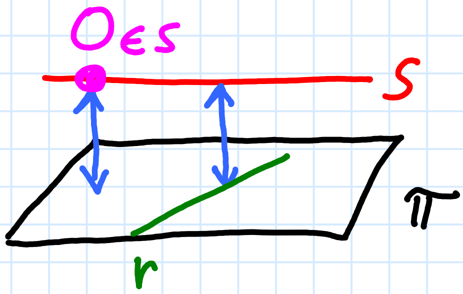
Lunedì 25 Gennaio ore 10:00

Titolo nota 25/01/2021

17/01/2017 (3)



$r: \; x - 5y - 90 = 3y + 2z = 0$

$s: \; 5x + z = x - 2y + 2z = 0$

$$\begin{array}{c} \begin{bmatrix} 1 & -5 & 0 \\ 0 & 3 & 2 \end{bmatrix} \\ \begin{matrix} \uparrow & \uparrow & \uparrow \\ l & m & n \end{matrix} \end{array} \quad \begin{aligned} l &= -10 \\ m &= -2 \\ n &= +3 \end{aligned}$$

param. dir. di r

$$\begin{bmatrix} 5 & 0 & 1 \\ 1 & -2 & 2 \end{bmatrix} \quad \begin{aligned} l' &= +2 \\ m' &= -9 \\ n' &= -10 \end{aligned}$$

param. dir di s

$\vec{v}_r = (-10, -2, 3) \leftarrow$ retta r

$\vec{v}_s = (2, 9, -10) \leftarrow$ retta s

$\vec{v}_r \not\parallel \vec{v}_s \Rightarrow r \not\parallel s$

(1) $\exists! \, \pi \in F(r) : \pi \parallel s$

(2) $O \in s$

(3) $d(r, s) = d(O, \pi) \leftarrow$ distanza punto-piano

$\pi \in F(r) : \quad \lambda \cdot (x - 5y - 90) + \mu \cdot (3y + 2z) = 0$

$$\pi: \underbrace{\lambda}_{a} \cdot x + \underbrace{(3 \cdot \mu - 5\lambda)}_{b} \cdot y + \underbrace{2\mu}_{c} \cdot z \; \underbrace{- 90\lambda}_{d} = 0$$

$$\pi \parallel s \iff \det\begin{bmatrix} \lambda & (3\mu-5\lambda) & 2\mu \\ 5 & 0 & 1 \\ 1 & -2 & 2 \end{bmatrix} \begin{matrix} \lambda & (3\mu-5\lambda) \\ 5 & 0 \\ 1 & -2 \end{matrix} = 0$$

$$3\mu - 5\lambda - 20\mu + 2\lambda - 10(3\mu - 5\lambda) = 0$$

$$3\mu - 5\lambda - 20\mu + 2\lambda - 30\mu + 50\lambda = 0$$

$$47\lambda - 47\mu = 0 \quad ; \quad \lambda - \mu = 0$$

scelgo $\lambda = 1$ et $\mu = 1$

$$\pi : x - 2y + 2z - 90 = 0$$

$$O(0,0,0)$$

$$d(O, \pi) = \frac{|0 - 2\cdot 0 + 2\cdot 0 - 90|}{\sqrt{1^2 + (-2)^2 + 2^2}} = \frac{90}{3}$$

$$\boxed{d(r,s) = 30}$$

$$C = \begin{bmatrix} 4 & (t-2) & -t \\ (t-4) & (2-t) & (t-1) \end{bmatrix}$$

$A = [4]$

$\det A = 4 \neq 0 \quad \forall t$

$rg\, A = 1 \quad \forall t$

$A' = [-t]$

$rg\, A' = 1 \quad \forall t \neq 0$

$$rg\, C = 1$$

$$\text{rg}\, C = 1 \iff \begin{cases} \det \begin{bmatrix} 4 & (t-2) \\ (t-4) & (2-t) \end{bmatrix} = 0 & \text{et} \\ \det \begin{bmatrix} 4 & -t \\ (t-4) & (t-1) \end{bmatrix} = 0 \end{cases}$$

$$\begin{cases} 4\cdot(2-t) + (2-t)(t-4) = 0 \\ 4\cdot(t-1) + t(t-4) = 0 \end{cases}$$

$$\begin{cases} (2-t)\cdot t = 0 & \rightarrow & t_1 = 0 \;;\; t_2 = +2 \\ t^2 - 4 = 0 & \rightarrow & t_3 = 2 \;;\; t_4 = -2 \end{cases}$$

L'unico valore reale di t è

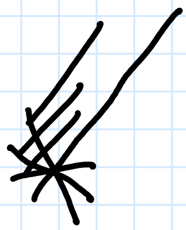
$$\boxed{t = +2}$$

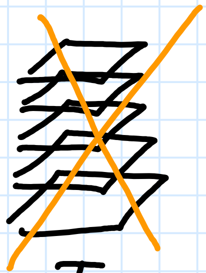
22/07/2015 (3) $t \in \mathbb{R}$

$\alpha: tx + t^2 \cdot y + t z = 0$

$\beta: x - 6y + t = 0$ $(1,-6,0)$

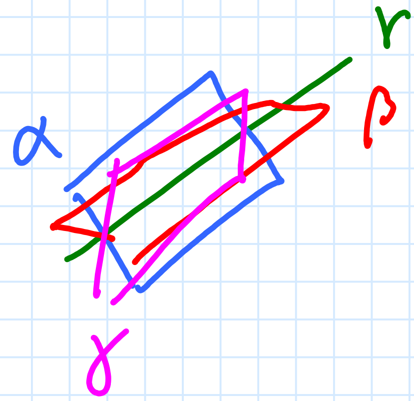
$\gamma: z + 6 = 0 \Rightarrow 0\cdot x + 0\cdot y + 1\cdot z + 6$
$(0,0,1)$

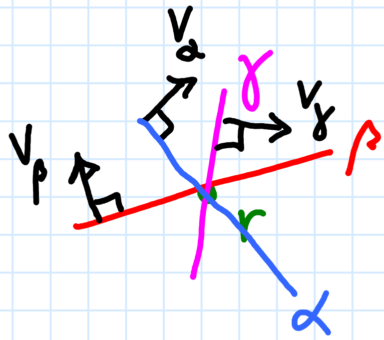




I

α, β, γ piani stesso fascio





$\vec{v}_\alpha = (t, t^2, t)$

$\vec{v}_\beta = (1, -6, 0)$

$\vec{v}_\alpha = (0, 0, 1)$

$$A = \left[\begin{array}{ccc} t & t^2 & t \\ \hline 1 & -6 & 0 \\ \hline 0 & 0 & 1 \end{array}\right]$$

$\text{rg}\, A < 3$

$\det A = 0$

$\det A = -6t - t^2 = 0$

$t \cdot (6 + t) = 0 \quad \begin{cases} \nearrow t_1 = 0 \\ \searrow t_2 = -6 \end{cases}$

$t_1 = 0 \quad \alpha : \boxed{0 \cdot x + 0 \cdot y + 0 \cdot z = 0}$ NON è un piano

quindi $t_1 = 0$ NON è una soluzione accettabile geometricamente

$\boxed{t_2 = -6} \Rightarrow -6x + 36y - 6z = 0$

$\rightarrow \alpha : x - 6y + z = 0$

$\rightarrow \beta : x - 6y - 6 = 0$

$\rightarrow \gamma : z + 6 = 0$

$$\underbrace{1}_{\alpha}\cdot(x-6y+z)+(-1)\cdot(\underbrace{z+6}_{\gamma}) = \underbrace{x-6y-6}_{\beta}$$

Risultato finale $\boxed{t=-6}$ è l'unico valore reale di t richiesto.

28/06/2017 (5)

$$3x^2+2\sqrt{3}\,xy+y^2+2\sqrt{3}\,x+2y+1=0$$

$$(?\cdot x+?\cdot y+?)^2=0$$

$$\underbrace{(\sqrt{3}\cdot x+y+1)^2=0}$$

$$(\underbrace{\sqrt{3}x+y+1}_{r})\cdot(\underbrace{\sqrt{3}x+y+1}_{s=r})=0$$

2 rette reali e coincidenti

19/07/2016 (5)

$$2x^2+2\sqrt{10}\,xy+5y^2-4x-2\sqrt{10}\,y+23=0$$

$$A = \begin{bmatrix} 2 & \sqrt{10} \\ \sqrt{10} & 5 \end{bmatrix} ; \quad p_A(\lambda) = \ldots = \lambda^2 - 7\lambda = \lambda \cdot (\lambda - 7)$$

$$\lambda_1 = 7 \; ; \; \lambda_2 = 0$$

autovettori relativi a $\lambda_1 = 7$

$$\begin{bmatrix} -5 & \sqrt{10} \\ \sqrt{10} & -2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad -5x + \sqrt{10}\,y = 0$$

$$-\sqrt{5}\cdot\cancel{\sqrt{5}}\,x + \cancel{\sqrt{5}}\cdot\sqrt{2}\,y = 0$$

$$-\sqrt{5}\,x + \sqrt{2}\,y = 0$$

$\vec{u} = (x, y) = (\sqrt{2}, \sqrt{5})$ auto vettore per $\lambda_1 = 7$

$$\|\vec{u}\| = \sqrt{7}$$

$\vec{v} = \left(\frac{\sqrt{2}}{\sqrt{7}}, \frac{\sqrt{5}}{\sqrt{7}} \right)$ auto VERSORE per $\lambda_1 = 7$

$$\Lambda = \begin{bmatrix} 7 & 0 \\ 0 & 0 \end{bmatrix} ; \quad C = \left[\begin{array}{c|c} \frac{\sqrt{2}}{\sqrt{7}} & -\frac{\sqrt{5}}{\sqrt{7}} \\ \frac{\sqrt{5}}{\sqrt{7}} & \frac{\sqrt{2}}{\sqrt{7}} \end{array} \right]$$

$\hookrightarrow$ rotazione

$$\begin{bmatrix} -4 & -2\sqrt{10} \end{bmatrix} \cdot \begin{bmatrix} \frac{\sqrt{2}}{\sqrt{7}} & -\frac{\sqrt{5}}{\sqrt{7}} \\ \frac{\sqrt{5}}{\sqrt{7}} & \frac{\sqrt{2}}{\sqrt{7}} \end{bmatrix} = \begin{bmatrix} -2\sqrt{14} & 0 \end{bmatrix}$$

$$-4 \cdot \frac{\sqrt{2}}{\sqrt{7}} - 2\,\frac{\sqrt{10}\cdot\sqrt{5}}{\sqrt{7}} = \frac{-4\sqrt{2} - 10\sqrt{2}}{\sqrt{7}} = \frac{-14\sqrt{2}}{\cancel{\sqrt{7}}} = -2\sqrt{14}$$

$$\cancel{\sqrt{7}}\cdot\sqrt{7}\cdot 2$$

$$-4\cdot\left(-\frac{\sqrt{5}}{\sqrt{7}}\right)-\frac{2\sqrt{10}\cdot\sqrt{2}}{\sqrt{7}}=\frac{4\cdot\sqrt{5}-4\sqrt{5}}{\sqrt{7}}=0$$

dopo la rotazione si ha che

$$7\cdot(x')^2+0\cdot(y')^2-2\sqrt{14}\,x'+0\cdot y'+23=0$$

$$\boxed{7(x')^2-2\sqrt{14}\cdot x'}+23=0$$

$$\boxed{7\cdot\left(x'-\frac{\sqrt{14}}{7}\right)^2-2}+23=0$$

traslazione $\quad x''=x'-\frac{\sqrt{14}}{7}$

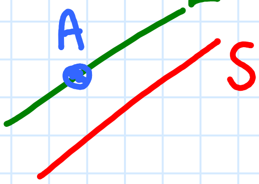
$$7\cdot(x'')^2+21=0$$

$$\boxed{(x'')^2+3=0}$$ equazione canonica

conica senza punti reali

$$(x''-\sqrt{3}\,i)\cdot(x''+\sqrt{3}\,i)=0$$

due rette immaginarie coniugate parallele

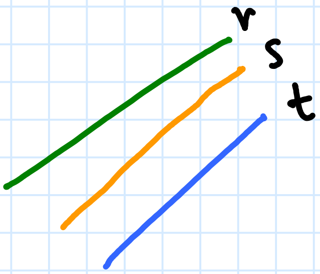


$r: 2x - y + 1 = 0$ $\quad A(0,1)$

$s: 2x - y + ? = 0$

$\neq 1$

$$(2x - y + 1)\cdot(2x - y + ?) = 0$$



$\mathscr{C}_1 = r \cup s$ $\quad$ asse x : $y = 0$

$\mathscr{C}_2 = s \cup t$

$\mathscr{C}_1 \neq \mathscr{C}_2$

$\mathscr{C}_1 : y\cdot(y-1) = 0$

$|\mathscr{C}_1 \cap \mathscr{C}_2| = \infty$ $\quad \mathscr{C}_2 : y\cdot(y+1) = 0$

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sist. lin. omog. $\quad$ 6 eq.

2 inc

unica sol.

$$\begin{cases} \boxed{2}\cdot x + \boxed{1}\cdot y = 0 \\ \boxed{3}\cdot x + \boxed{1}\cdot y = 0 \\ -x = 0 \\ \text{---} \\ \text{---} \\ \text{---} \end{cases}$$
