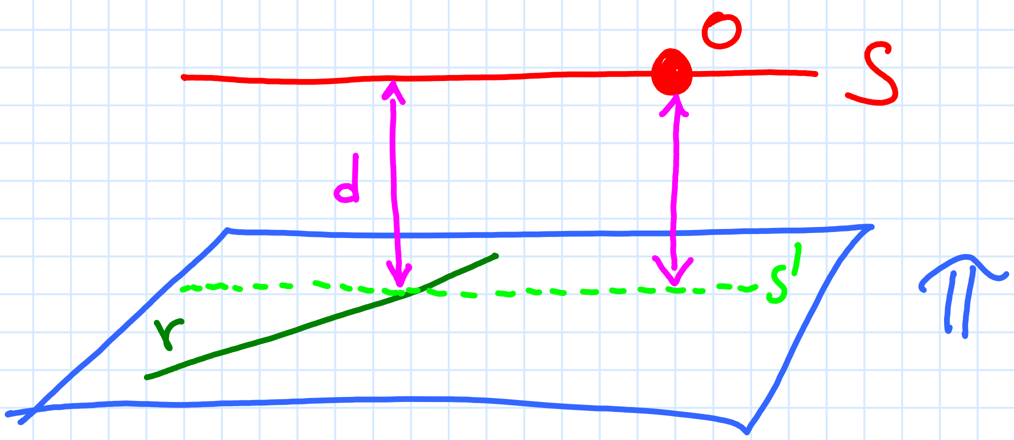
$$\vec{V}_r = (-10, -2, +3)$$

$$s \rightarrow \begin{bmatrix} 5 & 0 & 1 \\ 1 & -2 & 2 \end{bmatrix} \quad \begin{array}{l} +2 \\ -9 \\ -10 \end{array}$$

$$\vec{V}_s = (+2, -9, -10)$$

VEDO CHE $\vec{V}_r \not\parallel \vec{V}_s$

quindi r e S **NON** sono parallele



$$d(r,s) = d(O, \pi)$$

vi manca di trovare π : $\pi \in F(r)$ et $\pi // s$

$$\pi \in F(r) : \lambda(x - 5y - 90) + \mu(3y + 2z) = 0$$

$$\underbrace{\lambda}_{a} \cdot x + \underbrace{(3 \cdot \mu - 5\lambda)}_{b} \cdot y + \underbrace{(2\mu)}_{c} \cdot z \underbrace{- 90\lambda}_{d} = 0$$

$$\vec{V}_s = (l_s, m_s, n_s) = (2, -9, -10)$$

$$\pi // S \iff \vec{u}_\pi \cdot \vec{V}_s = 0 \iff al + bm + cn = 0$$

$$2 \cdot a - 9 \cdot b - 10c = 0$$

$$2 \cdot \lambda - 9(3\mu - 5\lambda) - 10 \cdot (2 \cdot \mu) = 0$$

$$47\lambda - 47\mu = 0 \qquad \lambda - \mu = 0$$

scelgo $\lambda = 1$ et $\mu = 1$

$$\pi : x - 2 \cdot y + 2 \cdot z - 90 = 0$$

$$d(r,s) = d(0,\pi) = \frac{|0 - 2\cdot 0 + 2 \cdot 0 - 90|}{\sqrt{1^2 + (-2)^2 + (2)^2}} = \frac{|-90|}{\sqrt{9}} = 30$$

$$\underbrace{d(r,s)}_{\text{sghembe}} = 30$$

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(4) r retta // asse X e passante per $A(1, 9, -3\sqrt{3})$. Trovare i $\underbrace{\text{piani}}$ α che contengono r e formano un angolo di $\frac{\pi}{6}$ RADIANTI con il $\underbrace{\text{piano}}$ β XY. $\to z = 0$

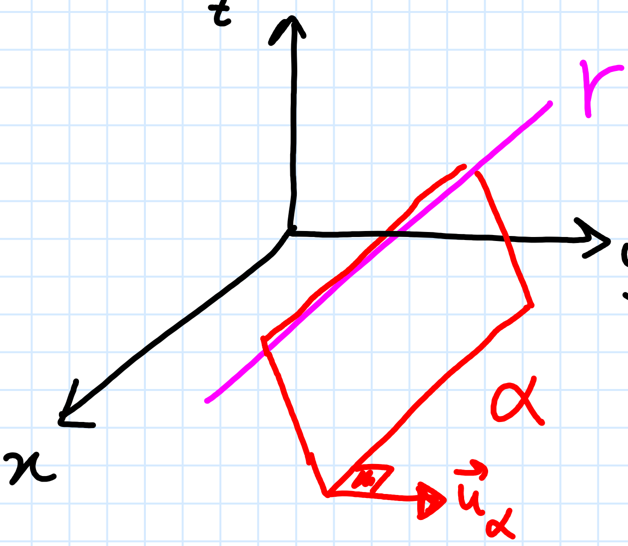
PROBLEMA: angolo tra 2 piani

$$\alpha : ax + by + cz + d = 0$$

$$\beta : a'x + b'y + c'z + d' = 0 \Rightarrow \underset{\sim}{\overset{a'}{0}} \cdot x + \underset{\sim}{\overset{b'}{0}} \cdot y + \underset{\sim}{\overset{c'}{1}} \cdot z + \underset{\sim}{\overset{d'}{0}} = 0$$

$$\boxed{\cos(\widehat{\alpha,\beta})} = \pm \frac{a\boxed{a}' + b\cdot\boxed{b}' + c\boxed{c}'}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{(\boxed{a'})^2+(\boxed{b'})^2+(\boxed{c'})^2}}$$

$\vec{u}_\alpha = (a,b,c) = (?,?,?)$; $\vec{u}_\alpha \perp \alpha$



$\vec{u}_\alpha \perp \alpha$

$\vec{u}_\alpha = (?\ ,\ ?\ ,\ ?)$

zero

$\vec{u}_\alpha = (0\ ,\ b\ ,\ c)$ dalla geometria del problema

$\vec{u}_\alpha \perp r \,//\, \text{asse}X \,//\, \vec{i} = (1,0,0)$

$$\vec{u}_\alpha \cdot \vec{i} = (a,b,c)\cdot(1,0,0) = 0$$

$$1\cdot a + 0\cdot b \cdot 0\cdot c = 0$$

$$a = 0$$

$(a',b',c') = (0,0,1)$

$$\frac{\sqrt{3}}{2} = \pm \frac{0\cdot 0 + b\cdot 0 + c\cdot 1}{\sqrt{0^2+b^2+c^2}\cdot\sqrt{0^2+0^2+1^2}}$$

$$\frac{\sqrt{3}}{2} = \pm \frac{c}{\sqrt{b^2+c^2}}$$

$3\cdot(b^2+c^2) = 4\cdot c^2$; $3b^2 - c^2 = 0$

$c^2 = 3b^2$ scelgo $\underline{b=1}$

$c^2 = 3 \Rightarrow c = \pm\sqrt{3}$

$\alpha_1 : \underbrace{0}_{a} \cdot x + \underbrace{1}_{b} \cdot y + \sqrt{3} \cdot z + d_1 = 0$

$\alpha_2 : \underbrace{0}_{a} \cdot x + \underbrace{1}_{1} \cdot y - \sqrt{3} \cdot z + d_2 = 0$

$A(1, 9, -3\sqrt{3}) \in \alpha_1 \Rightarrow 0 \cdot 1 + 1 \cdot 9 + \sqrt{3} \cdot (-3\sqrt{3}) + d_1 = 0 \Rightarrow$

$\Rightarrow d_1 = 0$

$$\boxed{\alpha_1 : \quad y + \sqrt{3} \cdot z = 0}$$

$A(1, 9, -3\sqrt{3}) \in \alpha_2 \Rightarrow 0 \cdot 1 + 1 \cdot 9 - \sqrt{3} \cdot (-3\sqrt{3}) + d_2 = 0 \Rightarrow$

$\Rightarrow 18 + d_2 = 0 \Rightarrow d_2 = -18$

$$\boxed{\alpha_2 : \quad y - \sqrt{3} \cdot z - 18 = 0}$$