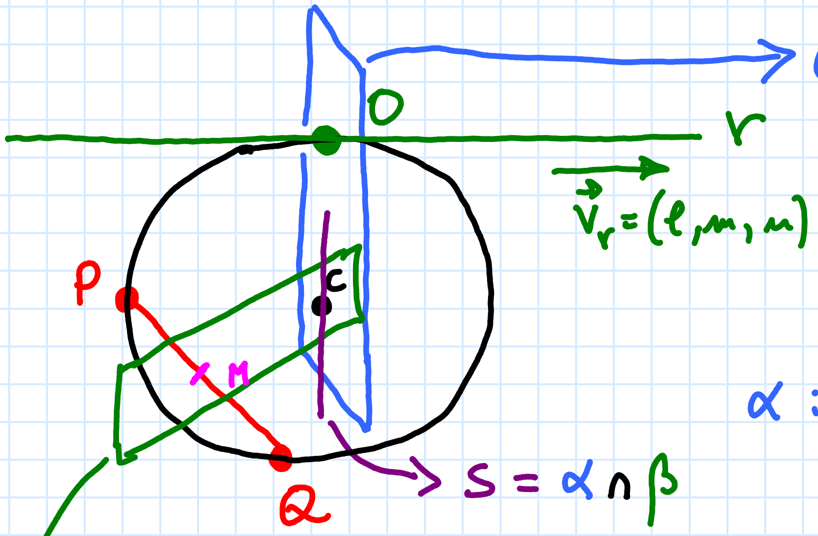
Lunedì 22 Febbraio - ore 10:00

Titolo nota 22/02/2021

Sfera tangente alla retta $r: x-y = 4x-z=0$ in $O(0,0,0)$ contenente i punti $P(0,1,-1)$ e $Q(1,1,1)$.



$\alpha: O \in \alpha$ et $\alpha \perp r$

C centro sfera

$C \in \alpha$

$\alpha: \ell x + my + nz = 0$

$s = \alpha \cap \beta$

$\beta: \beta \perp PQ$ et $M \in \beta$ M punto medio di PQ

$$s: \begin{cases} x = \ell_s t + x_0 \\ y = m_s t + y_0 \\ z = n_s t + z_0 \end{cases}$$

$$C(\ell_s t + x_0, m_s t + y_0, n_s t + z_0)$$

poi si trova il valore del parametro reale t usando le altre condizioni

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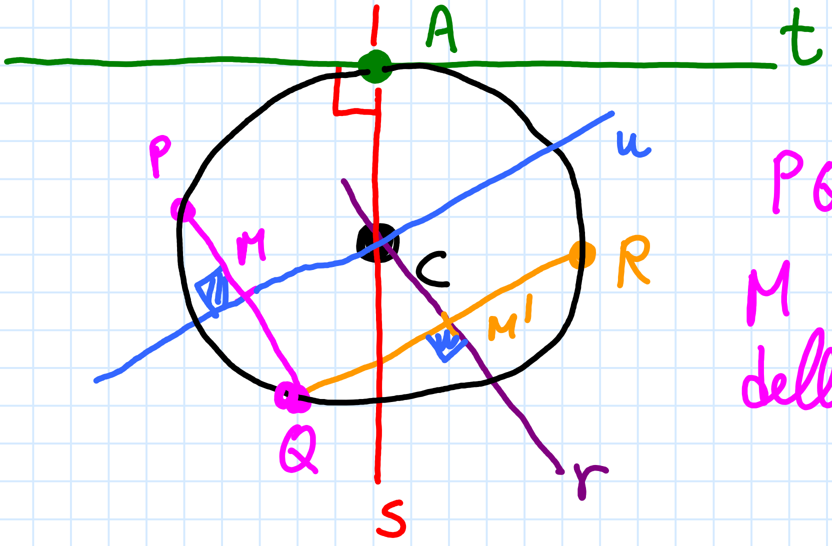
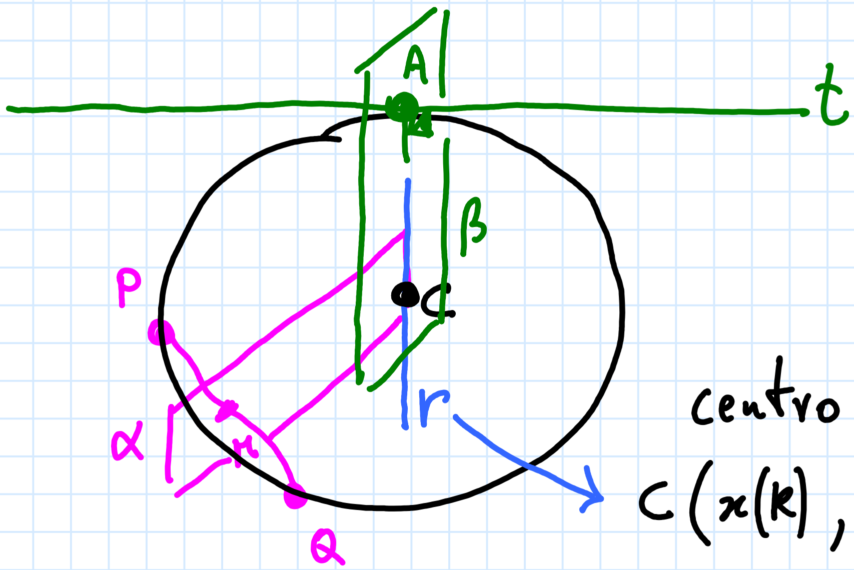
Circonferenza nel piano

$\downarrow$

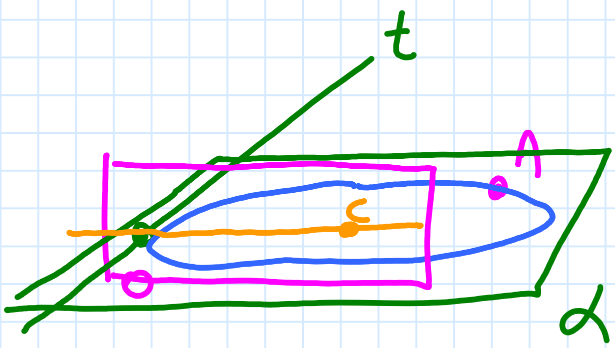


PQ è una corda
M punto medio della corda

Sfera (nello spazio ovviamente)



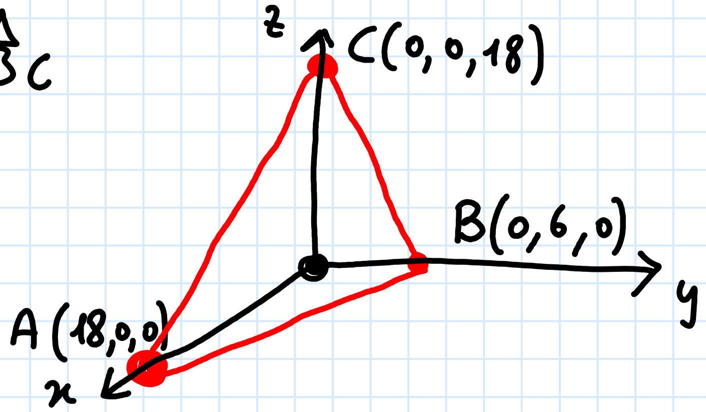
centro sfera
$C(x(k), y(k), z(k))$

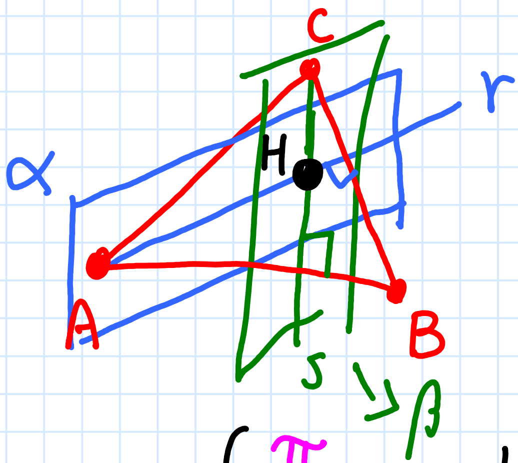


circonferenza nello spazio

Siano A, B e C i punti di intersezione del piano di equazione $\boxed{\pi: x+3y+z-18=0}$ con gli assi coordinati X, Y e Z rispettivamente.

Trovare le coordinate dell'ortocentro del triangolo $\hat{ABC}$





$\{H\} = r \cap s$

$r = \pi \cap \alpha \to$ da trovare

$s = \pi \cap \beta \to$ da trovare

$H: \begin{cases} \pi \\ \alpha \\ \beta \end{cases}$ sistema 3 eq. 3 incognite

$\alpha: A \in \alpha$ et $\alpha \perp [\vec{BC}]$

$B(0,6,0);\ C(0,0,18) \Rightarrow [\vec{BC}] = (0,-6,18)$

$[\vec{BC}] \perp \alpha \Rightarrow (a,b,c) = (0,-1,3)$

$\left.\begin{array}{l} \alpha: 0\cdot x - 1\cdot y + 3\cdot z + d = 0 \\ A(18,0,0) \in \alpha \Rightarrow d = 0 \end{array}\right| \Rightarrow \boxed{\alpha: y - 3z = 0}$

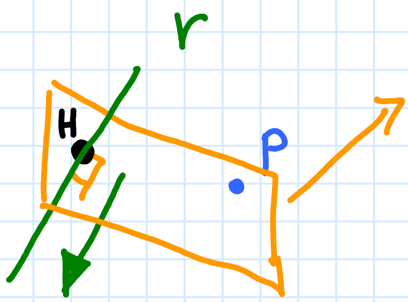
analogamente $\beta : C \in \beta$ et $\beta \perp [\vec{AB}]$

... → trovate equazione β

per trovare H mettete le 3 equazioni a sistema

Se esistono, trovare sull'asse X due punti A e B aventi distanza 5 dalla retta $r : x+2 = z+3 = 0$.

(problema distanza punto-retta)



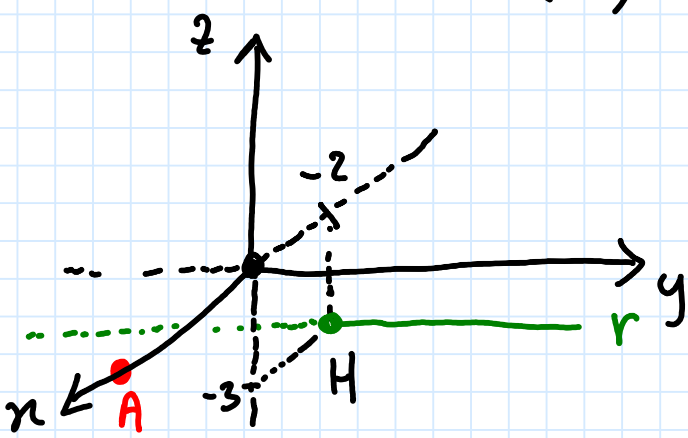
$\alpha : P \in \alpha$ et $\alpha \perp r$

$\{H\} = \alpha \cap r$

$d(P, r) = d(P, H)$ → formula distanza tra due punti

$r : \begin{cases} x = -2 \\ z = -3 \end{cases} \Rightarrow r \parallel \text{asse } Y \parallel (0,1,0)$

$A \in \text{asse } X \Rightarrow A(x, 0, 0)$



nel nostro caso

$\alpha : y = 0$

$H(-2, 0, -3)$

$A(x,0,0)$; $H(-2,0,-3)$

$$[d(A,H)]^2 = (x-(-2))^2 + (0-0)^2 + (0-(-3))^2 = 5^2$$

$$(x+2)^2 + 9 = 5^2$$

$$(x+2)^2 + 9 = 25$$

$$(x+2)^2 = 16 \Rightarrow x+2 = \pm 4 \begin{cases} x_1 = 2 \\ x_2 = -6 \end{cases}$$

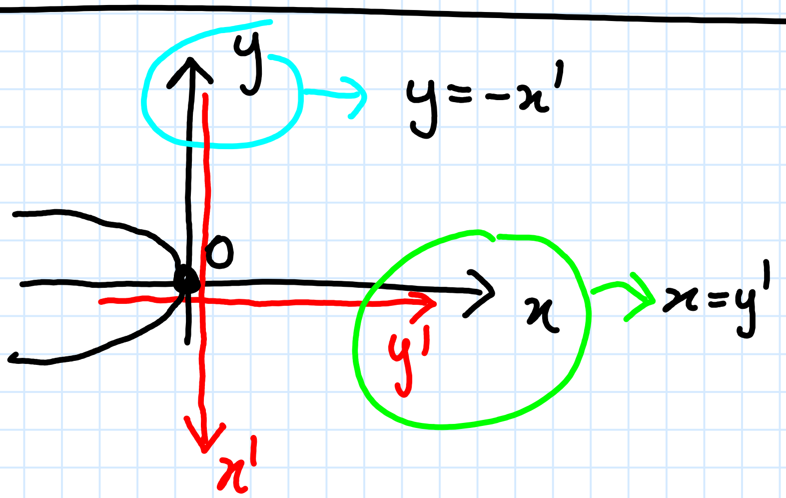
$A(2,0,0)$; $B(-6,0,0)$

$y^2 = -x$

$(-x')^2 = -y'$

$y' = -(x')^2$

parabola

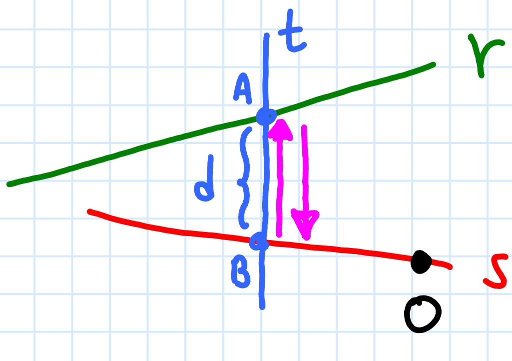


$r: 2x+7z-81 = y+3z = 0$

$s: 2x-7y = 2x-2y+z = 0$

d = distanza tra r e s

t retta minima distanza



$O(0,0,0) \in S$

trovare piano π tale che $\pi \in F(r)$ et $\pi // s$

$$d(r,s) = d(O,\pi) \rightarrow \text{formule distanza punto piano}$$

$$\pi \in F(r): \lambda(2x+7z-81)+\mu(y+3z)=0$$

$$\pi: \underbrace{(2\lambda)}_{a}\cdot x + \underbrace{\mu}_{b}\cdot y + \underbrace{(7\lambda+3\mu)}_{c}\cdot z - 81\lambda = 0$$

$$\pi // s \iff \det\begin{bmatrix} a & b & c \\ 2 & -7 & 0 \\ 2 & -2 & 1 \end{bmatrix} = 0$$

$$-7a-4c+14c-2b=0$$

$$-7a-2b+10c=0$$

$$-7(2\lambda)-2\mu+10(7\lambda+3\mu)=0$$

$$-14\lambda-2\mu+70\lambda+30\mu=0$$

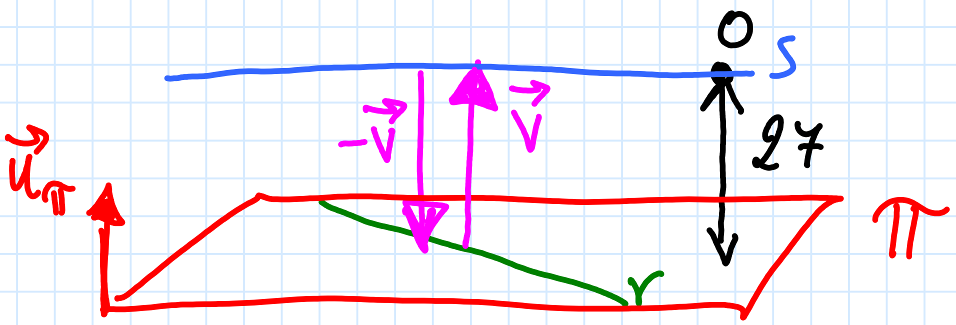
$$56\lambda+28\mu=0 \ ; \quad \boxed{2\lambda+\mu=0}$$

scelgo $\boxed{\lambda=1}$ e ottengo $\mu=-2$

$$\boxed{\pi: 2\cdot x - 2\cdot y + z - 81 = 0}$$

$\hookrightarrow \vec{u}_\pi(2,-2,1) \perp \pi$

$$d(O,\pi) = \frac{|2\cdot 0 - 2\cdot 0 + 0 - 81|}{\sqrt{2^2+(-2)^2+1^2}} = \frac{81}{3} = 27$$



$\|\vec{u}_\pi\| = 3$

$\|\vec{v}\| = 27$

$$\vec{v} = 9\,\vec{u}_\pi = 9\cdot(2,-2,1) = (18,-18,9)$$

$$-\vec{v} = (-18,+18,-9)$$

$\vec{v}$ e $-\vec{v}$ sono i due vettori che hanno la direzione della retta di minima distanza e lunghezza uguale alla distanza fra le 2 rette.