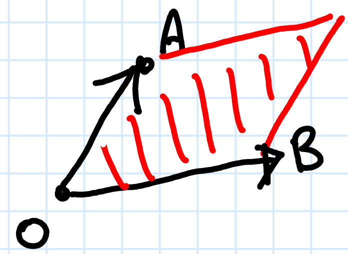
Lunedì 8 Febbraio - ore 10:00

Titolo nota 08/02/2021

$t \in \mathbb{R}$ $\quad O(0,0,0)$; $A(t,-t,t)$

$B(0,-12,12)$

area $A\hat{O}B = 156$



$[\vec{OA}] = (t,-t,t)$

$[\vec{OB}] = (0,-12,12)$

$$\| [\vec{OA}] \wedge [\vec{OB}] \| = 2 \cdot \text{area } A\hat{O}B = 2 \cdot 156 = 312$$

$$[\vec{OA}] \wedge [\vec{OB}] = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ t & -t & t \\ 0 & -12 & 12 \end{bmatrix} = (-12t)\vec{j} + (-12t)\vec{k}$$

$$\| [\vec{OA}] \wedge [\vec{OB}] \| = \sqrt{0^2 + (-12t)^2 + (-12t)^2} = 2 \cdot 156$$

$$\sqrt{2 \cdot 144 t^2} = 2 \cdot 156$$

$$\cancel{12}^{\,\cancel{6}\,3} \cdot |t| \cdot \sqrt{2} = \cancel{2}^{\,1} \cdot \cancel{156}^{\,26}$$

$$|t| = \frac{26}{\sqrt{2}} = 13\sqrt{2}$$

$$t = \pm 13\sqrt{2}$$

controllare i conti

$$8x^2 - 12xy + 17y^2 - 60x - 70y + 105 = 0$$

$$A = \begin{bmatrix} 8 & -6 \\ -6 & 17 \end{bmatrix}; \quad p_A(\lambda) = \dots = \lambda^2 - 25\lambda + 100 =$$

$$= (\lambda - 5)\cdot(\lambda - 20);$$

$$\lambda_1 = 5 \qquad \lambda_2 = 20$$

autovettori per $\lambda_1 = 5$

$$\begin{bmatrix} 3 & -6 \\ -6 & 12 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{cases} x - 2y = 0 \end{cases}$$

$\|(2,1)\| = \sqrt{5}$

$$x = 2y \qquad (x,y) = (2y, y) = y\,(2,1)$$

autoVERSORE $\frac{1}{\sqrt{5}}(2,1) = \left(\frac{2\sqrt{5}}{5}, \frac{\sqrt{5}}{5}\right)$

$\lambda_2 = 20$ <u>non</u> è necessario fare i conti

$$C = \left[\begin{array}{c|c} \frac{2\sqrt{5}}{5} & -\frac{\sqrt{5}}{5} \\ \frac{\sqrt{5}}{5} & \frac{2\sqrt{5}}{5} \end{array}\right]$$

è una rotazione

$$\begin{bmatrix} -60 & -70 \end{bmatrix} \cdot \begin{bmatrix} \frac{2\sqrt{5}}{5} & -\frac{\sqrt{5}}{5} \\ \frac{\sqrt{5}}{5} & \frac{2\sqrt{5}}{5} \end{bmatrix} = \begin{bmatrix} -38\sqrt{5} & -16\sqrt{5} \end{bmatrix}$$

dopo la rotazione

$$\underline{5(x')^2} + \underline{20(y')^2} \underline{- 38\sqrt{5}\,x'} - \underline{16\sqrt{5}\,y'} + 105 = 0$$

$$\boxed{5\cdot\left(1\cdot x' - \frac{19\sqrt{5}}{5}\right)^2 - 19^2} \boxed{+ 20\cdot\left(1\cdot y' - \frac{2\cdot\sqrt{5}}{5}\right)^2 - 16} + 105 = 0$$

traslazione
$$\begin{cases} x'' = x' - \dfrac{19\sqrt{5}}{5} \\ y'' = y' - \dfrac{2\sqrt{5}}{5} \end{cases}$$

$$\underbrace{-19^2 - 16 + 105}$$

$$5\cdot(x'')^2 + 20\cdot(y'')^2 - 272 = 0$$

$$\boxed{\frac{(x'')^2}{\left(\frac{272}{5}\right)} + \frac{(y'')^2}{\left(\frac{272}{20}\right)} = +1}$$

eq. canonica di un' ellisse