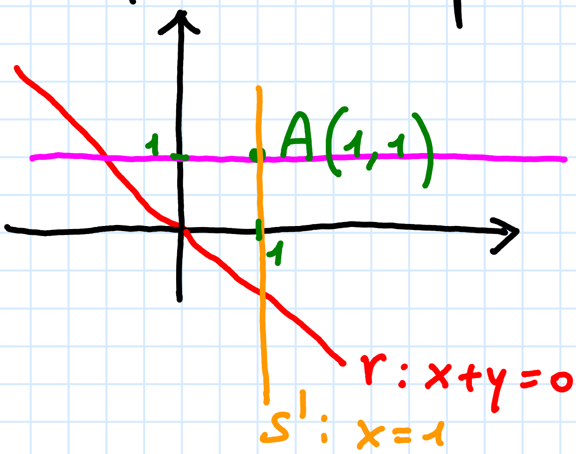
Lunedì 11 Gennaio ore 10:00

Titolo nota 11/01/2021

2 coniche distinte ognuna delle quali contenga tutti i punti della retta $r: x+y=0$ e passi per $A(1,1)$



$s: y-1=0$

$s': x-1=0$

$r \subseteq \mathcal{C} \Rightarrow \mathcal{C} = r \cup s$

$A \in \mathcal{C} \Rightarrow A \in s \Rightarrow s$ è una retta passante per A

$\mathcal{C} = r \cup s \Rightarrow \mathcal{C}: (x+y)\cdot(y-1)=0$

$\mathcal{C}' = r \cup s' \Rightarrow \mathcal{C}': (x+y)\cdot(x-1)=0$

$S = \{(x,y,z) \in \mathbb{R}^3 \mid x+6y-10z=0\}$ $\quad \dim S = 2$

$B = \{\underbrace{(t^3-6t^2,\, t^2-t,\, t)}_{u},\, \underbrace{(8,-3,-1)}_{v}\}$

valori di t per i quali B è base di S

$8+6(-3)-10(-1)=0$ OK $v \in S$

$(t^3-6t^2)+6(t^2-t)-10(t)=0$

$\underset{x}{\uparrow} \quad + 6 \underset{y}{\uparrow} \quad - 10 \underset{z}{\uparrow}$

$$t^3 - \cancel{6t^2} + \cancel{6t^2} - 6t - 10t = 0$$

$$t^3 - 16t = 0$$

$t_1 = 0$ $\quad$ $t_2 = -4$ $\quad$ $t_3 = +4$

$$t \cdot (t^2 - 16) = 0 \Rightarrow t \cdot (t+4) \cdot (t-4) = 0$$

$$u \in S \iff t \in \{-4, 0, +4\}$$

appartiene

$t_1 = 0 \Rightarrow u = (0,0,0)$ lin. DIP. $\Rightarrow$ B NON è BASE

$\hookrightarrow$ scartarlo

$\boxed{t_2 = -4} \Rightarrow u = (-160, +20, -4) \neq \vec{0}$

OK $\qquad v = (8, -3, -1) \neq \vec{0}$

ad occhio vedo che u e v non sono proporzionali, quindi SONO indip

$B = (u, v)$ base di S

$t_3 = +4 \Rightarrow u = (-32, 12, 4) \neq \vec{0}$

$\hookrightarrow$ scartare

$v = (8, -3, -1) \neq \vec{0}$

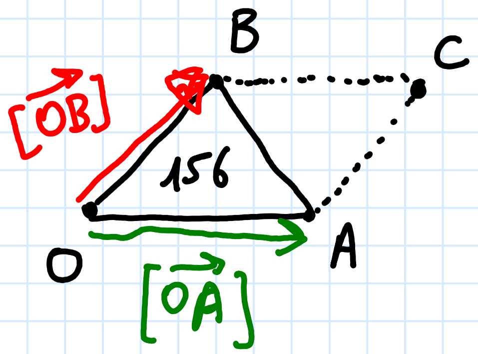
Vedo che $u = (-4) \cdot v \Rightarrow$ u e v DIP $\Rightarrow$

$\Rightarrow B$ NON è base di S

$\exists!$ valore di $t = -4$ per il quale B è base di S

$t \in \mathbb{R}$ $\quad O(0,0,0)$; $A(t,-t,t)$;

$B(0,-12,12)$ $\quad$ area $A\hat{B}O = 156$



$[\vec{OA}] = (t, -t, t)$

$[\vec{OB}] = (0, -12, 12)$

area $OBCA = \| [\vec{OA}] \wedge [\vec{OB}] \|$

area $OBA = \frac{1}{2} \cdot \| [\vec{OA}] \wedge [\vec{OB}] \|$

$$[\vec{OA}] \wedge [\vec{OB}] = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ t & -t & t \\ 0 & -12 & 12 \end{bmatrix} = 0 \cdot \vec{i} + (-12t)\vec{j} + (-12t) \cdot \vec{k}$$

$$\| [\vec{OA}] \wedge [\vec{OB}] \| = \sqrt{0^2 + (-12t)^2 + (-12t)^2}$$

$$\frac{1}{2} \cdot \sqrt{288 \cdot t^2} = 156$$

$$6 \cdot \sqrt{2 \cdot t^2} = 156$$

$$\sqrt{2\cdot t^2} = 26$$

$$4t^2 = (26)^2 = (2\cdot 13)^2 = 4\cdot 13^2$$

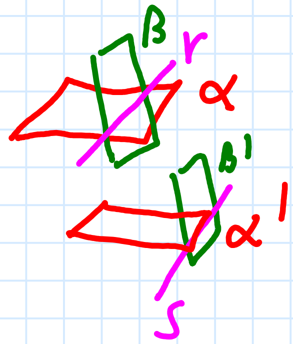
$$t^2 = 13^2 \qquad \boxed{t = \pm 13}$$

$$\boxed{r:\ \underbrace{x-3z}_{\alpha} = \underbrace{y}_{\beta} = 0}$$

$O(0,0,0)\in r$

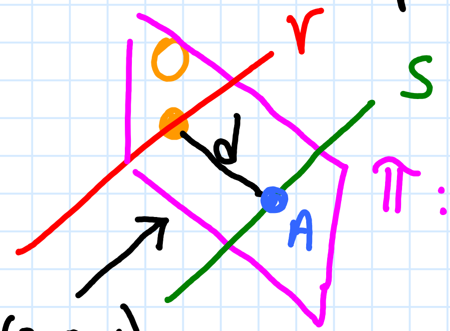
$$s:\ \underbrace{x-3z+10}_{\alpha'} = \underbrace{y-3}_{\beta'} = 0$$

$$\left.\begin{array}{l}\alpha \parallel \alpha' \\ \beta \parallel \beta'\end{array}\right\} \Rightarrow r \parallel s$$



$$r: \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \end{bmatrix} \to \begin{array}{l} l = +3 \\ m = 0 \\ n = +1 \end{array}$$

stessa cosa per s



$\vec{v} = (3,0,1)$

$\Pi: O \in \Pi$ et $\Pi \perp r$

$$\Pi: ax + by + cz + d = 0$$

$$a \to 3,\quad b \to 0,\quad c \to 1,\quad d \to 0$$

$$\Pi: 3x + z = 0$$

$$\{A\} = \Pi \cap s: \begin{cases} 3x + z = 0 \\ x - 3z + 10 = 0 \\ y - 3 = 0 \end{cases} \Rightarrow \begin{cases} y = 3 \\ z = -3x \\ x + 1 = 0 \end{cases}$$

$$\begin{cases} x = -1 \\ y = 3 \\ z = +3 \end{cases} \qquad A(-1, 3, 3)$$

$$d(r,s) = d(O,A) = \sqrt{(-1)^2 + 3^2 + 3^2} = \sqrt{19}$$

$$\boxed{d(r,s) = \sqrt{19}}$$

$\mathbb{R}^3$ se possibile

3 generatori tali che uno di essi sia il vettore $u = (1,0,2)$

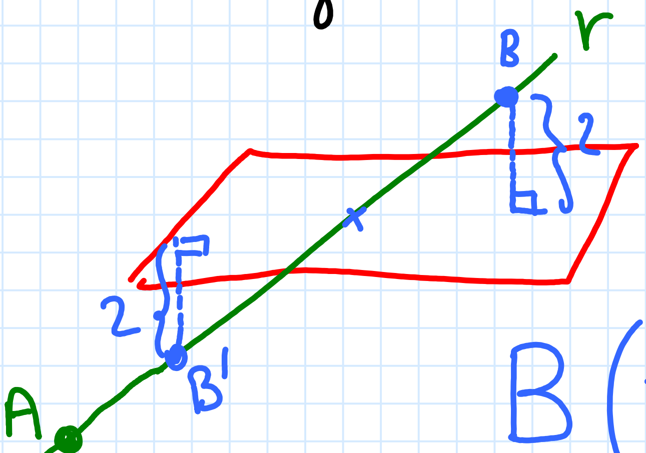
$v = (0,5,7)$

$w = (0,0,-11)$

$A(\underset{x_A}{1}, \underset{y_A}{-2}, \underset{z_A}{5})$ r passante per A

$r \,//\,$ asse $X \,//\, \vec{i} = (\underset{\ell}{1}, \underset{m}{0}, \underset{n}{0})$

2 punti su r tali che la loro distanza dal piano $\pi: x - 2y + 2z - 5 = 0$ sia uguale a 2



$\pi: x - 2y + 2z - 5 = 0$

$B, B' \in r: \begin{cases} x = 1t + 1 \\ y = 0 \cdot t + (-2) \\ z = 0 \cdot t + 5 \end{cases}$

$B(t+1, -2, 5)$

$$d(B, \pi) = 2$$

$$\frac{|(t+1)-2(-2)+2(5)-5|}{\sqrt{1^2+(-2)^2+2^2}} = 2$$

$$\frac{|t+\cancel{1}+\cancel{4}+10-\cancel{5}|}{3} = 2$$

$$|t+10| = 6 \;; \quad t+10 = \pm 6 \;; \quad t = -10 \pm 6$$

$$t_1 = -4 \;; \quad t_2 = -16$$

$$B(t+1, -2, 5)$$

$$t_1 = -4 \Rightarrow B_1(-3, -2, 5)$$

$$t_2 = -16 \Rightarrow B_2(-15, -2, 5)$$

i due punti richiesti.

Determinare eq. CARTESIANE di $\boxed{\text{piani}}$ passanti per retta r

$$r: 2x - 3z + 2 = y = 0$$

e formanti un angolo di $\frac{\pi}{6}$ rad col $\boxed{\text{piano}}$ $\alpha : x - z = 0$

Problema: angolo tra 2 piani

$$\cos \underset{\frac{\pi}{6}\,\text{rad}}{\underset{\downarrow}{\theta}} = \pm \frac{a \cdot a' + b \cdot b' + c \cdot c'}{\sqrt{a^2+b^2+c^2} \cdot \sqrt{(a')^2+(b')^2+(c')^2}}$$

$$\alpha : x - z = 0 \Rightarrow \alpha : \underbrace{1}_{a} \cdot x + \underbrace{0}_{b} \cdot y + \underbrace{(-1)}_{c} \cdot z = 0$$

$$\mathcal{F}(r) : \lambda \cdot (2x - 3z + 2) + \mu \cdot (y) = 0$$

$$\underbrace{(2\lambda)}_{a'} \cdot x + \underbrace{(\mu)}_{b'} \cdot y + \underbrace{(-3\lambda)}_{c'} \cdot z + \underbrace{2 \cdot \lambda}_{d'} = 0$$

$$\frac{\sqrt{3}}{2} = \pm \frac{1 \cdot (2\lambda) + 0 \cdot \mu + (-1) \cdot (-3\lambda)}{\sqrt{1^2+0^2+(-1)^2} \cdot \sqrt{(2\lambda)^2+\mu^2+(-3\lambda)^2}}$$

$$\frac{\sqrt{3}}{2} = \pm \frac{5\lambda}{\sqrt{2} \cdot \sqrt{13\lambda^2+\mu^2}}$$

$$\sqrt{6} \cdot \sqrt{13\lambda^2+\mu^2} = \pm 10\lambda$$

$$6 \cdot (13\lambda^2+\mu^2) = 100\lambda^2$$

$$78\lambda^2 + 6\mu^2 = 100\lambda^2$$

$$6\mu^2 = 22\lambda^2$$

$$3\mu^2 = 11\lambda^2$$

se fosse $\lambda = 0$, si avrebbe $\mu = 0$ ASSURDO

quindi $\lambda \neq 0$ scelgo io un valore A PIACERE per λ

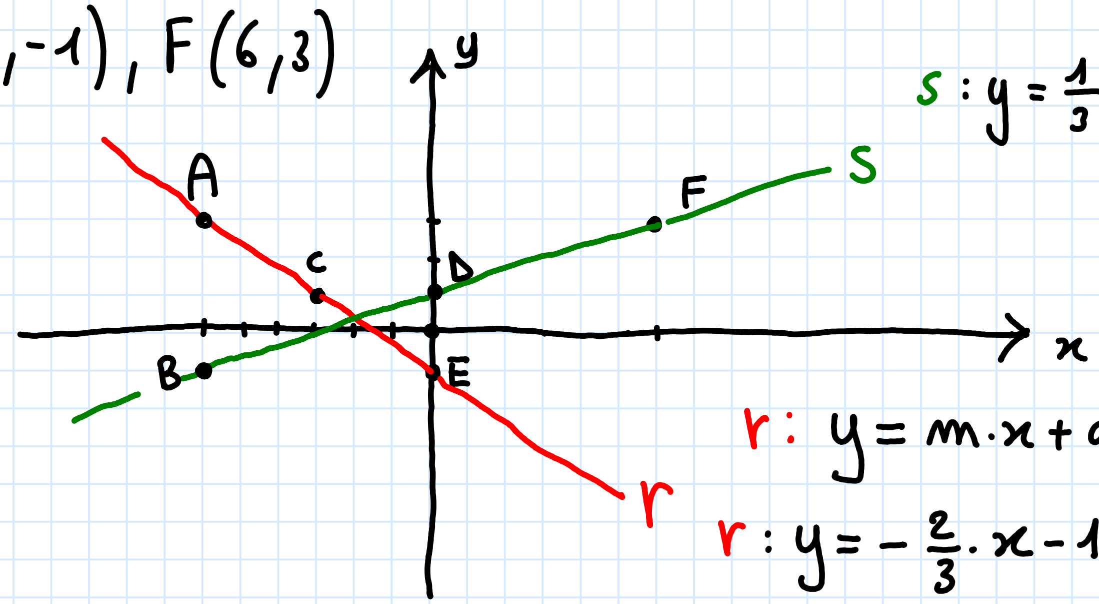
$$\boxed{\lambda = 3}$$

$$3 \cdot \mu^2 = 11 \cdot 9 = 99 \Rightarrow \mu^2 = 33 \Rightarrow \mu = \pm\sqrt{33}$$

1° piano $\lambda = 3$ et $\mu = \sqrt{33}$

2° piano $\lambda = 3$ et $\mu = -\sqrt{33}$

Conica $A(-6,3)$, $B(-6,-1)$, $C(-3,1)$, $D(0,1)$, $E(0,-1)$, $F(6,3)$



$s: y = \frac{1}{3}x + 1$

$r: y = m \cdot x + q$

$r: y = -\frac{2}{3} \cdot x - 1$

$r: \; y = -\frac{2}{3}x - 1 \qquad s: \; y = \frac{1}{3}x + 1$

$r: \; 2x + 3y + 3 = 0 \quad ; \quad s: \; x - 3y + 3 = 0$

$$\mathscr{C} = r \cup s \Rightarrow \boxed{(2x+3y+3)\cdot(x-3y+3) = 0}$$

$$A = \begin{bmatrix} -1 & 0 & h \\ -3 & 2 & -2 \\ 0 & k & 1 \end{bmatrix}$$

$h \in \mathbb{R}$, $k \in \mathbb{R}$ tali che $v = (2, 3, -3)$ è un autovettore di A

Scrivere anche il suo autovalore λ

$\boxed{\text{DEF.}}$ $A \cdot v = \lambda \cdot v$

$$\begin{bmatrix} -1 & 0 & h \\ -3 & 2 & -2 \\ 0 & k & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix} = \lambda \cdot \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} -2-3h \\ 6 \\ 3k-3 \end{bmatrix} = \begin{bmatrix} 2\lambda \\ 3\lambda \\ -3\lambda \end{bmatrix} \Leftrightarrow \begin{cases} -2-3h = 2\lambda \\ \boxed{6 = 3\lambda} \Rightarrow \boxed{\lambda = 2} \\ 3k-3 = -3\lambda \end{cases}$$

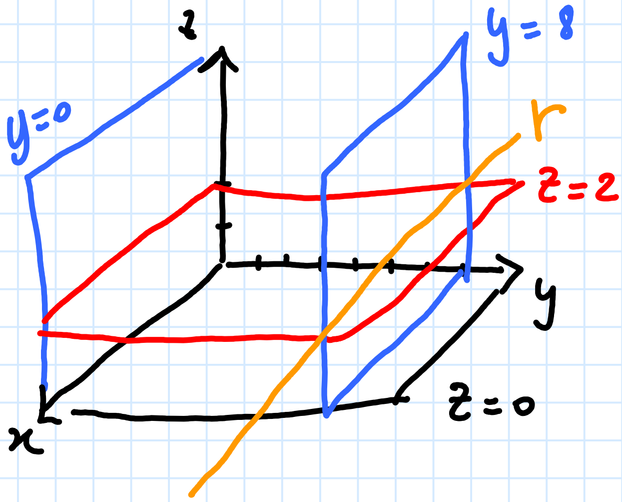
$$\begin{cases} -2-3h = 4 \\ 3k-3 = -6 \end{cases} \qquad \begin{cases} 3h = -6 \\ 3k = -3 \end{cases} \qquad \begin{cases} h = -2 \\ k = -1 \end{cases}$$

$(h,k)=(-2,-1)$; $\lambda=2$

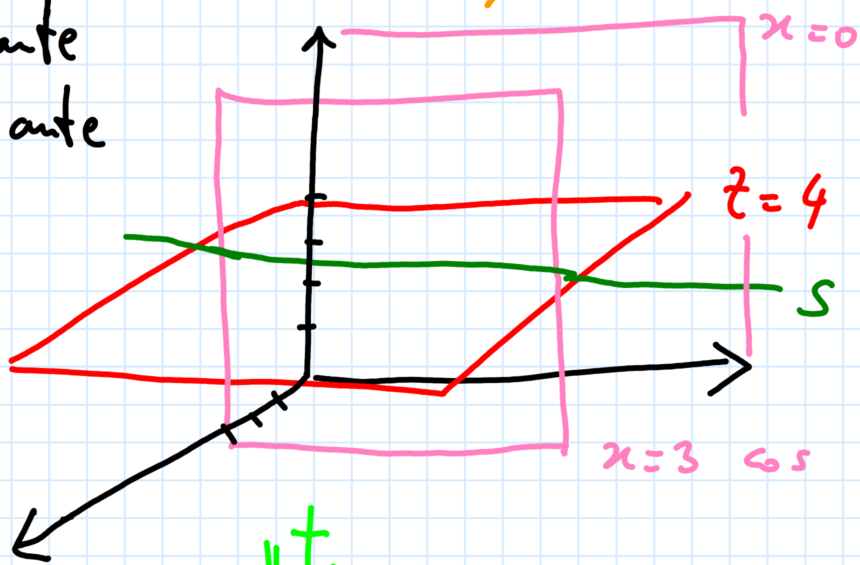
$r:\ z-2 = y-4z=0$ // asse X

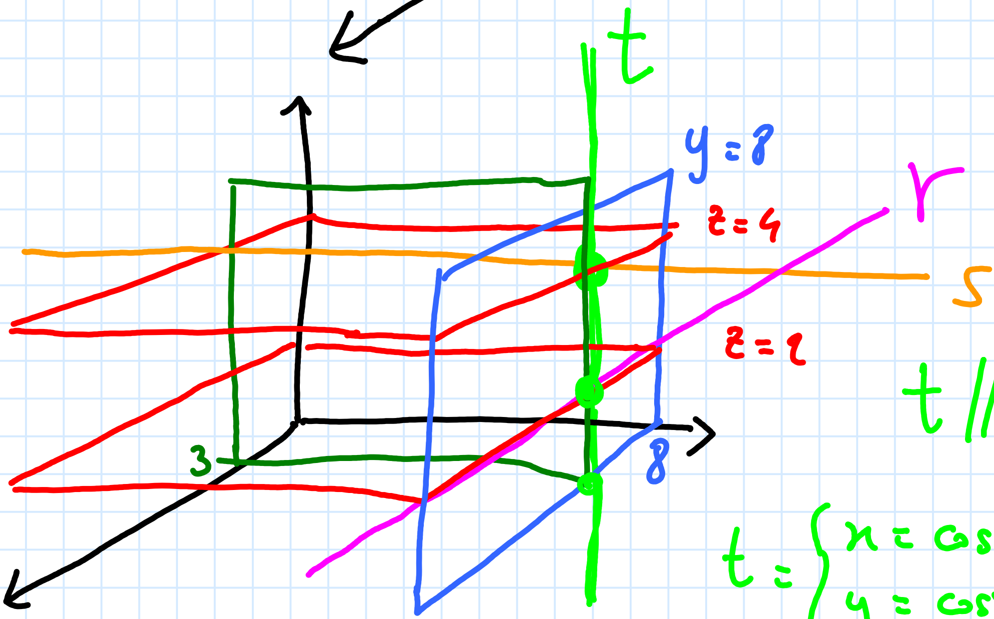
$s:\ z-4 = 4x-3z=0$ // asse Y

$r: \begin{cases} z=2 & \text{costante} \\ y=8 & \text{costante} \end{cases}$



$s: \begin{cases} z=4 & \text{costante} \\ x=3 & \text{costante} \end{cases}$





t // asse z

$t: \begin{cases} x = \text{cost} = 3 \\ y = \text{cost} = 8 \end{cases}$

$\boxed{t:\ x-3=y-8=0}$