17 Dicembre

Buongiorno!

Titolo nota 17/12/2020

27/01/2011 (2)

$t \in \mathbb{R}$ soluzioni del sistema lineare (x, y)

$$12x - tx + ty - t = tx - 12x - 2y + t = 0$$

$$\begin{cases} (12-t)\cdot x + t\cdot y = t \\ (t-12)\cdot x - 2y = -t \end{cases} ; \quad \begin{cases} (12-t)\cdot x + t\cdot y = t \\ (12-t)\cdot x + 2\cdot y = t \end{cases}$$

2 eq, 2 inc. (candidato CRAMER)

$$A = A(t) = \begin{bmatrix} (12-t) & t \\ (12-t) & 2 \end{bmatrix} ; \quad \det A = 2\cdot(12-t) - t(12-t) = (12-t)\cdot(2-t)$$

$\forall t \in \mathbb{R} - \{2, 12\}$ $\det A \neq 0 \Rightarrow rg\, A = 2 = MAX$

$\rightarrow$ è un sistema di CRAMER ovvero ha un'unica soluzione (una per ogni t)

$$\begin{cases} (12-t)\cdot x + t\cdot y = t \\ (12-t)\cdot x + 2\cdot y = t \end{cases}$$

$t \neq 2$ et $t \neq 12$

$$x = \frac{\det\begin{bmatrix} t & t \\ t & 2 \end{bmatrix}}{\det A} ; \quad y = \frac{\det\begin{bmatrix} (12-t) & t \\ (12-t) & t \end{bmatrix}}{\det A}$$

2 righe uguali

$\det A = (12-t)\cdot(2-t)$

$$x=\frac{2t-t^2}{(12-t)(2-t)}\;;\quad y=\frac{0}{(12-t)(2-t)}$$

$$x=\frac{t\cancel{(2-t)}}{(12-t)\cancel{(2-t)}}\;;\quad y=0 \text{ fisso } \left(\cancel{\forall t\in\mathbb{R}}\ \begin{matrix}t\neq 2\\ t\neq 12\end{matrix}\right)$$

$\forall t\in\mathbb{R}-\{2,12\}$ l'unica soluzione del sistema è la coppia ordinata

$$(x,y)=\left(\frac{t}{12-t},\,0\right)$$

$\boxed{t=2}$

$$\begin{cases}10\cdot x+2\cdot y=2\\ 10\cdot x\cancel{+2\cdot y=2}\end{cases}$$ è la stessa

$5x+y=1$; $y=1-5x$ $\quad x$ libera

$(x,y)=(x,1-5\cdot x)=x(1,-5)+(0,1)\quad\forall x\in\mathbb{R}$

ho ∞^1 soluzioni

$\boxed{t=12}$

$$\begin{cases}0\cdot x+12\cdot y=12\\ 0\cdot x+2y=12\end{cases}\qquad\begin{cases}y=1\\ y=6\end{cases}\Rightarrow 1=6 \;\text{↯}$$

NON ci sono soluzioni

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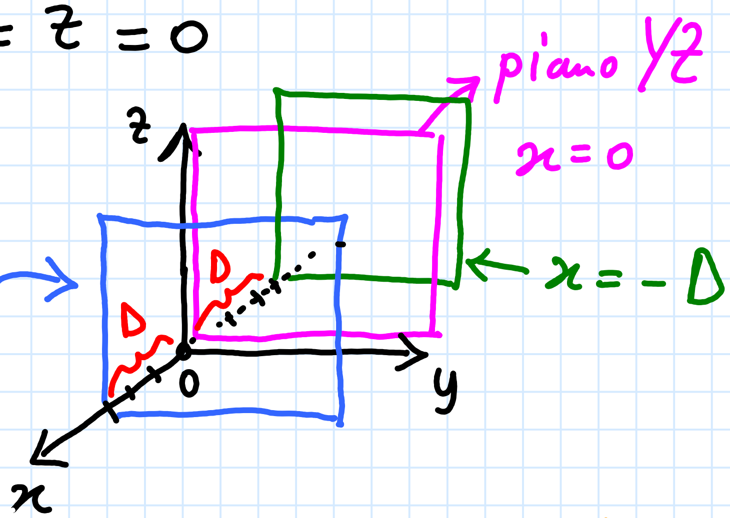
(3) piani paralleli piano YZ e aventi da esso distanza uguale a quella tra l'origine e la retta

$r: 5x-3y-34 = z = 0$

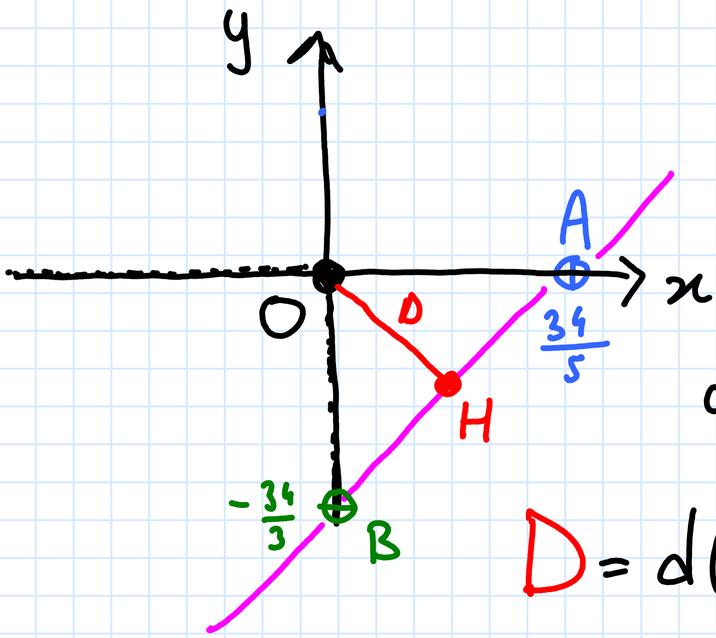
$D = d(O,r)$



$r: 5x-3y-34 = \boxed{z=0}$ → piano XY

$A = r \cap$ asse X

$B = r \cap$ asse Y



$$d(O,r) = \frac{|5\cdot 0 - 3\cdot 0 - 34|}{\sqrt{5^2+(-3)^2}} = ?$$

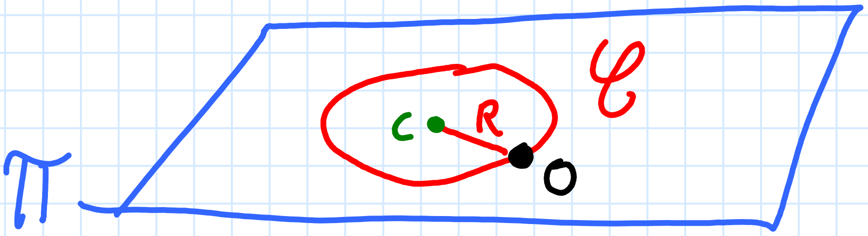
$D = d(O,r) = \sqrt{34}$

$x = \sqrt{34}$ $\qquad$ $x = -\sqrt{34}$

14/01/2009 (9)

circonferenza che giace sul piano $\pi: 3x+y-2z=0$ e ha il centro nel punto $C(0,2,1)$ $\quad R=\sqrt{5}$

e passa per l'origine $O(0,0,0)$



$\mathscr{C} = \pi \cap \mathscr{S}$

??

chiave $\mathscr{S}$ di centro C e passante per O

$$\mathscr{S}: x^2+y^2+z^2+ax+by+cz+d=0$$

$$O(0,0,0) \in \mathscr{S} \Longrightarrow d=0$$

$$C(\underset{x_c}{0},\underset{y_c}{2},\underset{z_c}{1}) \text{ centro } \mathscr{S} \Longrightarrow \begin{cases} a \overset{DEF.}{=} -2x_c = 0 \\ b \overset{DEF}{=} -2y_c = -4 \\ c \overset{DEF}{=} -2z_c = -2 \end{cases}$$

$$\mathscr{S}: x^2+y^2+z^2-0\cdot x-4\cdot y-2\cdot z+0=0$$

$$\mathscr{C}: \underbrace{3x+y-2z}_{\pi} = \underbrace{x^2+y^2+z^2-4y-2z}_{\mathscr{S}} = 0$$

$C(0,2,1)$ $\qquad R=\sqrt{5}$

$$\mathscr{S}: (x-0)^2+(y-2)^2+(z-1)^2=(\sqrt{5})^2$$

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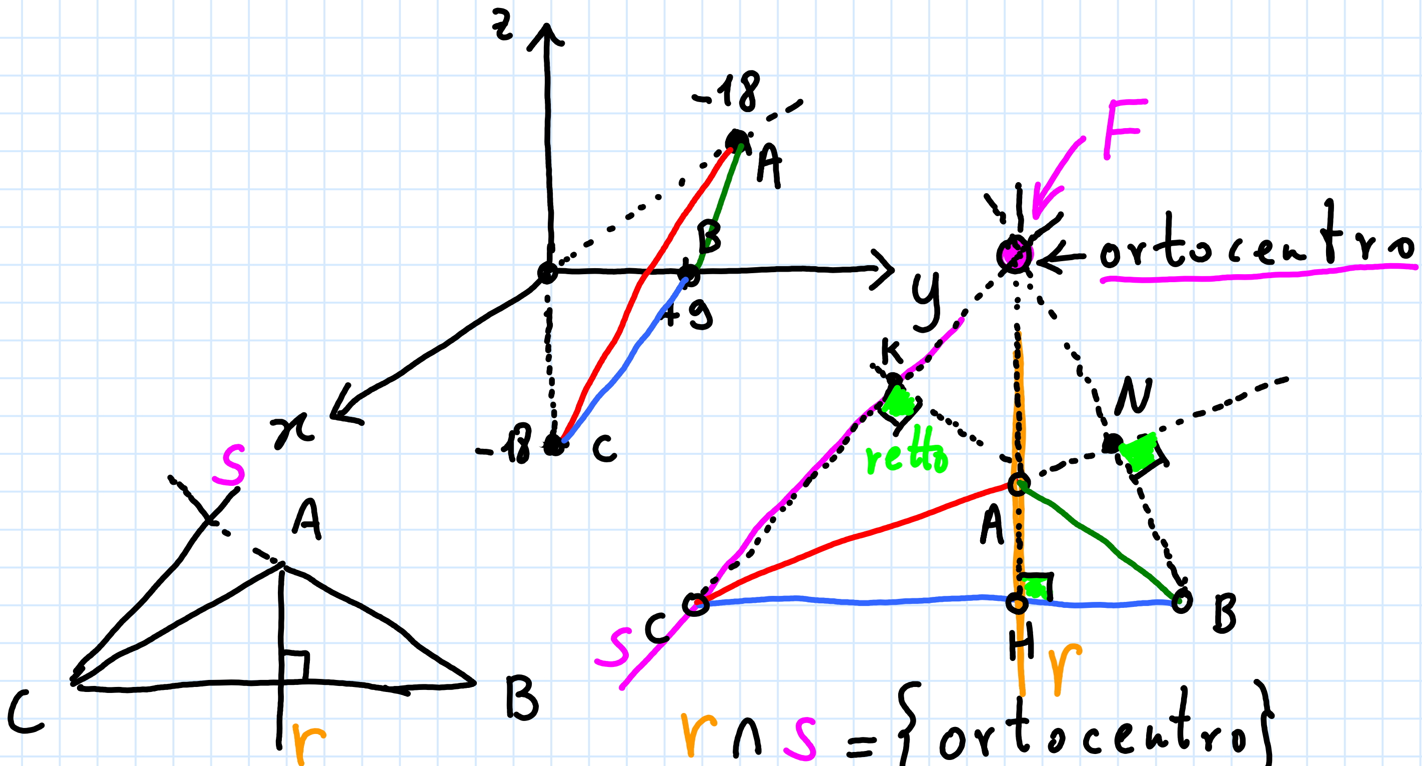
3) Siano A, B e C i punti di intersezione del piano di equazione $\underset{\pi}{\boxed{x-2y+z+18=0}}$ con gli assi coordinati X, Y e Z rispettivamente. Determinare l'**ortocentro** (punto d'incontro delle altezze) del triangolo ABC.



$r \cap s = \{\text{ortocentro}\}$

due rette nello SPAZIO

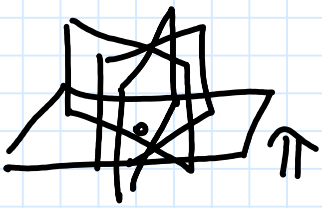
ognuna delle due si rappresenta come intersezione di due PIANI

π è il piano passante per A, B e C

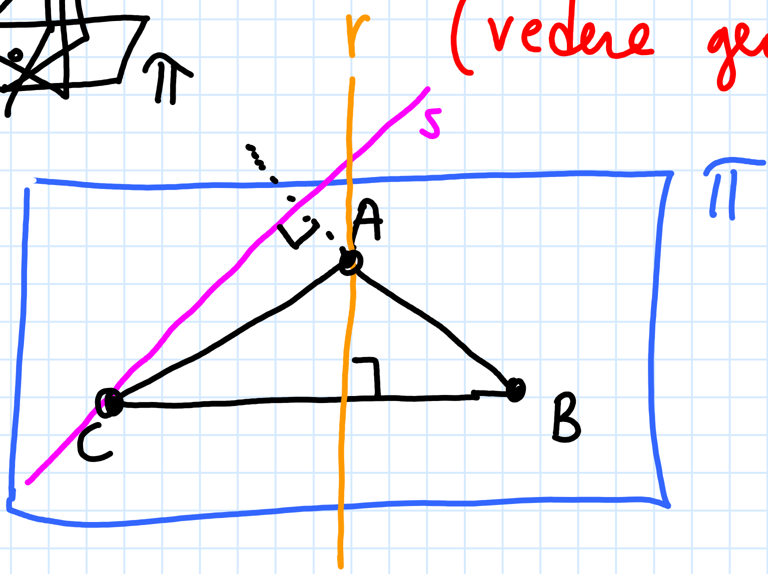
$r = \pi \cap \alpha$ $\quad$ α da trovare

$s = \pi \cap \beta$ $\quad$ β da trovare

chiave del problema



(vedere geometricamente α e β)



super chiave

$\alpha : A \in \alpha$ et $\alpha \perp [\overrightarrow{CB}]$

$\beta : C \in \beta$ et $\beta \perp [\overrightarrow{AB}]$

$$\left.\begin{array}{l} C(0,0,-18) \\ B(0,9,0) \\ A(-18,0,0) \end{array}\right| \Rightarrow [\overrightarrow{CB}] = (0,9,18) = 9\cdot(0,1,2)$$

$$\alpha : 0\cdot(x-(-18)) + 1\cdot(y-0) + 2\cdot(z-0) = 0$$

$$\boxed{\alpha : y + 2z = 0}$$

$$[\overrightarrow{AB}] = (18,9,0) = 9\cdot(2,1,0)$$

$$\beta : 2\cdot(x-0) + 1\cdot(y-0) + 0\cdot(z-(-18)) = 0$$

$$\boxed{\beta : 2x + y = 0}$$

$$\pi : x - 2y + z + 18 = 0$$

$$r : y + 2z = x - 2y + z + 18 = 0$$

$$s : 2x + y = x - 2y + z + 18 = 0$$

$$\{\text{ortocentro}\} = r \cap s : \begin{cases} y + 2z = 0 \\ 2x + y = 0 \\ x - 2y + z + 18 = 0 \end{cases} \rightsquigarrow$$

$\rightsquigarrow$ fate i conti ortocentro $(-3, 6, -3)$

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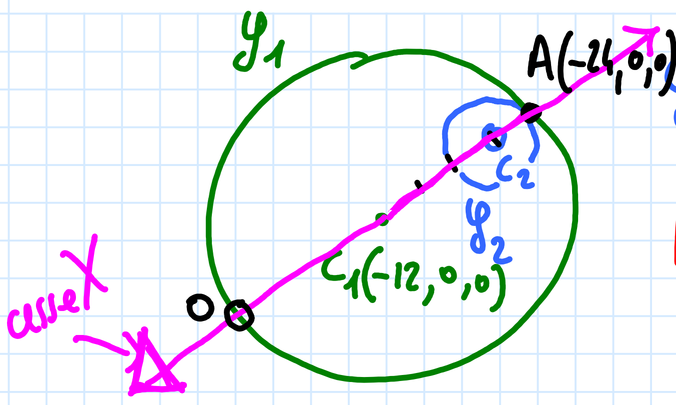
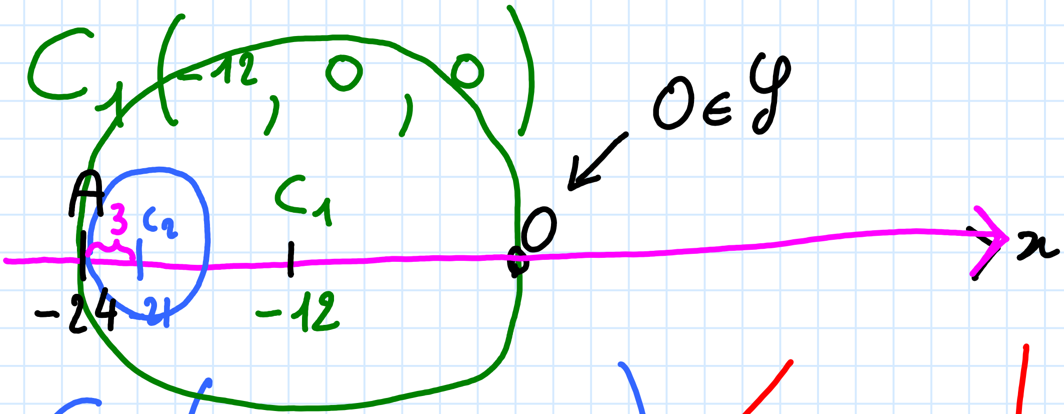
6) Si consideri la sfera di equazione $S_1 : x^2 + y^2 + z^2 + 24x = 0$. Scrivere l'equazione della sfera S_2 tangente internamente a S_1 nel punto $A(-24, 0, 0)$ e avente il raggio uguale a 1/4 del raggio di S_1.



$\mathcal{S}_2$ tang INT a $\mathcal{S}_1$

$$R_2 = \frac{1}{4} R_1$$



$$C_2(\underbrace{-21}_{x_{C_2}}, \underbrace{0}_{y_{C_2}}, \underbrace{0}_{z_{C_2}}) \longleftarrow \text{chiave}$$

$A \in \mathcal{S}_2$

$a \overset{DEF}{=} -2x_c$

$b \overset{DEF}{=} -2y_c$

$c \overset{DEF}{=} -2z_c$

$$\mathcal{S}_2 : x^2+y^2+z^2+\underset{\downarrow \atop +42}{a}x+\underset{\downarrow \atop 0}{b}y+\underset{\downarrow \atop 0}{c}z+d=0$$

$$x^2+y^2+z^2+42x+d=0$$

$$A(-24,0,0) \in \mathcal{S}_2 \Rightarrow (-24)^2+0^2+0^2+42(-24)+d=0$$

$$\Rightarrow \text{fate voi} \Rightarrow d=\underbrace{432}_{\text{mi fido}}$$

$$\mathcal{S}_2 : x^2+y^2+z^2+42 \cdot x+432=0$$

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3) Sia A il punto di intersezione tra il piano α $3x + 17y + z - 9 = 0$ e l'asse X. Sia r la retta per l'origine O e perpendicolare al piano β $5y + 2z + 11 = 0$. Sia h la distanza di A dalla retta r. Siano B e C i punti di r aventi distanza 2h da A. Calcolare l'**area** del triangolo ABC.

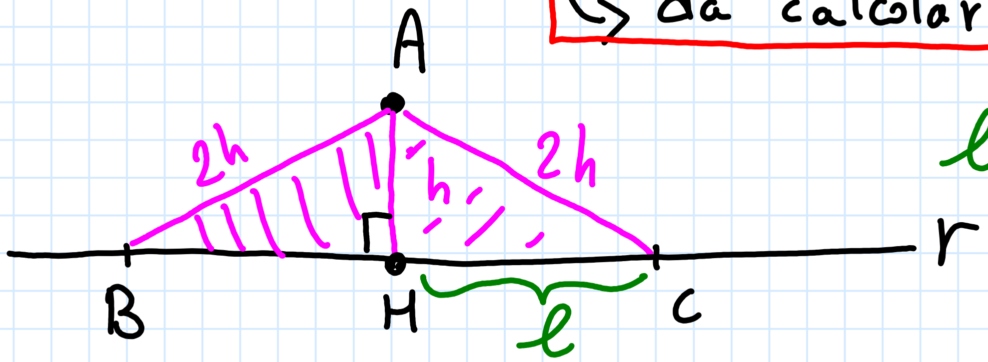
asse $X : y=z=0 \Rightarrow A(3,0,0)$

$\beta : 0 \cdot x+5y+2z+11=0$ $\qquad O(0,0,0) \in r$ et $r \perp \beta$

$$r : \begin{cases} x = 0t+0 \\ y = 5t+0 \\ z = 2t+0 \end{cases} \qquad r. \begin{cases} x = \boxed{0} \\ y = \boxed{5}t \\ z = \boxed{2}t \end{cases} \qquad \begin{array}{l} t = \frac{1}{2}z \\ y = \frac{5}{2}z \end{array}$$

$$r: \begin{cases} x=0 \\ 2y-5z=0 \end{cases}$$

$$\boxed{h \overset{DEF}{=\!=} d(r,A) \;\hookrightarrow \text{da calcolare}}$$



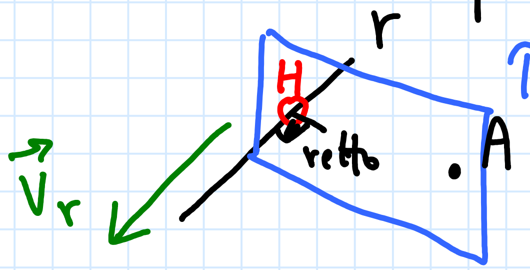
$\ell = ? = \sqrt{3}\,h$

$$\text{Area } \overset{\triangle}{ABC} = ? = 2 \cdot \text{area } \overset{\triangle}{AHC} = \ell \cdot h = \sqrt{3} \cdot h^2$$

$d(A,r) = ?$ $\quad A(3,0,0)$

$r: \; 2y - 5z = x = 0$

$\hookrightarrow$ distanza punto-retta ??



$\pi: A \in \pi$ et $\pi \perp r$

$\vec{u}_\pi \perp \pi$

prendo come $\vec{u}_\pi = \vec{v}_r$

$r: \; 2y - 5z = x = 0 \implies \vec{v}_r = ? = (0,5,2)$ lo avevamo

$$\pi: \; 0 \cdot (x-3) + 5(y-0) + 2(z-0) = 0$$

$$\pi: \; 5 \cdot y + 2z = 0$$

$$\pi \wedge r : \begin{cases} 5y+2z=0 \\ x=0 \\ 2y-5z=0 \end{cases} \Longrightarrow H(0,0,0)$$

$$d(A,r) = d(\underset{(3,0,0)}{A}, \underset{(0,0,0)}{H}) = 3 = h$$

$$\text{area } \overset{\triangle}{ABC} = 9\sqrt{3}$$

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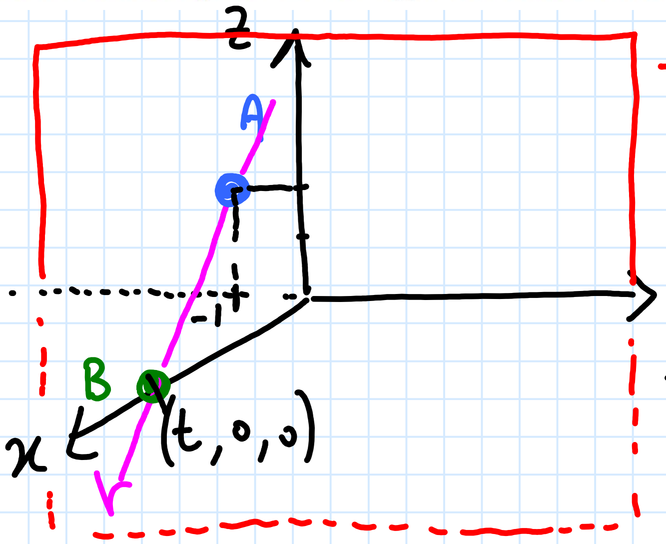
4) Siano A(0,–1, 2) e B(t, 0, 0). Trovare i valori del parametro reale t per i quali la retta r passante per A e B forma col piano YZ un angolo di π/3 radianti.



→ piano YZ : $x=0$ (π)

$$\underbrace{1}_{a}\cdot x + \underbrace{0}_{b}\cdot y + \underbrace{0}_{c}\cdot z + 0 = 0$$

PROBLEMA: angolo retta-piano

$$\underbrace{\sin(\pi, r)}_{\sin(60°)} = \frac{|al + bm + cn|}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{l^2+m^2+n^2}}$$

$$[\overrightarrow{AB}] = (\underbrace{t}_{l}, \underbrace{1}_{m}, \underbrace{-2}_{n})$$

$A(0,-1,2)$

$B(t,0,0)$

$a = 1$

$b = 0$

$c = 0$

$$\frac{\sqrt{3}}{2} = \frac{|1\cdot t + 0\cdot 1 + 0(-2)|}{\sqrt{t^2+1^2+(-2)^2}\cdot\sqrt{1^2+0^2+0^2}}$$

$$\frac{\sqrt{3}}{2} = \frac{|t|}{\sqrt{t^2+5}} \quad ; \quad 3\cdot(t^2+5) = 4\cdot t^2$$

$$\boxed{t^2 = 15} \qquad \boxed{t = \pm\sqrt{15}}$$

$$B_1(\sqrt{15}, 0, 0) \quad ; \quad B_2 = (-\sqrt{15}, 0, 0)$$

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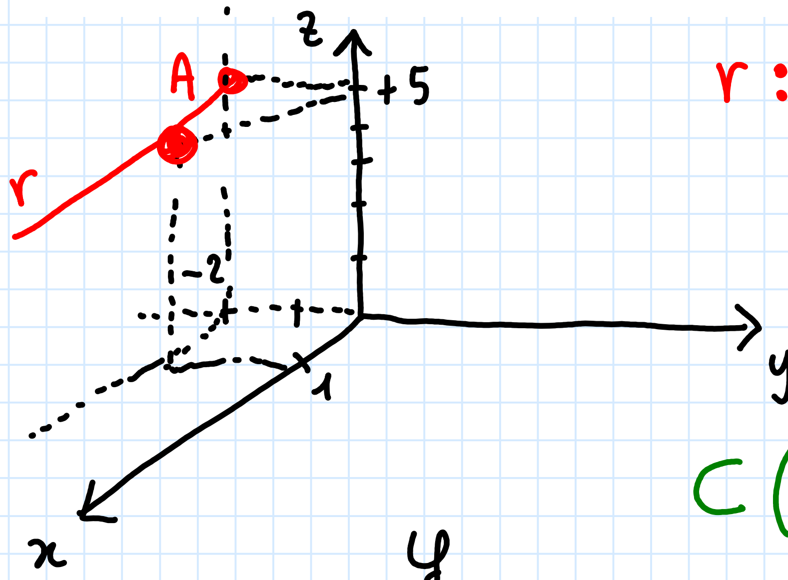
6) Scrivere le equazioni delle **sfere** di raggio 2, aventi il centro $\underbrace{\text{sulla retta}}_{r}$ passante per A(1,–2, 5) e parallela all'asse X e tangenti al piano π : x – 2y + 2z – 5 = 0.



$$r:\ y+2 = z-5 = 0$$

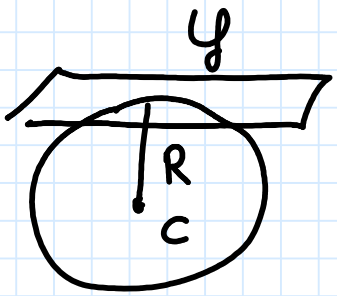
$$\downarrow \qquad\qquad \downarrow$$

$$y=-2 \qquad z=5$$

fisso   fisso

x variabile

$$C(x, -2, 5)$$



π tangente

$$d(C, \pi) = R$$

$$C(x_c, -2, 5) \ ; \ \pi: x - 2y + 2z - 5 = 0 \ ; \ R = 2$$

$$\frac{|x_c - 2(-2) + 2\cdot(5) - 5|}{\sqrt{1^2 + (-2)^2 + 2^2}} = 2$$

$$\frac{|x_c+9|}{3}=2$$

$$|x_c+9|=6$$

$$x_c+9=\pm 6 \qquad x_c=-9\pm 6 \begin{cases} x_{c_1}=-15 \\ x_{c_2}=-3 \end{cases}$$

$C_1(-15,-2,5)$ $\quad R_1=2$

$C_2(-3,-2,5)$ $\quad R_2=2$

$$\boxed{\begin{aligned} \varphi_1 &: (x+15)^2+(y+2)^2+(z-5)^2=2^2 \\ \varphi_2 &: (x+3)^2+(y+2)^2+(z-5)^2=2^2 \end{aligned}}$$