3 Maggio 2021 ore 15:00 – 18:00

Titolo nota 03/05/2021

$$A = \begin{bmatrix} \boxed{0} & (t^2-16) & t\cdot(t-4) & \boxed{0} \\ \boxed{0} & (t^2-4t) & (t-4)(t+4) & \boxed{0} \end{bmatrix}_{2\times 4}$$

rg A con t parametro reale

$$A' = \begin{bmatrix} (t-4)(t+4) & t\cdot(t-4) \\ t\cdot(t-4) & (t-4)(t+4) \end{bmatrix}_{2\times 2}$$

$$\text{rg } A' = 2 = \max \iff \det A' \neq 0$$

$$\det A' = (t-4)^2\cdot(t+4)^2 - t^2\cdot(t-4)^2 =$$

$$= (t-4)^2\cdot\left[(t+4)^2 - t^2\right] = (t-4)^2\cdot\left[\cancel{t^2} + 16 + 8t - \cancel{t^2}\right] =$$

$$= (t-4)^2\cdot 8\cdot(2+t)$$

$$\det A' = 8\cdot(2+t)\cdot(t-4)^2$$

$$\det A' = 0 \iff t=-2 \text{ oppure } t=4$$

(1) $\forall t \in \mathbb{R} - \{-2, 4\}$ rg $A' = 2$

(2) $t=-2$ $\quad A' = \begin{bmatrix} -12 & +12 \\ +12 & -12 \end{bmatrix} \begin{matrix} R_1 \\ R_2 \end{matrix}$ $\quad R_1 = -R_2$

$t = -2 \quad rg\, A' = 1$

(3) $t = 4 \qquad A' = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \; ; \; rg\, A' = 0$

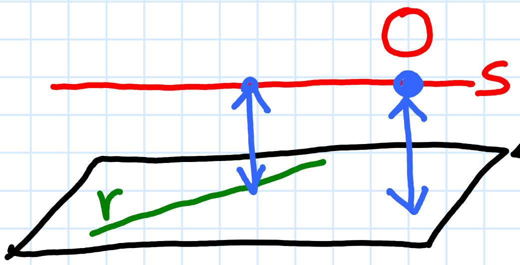
FINE

$$r : 2x - 3y + 12 = y + z = 0$$

$$s : \underbrace{x + 7z}_{} = \underbrace{2x - y + 2z}_{} = 0 \qquad O \in s$$

$d(r,s) = ?$

parametri direttori retta minima distanza?



piano α : $\alpha \in \mathcal{F}(r)$ et $\alpha // s$

$$\mathcal{F}(r) : \quad \lambda \cdot (2x - 3y + 12) + \mu \cdot (y + z) = 0$$

$$\alpha : \underbrace{(2\lambda)}_{a} \cdot x + \underbrace{(\mu - 3\lambda)}_{b} \cdot y + \underbrace{\mu}_{c} \cdot z + \underbrace{12\lambda}_{d} = 0$$

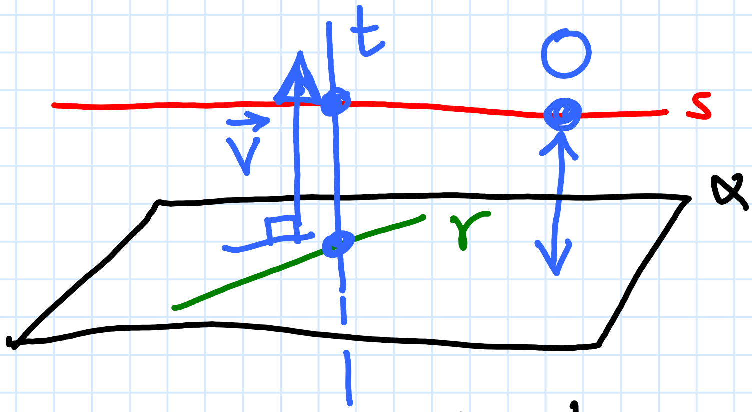
$$\alpha // s \overset{\text{teorema}}{\Longleftrightarrow} 0 = \det \begin{bmatrix} 2\lambda & (\mu - 3\lambda) & \mu \\ 1 & 0 & 7 \\ 2 & -1 & 2 \end{bmatrix} \begin{matrix} 2\lambda & (\mu - 3\lambda) \\ 1 & 0 \\ 2 & -1 \end{matrix}$$

$$14(\mu - 3\lambda) - \mu + 14\lambda - 2(\mu - 3\lambda) =$$

$$= 12(\mu - 3\lambda) - \mu + 14\lambda = 11\mu - 22\lambda = 0$$

$$\boxed{\mu - 2\lambda = 0}$$ una sua autosoluzione $(\lambda, \mu) = (1, 2)$

$$\boxed{\alpha : 2x - y + 2z + 12 = 0}$$



$\vec{v} = (2, -1, 2)$ parametri direttori della retta di minima distanza

$$d(r,s) = d(O,\alpha) \overset{\text{formula}}{=} \frac{|2\cdot 0 - 0 + 2\cdot 0 + 12|}{\sqrt{2^2 + (-1)^2 + 2^2}} = \frac{12}{3}$$

$$\boxed{d(r,s) = 4}$$

FINE

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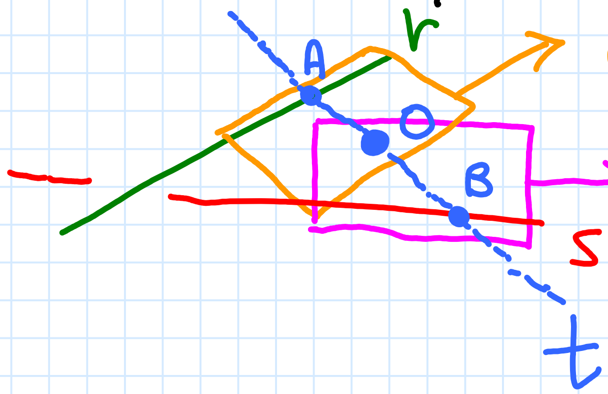
parametri direttori retta passante per l'origine $O(0,0,0)$ che si appoggia alla retta

$r : z = 4x - y + 7z + 11 = 0$ $\quad r: \begin{cases} \boxed{z=0}\ \alpha \\ 4x-y+7z+11=0 \end{cases}$

e alla retta

$s : 11x + y - 3z - 14 = \boxed{x + 3y = 0}\ \beta$



$\alpha : \alpha \in \mathcal{F}(r)$ et $O \in \alpha$

$\beta : \beta \in \mathcal{F}(s)$ et $O \in \beta$

$t = \alpha \cap \beta$

$t : \begin{cases} z = 0 \\ x + 3y = 0 \end{cases}$

$$\begin{bmatrix} 0 & 0 & 1 \\ 1 & 3 & 0 \end{bmatrix} \quad \begin{aligned} l &= -3 = -3 \\ m &= -(-1) = +1 \\ n &= 0 = 0 \end{aligned}$$

$(l, m, n) = (-3, +1, 0)$

FINE

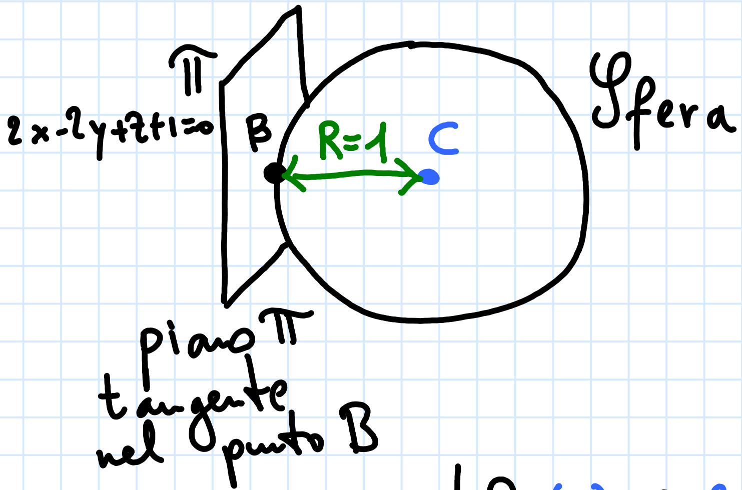
Sfere di raggio 1 aventi il centro sulla retta r passante per $A(-3,2,5)$ e parallela all'asse z e tangenti al piano $\pi : 2x - 2y + z + 1 = 0$

$r \parallel \text{asse } z \parallel \vec{k} = (0,0,1)$ $\quad (l_r, m_r, n_r) = (0,0,1)$

$A(-3,2,5) \in r$

$$r : \begin{cases} x = 0t + (-3) \\ y = 0t + 2 \\ z = 1t + 5 \end{cases} \qquad r : \begin{cases} x = -3 \\ y = 2 \\ z = t + 5 \end{cases}$$

$$C_{sfera} \in r \Rightarrow C(-3, 2, t+5)$$



$$d(C, \pi) = \text{raggio}$$

$$C(-3, 2, t+5)$$

$$d(C, \pi) = \frac{|2\cdot(-3) - 2\cdot 2 + (t+5) + 1|}{\sqrt{2^2 + (-2)^2 + 1^2}} = 1$$

$$\frac{|\cancel{-6} - 4 + t + \cancel{5} + \cancel{1}|}{\sqrt{9}} = 1$$

$$|t - 4| = 1 \cdot 3 = 3$$

$$|t - 4| = 3$$

$$t - 4 = \pm 3 \qquad t_1 = 4 + 3 = 7$$

$$t = 4 \pm 3$$

$$t_2 = 4 - 3 = 1$$

$$C(-3, 2, t+5) \rightarrow \boxed{\begin{matrix} C_1(-3, 2, 12) \\ C_2(-3, 2, 6) \end{matrix}}$$ i due centri richiesti