Lunedì 1 Febbraio ore 10:00

Titolo nota 01/02/2021

Buongiorno!!!

$$m_g(\lambda_1) \overset{DEF.}{=\!=} \dim(E_{\lambda_1})$$

$$A_{n\times n} \qquad \dim(E_{\lambda_1}) = n - rg(A - \lambda_1 \cdot I_n)$$

$$n=3 \qquad 3 - \underbrace{rg(A-\lambda_1 \cdot I_3)}_{1} = 2$$

$$A = \begin{bmatrix} (t^2-1) & t\cdot(t+1) & 0 \\ (t^2+t) & (t-1)(t+1) & 0 \end{bmatrix}_{2\times 3} \qquad t \in \mathbb{R}$$

rango $A = ?$

$$0 \leq rg A \leq 2 \qquad B = \begin{bmatrix} (t^2-1) & t\cdot(t+1) \\ (t^2+t) & (t-1)(t+1) \end{bmatrix}$$

$$\forall t \in \mathbb{R} \quad rg B = rg A$$

$$rg B = 2 = \max \iff \det B \neq 0$$

$$\det B = (t-1)^2 \cdot (t+1)^2 - t^2 \cdot (t+1)^2 =$$

$$= (t+1)^2 \cdot \left[(t-1)^2 - t^2 \right] = (t+1)^2 \cdot (1-2t);$$

$\det B = 0 \iff t = -1$ vel $t = \frac{1}{2}$

(1) $\forall t \in \mathbb{R} - \{-1, +\frac{1}{2}\}$ $\quad rg\,A = rg\,B = 2$

(2) $\boxed{t = -1} \Rightarrow A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow rg\,A = 0$

(3) $\boxed{t = \frac{1}{2}} \Rightarrow A = \begin{bmatrix} -\frac{3}{4} & \frac{3}{4} & 0 \\ \frac{3}{4} & -\frac{3}{4} & 0 \end{bmatrix} \Rightarrow rg\,A = 1$

- $r \parallel$ asse Y ; $A(-\sqrt{3}, 2, 3) \in r$;
- piani per r (fascio di piani per r)
- tali che angolo $\boxed{\frac{\pi}{6}\text{ rad}} \to 30°$ con piano YZ

asse $Y \Rightarrow (\ell, m, n) = (0, 1, 0)$

$$r: \begin{cases} x = 0 \cdot t + (-\sqrt{3}) \\ y = 1 \cdot t + 2 \\ z = 0 \cdot t + 3 \end{cases} ; \quad r: \begin{cases} x = -\sqrt{3} \\ y = t + 2 \\ z = 3 \end{cases} ; \quad r: \begin{cases} x + \sqrt{3} = 0 \\ z - 3 = 0 \end{cases}$$

$$F(r): \lambda \cdot (x + \sqrt{3}) + \mu (z - 3) = 0$$

$$\underbrace{\lambda}_{a} \cdot x + \underbrace{0}_{b} \cdot y + \underbrace{\mu}_{c} \cdot z + \underbrace{(\sqrt{3}\lambda - 3\mu)}_{d} = 0$$

piano YZ : $x = 0$; $\underbrace{1}_{a'} \cdot x + \underbrace{0}_{b'} \cdot y + \underbrace{0}_{c'} \cdot z + \underbrace{0}_{d'} = 0$

angolo θ tra 2 piani

$$\cos\theta = \pm \frac{a\cdot a' + b\cdot b' + c\cdot c'}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{(a')^2+(b')^2+(c')^2}}$$

$$\frac{\sqrt{3}}{2} = \pm \frac{\lambda}{\sqrt{\lambda^2+\mu^2}\cdot\sqrt{1^2}}$$

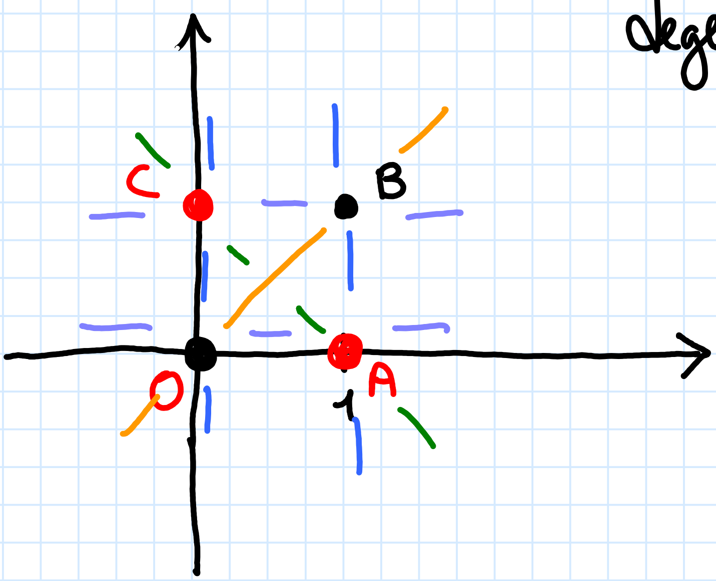
$$\sqrt{3}\cdot\sqrt{\lambda^2+\mu^2} = \pm 2\lambda$$

$$3\cdot(\lambda^2+\mu^2) = 4\lambda^2 \quad ; \quad \boxed{\lambda^2 = 3\mu^2}$$

scelgo $\mu = 1 \Rightarrow \lambda^2 = 3 \Rightarrow \lambda = \pm\sqrt{3}$

piano (1) con $\lambda = \sqrt{3}$ et $\mu = 1$

piano (2) con $\lambda = -\sqrt{3}$ et $\mu = 1$



degeneri = unione 2 rette

$r \cup s$

$\mathcal{C}_1$: —?

$\mathcal{C}_2$: —?

$\mathcal{C}_3$: —?

$r \,//\,$ asse z ; $A(4\sqrt{3}, 12, -2\sqrt{3})$

piani per r

angolo $\frac{\pi}{3}$ rad asse X

asse $z \Rightarrow (l, m, n) = (0, 0, 1)$

$$r: \begin{cases} x = 0 \cdot t + 4\sqrt{3} \\ y = 0 \cdot t + 12 \\ z = 1 \cdot t - 2\sqrt{3} \end{cases} ; \quad r: \begin{cases} x - 4\sqrt{3} = 0 \\ y - 12 = 0 \end{cases}$$

$$F(r): \lambda \cdot (x - 4\sqrt{3}) + \mu \cdot (y - 12) = 0$$

$$\underbrace{\lambda}_{a} \cdot x + \underbrace{\mu}_{b} \cdot y + \underbrace{0}_{c} \cdot z \underbrace{- (4\sqrt{3}\lambda + 12\mu)}_{d} = 0$$

asse $X \Rightarrow (l, m, n) = (1, 0, 0)$

angolo θ retta - piano

$$\sin\theta = \frac{|a \cdot l + b \cdot m + c \cdot n|}{\sqrt{a^2 + b^2 + c^2} \cdot \sqrt{l^2 + m^2 + n^2}}$$

$$\theta = \frac{\pi}{3} \text{ rad} = 60° \Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$$

sostituiamo tutto nella formula e facciamo i conti. E così troviamo λ e μ

$\boxed{\text{Parabola per } A(0,1)}$ $\quad F(1,1)$

$r: x + y = 0$ direttrice

$$d(A, F) \overset{?}{=} d(A, r)$$

$d(A, F) = 1$

$d(A, H) = \text{conti} = ?$?