Martedì 16 Febbraio - ore 10:00

Titolo nota 16/02/2021

Hp. A invertibile

Th. A^T è invertibile e, inoltre $(A^T)^{-1} = (A^{-1})^T$

A invertibile $\Leftrightarrow \det A \neq 0 \Leftrightarrow \det(A^T) = \det A \neq 0 \Leftrightarrow$

$\Leftrightarrow \det(A^T) \neq 0 \Leftrightarrow A^T$ è invertibile

devo provare che l'inversa della trasposta è la trasposta dell'inversa

chiamo $C \overset{DEF}{=\!=} (A^{-1})^T$ e provo che C è l'inversa della trasposta

ovvero $(A^T) \cdot C \overset{①}{=} I_n \overset{②}{=} C \cdot (A)^T$

① $A^T \cdot (\underbrace{\boxed{A^{-1}}}_{B})^T = (\underbrace{\boxed{A^{-1}}}_{B} \cdot A)^T = (I_n)^T = I_n$

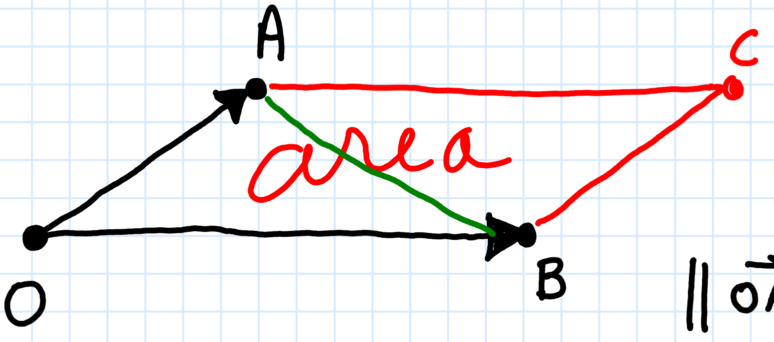
proprietà

$A^T \cdot B^T = (B \cdot A)^T$

② in modo analogo

$t \in \mathbb{R}$ $\quad$ $\text{area}(\triangle OAB) = 168$

$O(0,0,0)$; $A(t,-t,t)$; $B(0,-14,14)$



$[\overrightarrow{OA}] = (t,-t,t)$

$[\overrightarrow{OB}] = (0,-14,14)$

$\| \overrightarrow{OA} \wedge \overrightarrow{OB} \| = \text{area } OACB$

$$\| \overrightarrow{OA} \wedge \overrightarrow{OB} \| = 2 \cdot 168$$

$$\det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ t & -t & t \\ 0 & -14 & 14 \end{bmatrix} = 0\vec{i} - 14t\,\vec{j} - 14t\,\vec{k}$$

$$\overrightarrow{OA} \wedge \overrightarrow{OB} = (0, -14t, -14t)$$

$$\| \overrightarrow{OA} \wedge \overrightarrow{OB} \| = \sqrt{0^2 + (-14t)^2 + (-14t)^2} =$$

$$= \sqrt{2 \cdot 14^2 \cdot t^2} = 14|t| \cdot \sqrt{2}$$

$$\overset{7 \; 1}{\cancel{14}} \cdot |t| \cdot \sqrt{2} = \overset{1}{\cancel{2}} \cdot \overset{24}{\cancel{168}}$$

$$|t| \cdot \sqrt{2} = 24 = 2 \cdot 12$$

$$|t| = 12 \cdot \sqrt{2} ; \quad \boxed{t = \pm 12 \cdot \sqrt{2}}$$

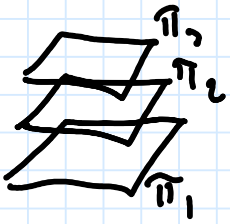
$t \in \mathbb{R}$ stesso fascio

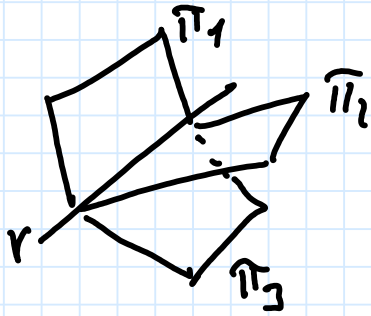
$\pi_1 : tx + t^2 y + tz = 0$ (π_1 piano $\Rightarrow t \neq 0$)

$\pi_2 : x + 5y + t = 0 \Rightarrow \vec{u}_{\pi_2} = (1,5,0)$

$\pi_3 : z - 5 = 0 \Rightarrow \vec{u}_{\pi_3} = (0,0,1)$

2 tipi di fasci di piani → improprio (I) → proprio (II)

(I) 

(II) 

vedo che $\pi_2 \not\parallel \pi_3$ quindi si tratta di un fascio PROPRIO

$r \overset{DEF}{=} \pi_2 \cap \pi_3$

$\pi_1 \in F(r)$

$$A = \begin{bmatrix} t & t^2 & t & 0 \\ 1 & 5 & 0 & t \\ 0 & 0 & 1 & -5 \end{bmatrix} \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array} \qquad R_3 = \lambda R_1 + \mu R_2$$

$\boxed{rg\, A = 2}$ (con teorema orlati o riduzione a gradino)

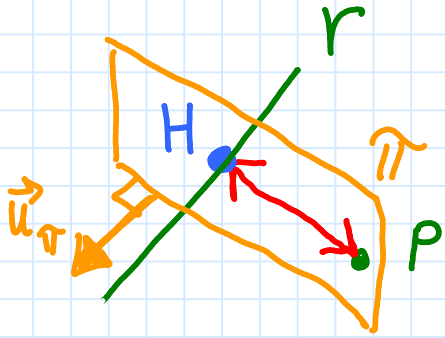
↳ facendo i conti ..... si trova $\boxed{t = 5}$

$t = 0$ NON è accettabile in quanto non si avrebbe un piano

Se esistono, $\boxed{A \text{ e } B}$ distanti 5 dalla retta $r: x+2 = z+3 = 0$ → sull'asse X

P generico punto asse X : $y = z = 0$

$P(x, 0, 0)$

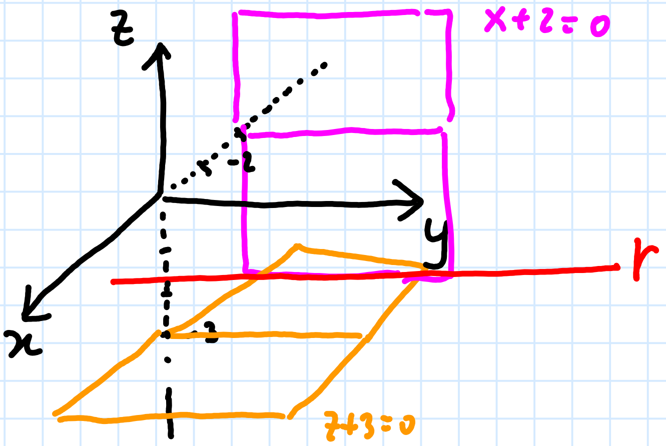


$\underbrace{d(P, r)}$ $= 5$

↳ distanza punto-retta

(I) trovo H

(II) $d(P, H) = 5$



$r: \begin{cases} x+2=0 \\ z+3=0 \end{cases}$

r // asse Y $(0,1,0)$

$\vec{u}_\pi = (\underset{a}{0}, \underset{b}{1}, \underset{c}{0})$ e π passa per $P(\underset{x_0}{x}, \underset{y_0}{0}, \underset{z_0}{0})$

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$$\boxed{\pi : y = 0}$$

$\{H\} = \pi \wedge r : \begin{cases} y=0 \\ x+2=0 \\ z+3=0 \end{cases} \rightarrow H(-2, 0, -3)$

$P(x, 0, 0)$

$d(P,H)=5$

$$\sqrt{(x-(-2))^2+(0-0)^2+(0-(-3))^2}=5$$

$$(x+2)^2+9=25$$

$$(x+2)^2=16$$

$$x+2=\pm 4$$

$$x=-2\pm 4 \begin{cases} x_1=-6 \rightarrow A(-6,0,0) \\ x_2=+2 \rightarrow B(2,0,0) \end{cases}$$

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retta r per $A(0,0,3)$ $\quad r \parallel \underbrace{\pi: 2x-3y+z=0}_{\vec{u}_\pi=(2,-3,1)}$

$r \perp s: \ x+3=3y+z-6=0$

(l,m,n) parametri direttori di r

$$s:\begin{cases} x+3=0 \\ 3y+z-6=0 \end{cases} \qquad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix} \rightarrow \begin{matrix} 0 \\ -1 \\ +3 \end{matrix}$$

$r \perp s \iff (l,m,n)\bullet(0,-1,3)=0 \iff$

$\iff -m+3n=0 \iff \boxed{m=3n}$

$(l,3n,n)\bullet(2,-3,1)=0$

$2l-9n+n=0 \Rightarrow \boxed{l=4n}$

$(4m, 3m, m) \quad \forall m \neq 0$

scelgo $m = 1$ e ottengo $(4, 3, 1)$

$$\begin{array}{ccc} (4, & 3, & 1) \\ \uparrow & \uparrow & \uparrow \\ l & m & n \end{array}$$

$A(0,0,3) \in r$

$$r: \begin{cases} x = 4 \cdot t + 0 \\ y = 3 \cdot t + 0 \\ z = 1 \cdot t + 3 \end{cases}$$

$$\boxed{r: \begin{cases} x = 4t \\ y = 3t \\ z = t+3 \end{cases}}$$

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sist. lineare 2 eq. 4 inc. w, x, y, z

tale che $\boxed{(0,0,0,0)}$ banale e $(1,2,1,2)$ siano due delle sue soluzioni. Se non è possibile motivare la risposta.

$$\begin{cases} \boxed{1} w + \boxed{0} x + \boxed{-1} y + \boxed{0} z = 0 \\ \boxed{0} w + \boxed{1} x + \boxed{0} y + \boxed{-1} z = 0 \end{cases}$$

candidato → 2 equazioni → 4 incognite → omogeneo

$$\begin{cases} w - y = 0 \\ x - z = 0 \end{cases}$$

$$\begin{array}{cccc} (1, & 2, & 1, & 2) \\ \uparrow & \uparrow & \uparrow & \uparrow \\ w & x & y & z \end{array}$$

FUNZIONA !!!

Sia $S = \{(x,y,z) \in \mathbb{R}^3 \mid x + 6y - 10z = 0\}$

$B = \{(t^3 - 6t^2, t^2 - t, t), (8, -3, -1)\}$

$t \in \mathbb{R} \mid B$ base di S

$u = (t^3 - 6t^2, t^2 - t, t)$; $v = (8, -3, -1)$

condizione **NECESSARIA** $u \in S$ et $v \in S$

$v \in S$? $8 + 6(-3) - 10(-1) = 0$? sì

$u \in S \Rightarrow (t^3 - 6t^2) + 6(t^2 - t) - 10t = 0$ ↑ lo imponiamo

$t^3 - \cancel{6t^2} + \cancel{6t^2} - 6t - 10t = 0$

$t \cdot (t-4) \cdot (t+4) = 0$ → $t_1 = 0$, $t_2 = 4$, $t_3 = -4$ } candidati

$\boxed{t_1 = 0} \Rightarrow u = (0,0,0)$ vettore **NULLO**

$B = \{(0,0,0), (8,-3,-1)\}$ **NO**

DIP.

$t_1 = 0$ si scarta

$\boxed{t_2 = 4} \Rightarrow u = (-32, 12, 4)$, $v = (8, -3, -1)$ $\Rightarrow u = (-4)v$

si scarta

u e v sono LIN. DIP. $\Rightarrow$ NO BASE

$\boxed{t_3 = -4} \Rightarrow \begin{array}{l} \vec{u} = (-160,\ 20,\ -4) \\ \vec{v} = (8,\ -3,\ -1) \end{array}\Big]$ INDIP

$B = \{\vec{u}, \vec{v}\}$ sono 2 vettori di S linearmente indipendenti

Siccome $\dim S = 2$

$\boxed{t_3 = -4}$ unico valore reale per cui l'insieme B è una base di S

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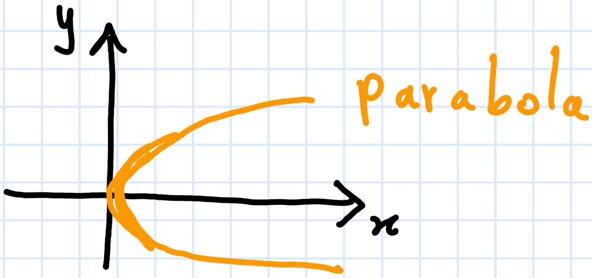
(1) $y^2 = 0$

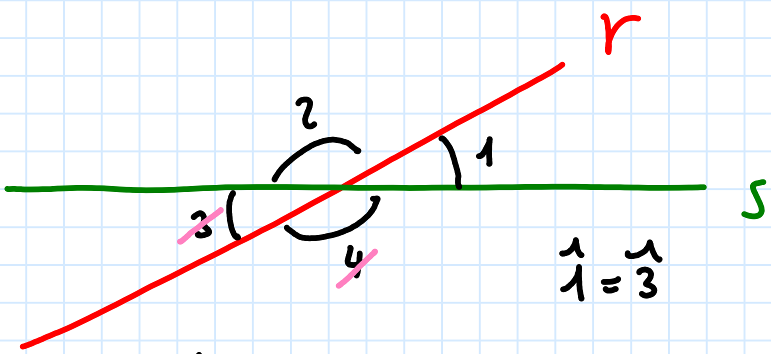
$r:\ y = 0$ $\qquad r \cup s:\ y \cdot y = 0$

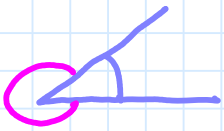
$s \equiv r:\ y = 0$

unione di 2 rette reali coincidenti

(2) $y^2 = x$



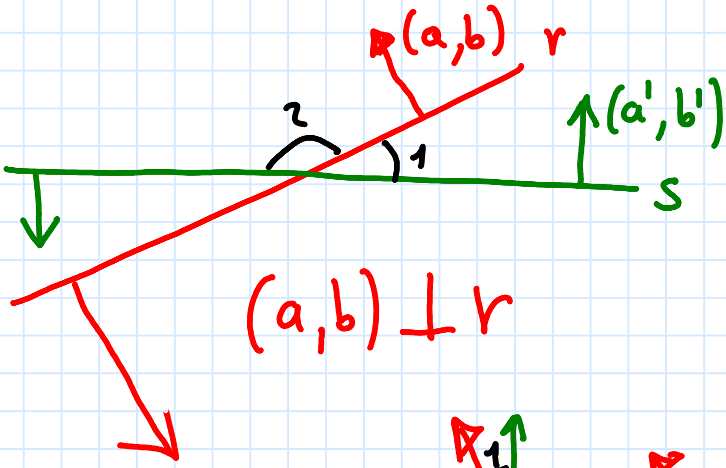




$\hat{1} = \hat{3}$ $\qquad \hat{2} = \hat{4}$

$\hat{1} + \hat{2} =$ angolo piatto

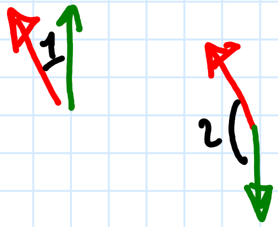
$\cos \hat{2} = -\cos \hat{1}$



$r: \; ax+by+c=0$

$s: \; a'x+b'y+c'=0$

$(a,b) \perp r$ $\qquad$ $(a',b') \perp s$

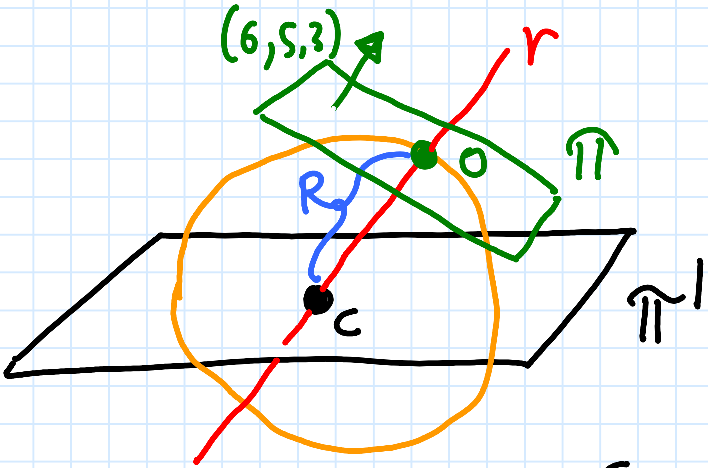


$$\cos(\widehat{r,s}) = \pm \frac{aa'+bb'}{\sqrt{a^2+b^2}\cdot\sqrt{(a')^2+(b')^2}}$$

Sfera tang. piano $\pi: 6x+5y+3z=0$ in $O(0,0,0)$ e avente il centro sul piano $\pi': 2y-5=0$

$\vec{u}_\pi \perp \pi$ $\qquad$ $\vec{u}_\pi = (6,5,3)$

$\vec{u}_{\pi'} \perp \pi'$ $\qquad$ $\vec{u}_{\pi'} = (0,2,0)$ $\Rightarrow \pi \not\parallel \pi'$



$$r: \begin{cases} x = 0 + 6t \\ y = 0 + 5t \\ z = 0 + 3t \end{cases}$$

$$C: \begin{cases} x = 6t \\ y = 5t \\ z = 3t \\ 2y-5=0 \end{cases} \to 10t - 5 = 0 \Rightarrow t = \frac{1}{2}$$

$$C\left(3, \frac{5}{2}, \frac{3}{2}\right) ; \quad O(0,0,0)$$

$$R = d(C,O) = \sqrt{9 + \frac{25}{4} + \frac{9}{4}} = \sqrt{\frac{70}{4}}$$

$$(x-x_c)^2 + (y-y_c)^2 + (z-z_c)^2 = R^2$$

sfera richiesta $\rightarrow$ $\boxed{(x-3)^2 + \left(y-\frac{5}{2}\right)^2 + \left(z-\frac{3}{2}\right)^2 = \frac{35}{2}}$

$$3x^2 - 4xy + y^2 + \dots \quad \dots$$

$$A = \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\boxed{X^T \cdot A \cdot X} = \boxed{3x^2 - 4xy + y^2}$$ complesso 2° grado

$$[x \quad y] \cdot \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = [(3x-2y) \quad (-2x+y)] \begin{bmatrix} x \\ y \end{bmatrix} =$$

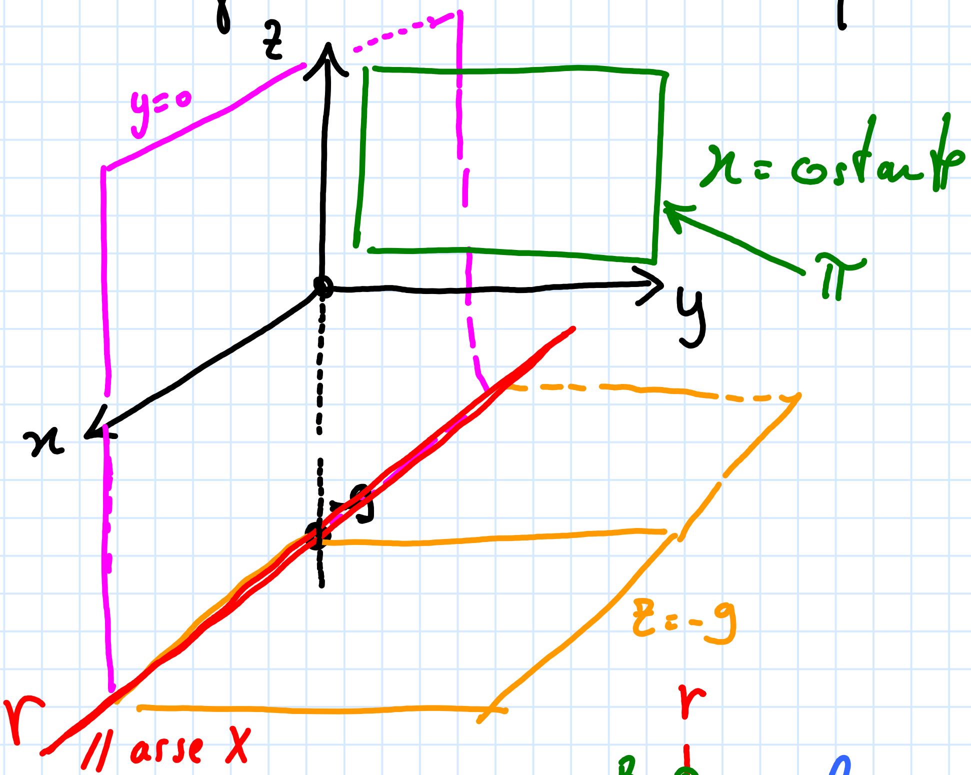
$$= (3x-2y) \cdot x + (-2x+y) \cdot y =$$

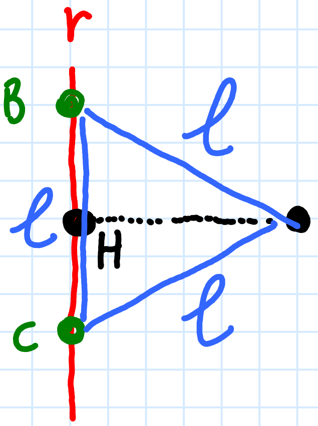
$$= 3x^2 \underbrace{-2xy - 2xy} + y^2 =$$

$$= 3x^2 - 4xy + y^2 ;$$

Sia $A(1, \sqrt{47}, -10)$. Sulla retta $r: y = z + 9 = 0$ trovare due punti B e C tali che il triangolo $\overset{\triangle}{ABC}$ sia equilatero.

$r: \begin{cases} y = 0 \\ z = -9 \end{cases}$





$A \notin r$

$A(1, \sqrt{47}, -10)$

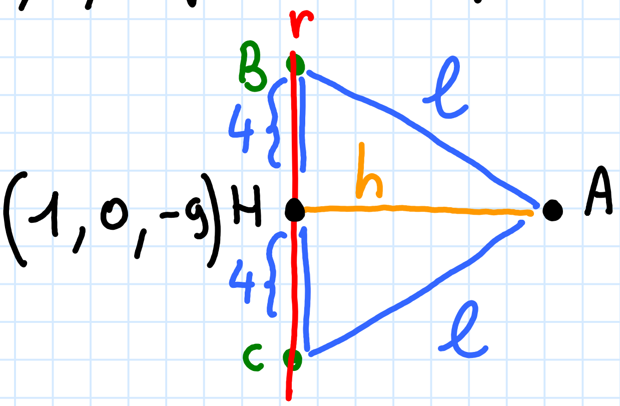
H proiezione ortogonale del punto A sulla retta $r: \begin{cases} y = 0 \\ z = -9 \end{cases}$

$\pi : x = \text{costante}$; $A \in \pi \Rightarrow 1 = \text{cost.}$

$\boxed{\pi : x = 1}$ $\qquad \{H\} = r \cap \pi : \begin{cases} y = 0 \\ z = -9 \\ x = 1 \end{cases}$

$$\left.\begin{array}{l} H(1, 0, -9) \\ A(1, \sqrt{47}, -10) \end{array}\right] \Rightarrow d(A, H) = \sqrt{(1-1)^2 + (\sqrt{47}-0)^2 + (-10+9)^2}$$

$$d(A,H) = \sqrt{48} = 4\sqrt{3}$$



$h = 4\sqrt{3}$

$\ell = \ldots\ldots = 8$

↑ ripassare

$r: \begin{cases} y = 0 \\ z = -9 \end{cases}$ $B, C \in r$

$B(x_B, 0, -9)$

$C(x_c, 0, -9)$

$x_B = 1 - 4 = -3$

$x_c = 1 + 4 = 5$

$$\boxed{\begin{matrix} B(-3, 0, -9) \\ C(5, 0, -9) \end{matrix}}$$

$A(1, \sqrt{47}, -10)$

Verifica:

$$\left.\begin{matrix} d(A,B) = 8 \\ d(A,C) = 8 \\ d(B,C) = 8 \end{matrix}\right] \longrightarrow \text{equilatero}$$