Lunedì 17 Maggio ore 15:00–18:00

Titolo nota 17/05/2021

piano α passante per $A(9, \sqrt{5}, 23)$
parallelo alla retta $r: x+3y-7 = 2y+z+9=0$
e perpendicolare al piano $\pi: 3x+2z+13=0$

$a'=3 \quad b'=0 \quad c'=2$

$$r: \begin{bmatrix} 1 & 3 & 0 \\ 0 & 2 & 1 \end{bmatrix} \quad \begin{matrix} l = 3 \\ m = -(1) = -1 \\ n = 2 \end{matrix}$$

$$\alpha \in S(A) : a(x-9)+b(y-\sqrt{5})+c(z-23)=0$$

$a=? \quad b=? \quad c=?$

$$\alpha \parallel r \iff a\cdot l + b\cdot m + c\cdot n = 0$$

$$\boxed{3a - b + 2c = 0}$$

$$\alpha \perp \pi \iff a\cdot a' + b\cdot b' + c\cdot c' = 0$$

$$\boxed{3a + 2c = 0}$$

$$\begin{cases} 3a - b + 2c = 0 \\ 3a + 2c = 0 \end{cases} \qquad \begin{cases} b = 0 \\ 3a + 2c = 0 \end{cases}$$

quindi, una (delle infinite) autosoluzione
è $(a,b,c) = (2, 0, -3)$

$$2\cdot(x-9) + 0\cdot(y-\sqrt{5}) - 3\cdot(z-23) = 0$$

$$2x - 18 - 3z + 6g = 0$$

$$\alpha: 2x - 3z + 51 = 0$$

Scrivere piano α perpendicolare al piano $\pi: 9x + 3y + 25z + 5 = 0$, parallelo alla retta $r: x - 3y + 13 = z - 4y + 17 = 0$ passante per il punto $A(9, 3, -1)$

$$r: \begin{bmatrix} 1 & -3 & 0 \\ 0 & -4 & 1 \end{bmatrix} \begin{array}{l} l = -3 \\ m = -(1) = -1 \\ n = -4 \end{array}$$

$\vec{V}_r \parallel r \qquad \vec{V}_r = (-3, -1, -4)$

$-\vec{V}_r \parallel r \qquad -\vec{V}_r = (\underset{l}{3}, \underset{m}{1}, \underset{n}{4})$

$\pi: 9x + 3y + 25z + 5 = 0$

$a' = 9 \qquad b' = 3 \qquad c' = 25$

$$\alpha: \underset{?}{a} \cdot (x - 9) + \underset{?}{b} \cdot (y - 3) + \underset{?}{c} \cdot [z - (-1)] = 0$$

$\alpha \parallel r \iff a \cdot l + b \cdot m + c \cdot n = 0$

$$\boxed{3 \cdot a + b + 4c = 0}$$

$\alpha \perp \pi \iff a \cdot a' + b \cdot b' + c \cdot c' = 0$

$$\boxed{9a+3b+25c=0}$$

$$\begin{cases} 3a+b+4c=0 \\ 9a+3b+25c=0 \end{cases} \rightarrow \begin{cases} b=-3a-4c \\ 9a+3(-3a-4c)+25c=0 \end{cases}$$

$$\begin{cases} b=-3a-4c \\ \cancel{9a}-\cancel{9a}-12c+25c=0 \end{cases} \quad \begin{cases} b=-3a-4c \\ 13c=0 \end{cases} \quad \begin{cases} c=0 \\ b=-3a \end{cases}$$

$$(a,b,c)=(1,-3,0)$$

$$1\cdot(x-9)-3(y-3)+0\cdot(z-(-1))=0$$

$$\boxed{\alpha: x-3y=0}$$

$A(-3,2,5)$; asse z;

$r \parallel$ asse z e r passante per A

Sulla retta r trovare i punti che hanno distanza 1 dal piano

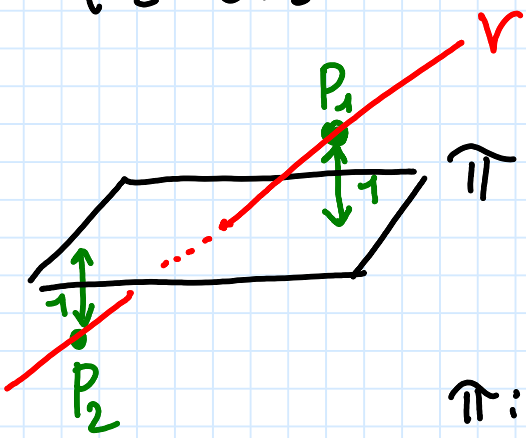
$\pi: 2x-2y+z-5=0.$

$$r \parallel \text{asse } z \parallel \vec{k} = (0,0,1) \Rightarrow r \parallel \vec{k} = (\underset{\ell}{0}, \underset{m}{0}, \underset{n}{1})$$

$$A(\underset{x_0}{-3}, \underset{y_0}{2}, \underset{z_0}{5}) \in r$$

$$r: \begin{cases} x = \ell \cdot t + x_0 \\ y = m \cdot t + y_0 \\ z = n \cdot t + z_0 \end{cases}$$

$$r: \begin{cases} x = -3 \\ y = 2 \\ z = t + 5 \end{cases} \qquad P \in r \qquad P(-3, 2, \underset{\text{varia}}{t+5})$$



$$\boxed{d(P, \pi) = 1}$$

$\pi: 2x - 2y + z - 5 = 0$

$P(-3, 2, t+5)$

$$d(P, \pi) = \frac{|2(-3) - 2(2) + (t+5) - 5|}{\sqrt{2^2 + (-2)^2 + 1^2}} = 1$$

$$\frac{|-6 - 4 + t + \cancel{5} - \cancel{5}|}{\sqrt{9}} = 1$$

$$|t - 10| = 3 \;; \quad t - 10 = \pm 3 \;; \quad t = 10 \pm 3 \begin{cases} \nearrow t_1 = 13 \\ \searrow t_2 = 7 \end{cases}$$

$t_1 = 13 \rightarrow P_1(-3, 2, 18)$

$t_2 = 7 \rightarrow P_2(-3, 2, 12)$

$r: 3x+4y+2z+4 = x+2 = 0$ → NON COMPLANARI (sghembe)

$\boxed{s: 2y+z = x+2y+z = 0}$ → COMPLANARI

$O(0,0,0) \in s$

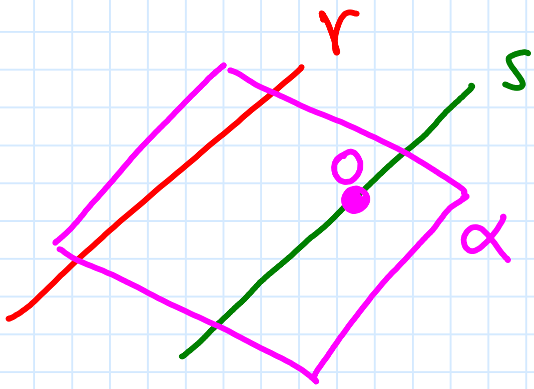
Se sono sghembe, calcolare la loro distanza.
Altrimenti, trovare il piano che le contiene.

$$r: \begin{bmatrix} 3 & 4 & 2 \\ 1 & 0 & 0 \end{bmatrix} \begin{cases} l = 0 \\ m = -(-2) = +2 \\ n = -4 \end{cases} \quad (0,2,-4)$$

$$s: \begin{bmatrix} 0 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} = \begin{cases} l = 0 \\ m = -(-1) = +1 \\ n = -2 \end{cases} \quad (0,1,-2)$$

$$(0,2,-4) = 2\cdot(0,1,-2)$$

r e s sono <u>PARALLELE</u>



α contenente r e s

$\alpha \in \mathcal{F}(r)$ et $O(0,0,0) \in \alpha$

$r: 3x+4y+2z+4 = x+2 = 0$

$\mathcal{F}(r): \lambda(3x+4y+2z+4) + \mu(x+2) = 0$

$O(0,0,0) \in r \Rightarrow \lambda(3\cdot 0+4\cdot 0+2\cdot 0+4)+\mu(0+2)=0$

$\Rightarrow 4\lambda+2\mu=0 \Rightarrow \underbrace{2\lambda+\mu}=0 \Rightarrow$

scelgo $(\lambda,\mu)=(1,-2)$

$\alpha: 1\cdot(3x+4y+2z+4)-2\cdot(x+2)=0$

$$\boxed{\alpha: x+4y+2z=0}$$

$r: 3x+4y+2z+4 = x+2 = 0$

$s: 2y+z = x+2y+z = 0$

$$C = \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 3 & 4 & 2 & 4 \\ 1 & 0 & 0 & 2 \end{bmatrix} \quad A \qquad rg\, C = ?$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & -2 & -1 & 4 \\ 0 & -2 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Longrightarrow rg\, C < 3 \Rightarrow \text{complanari}$$

$\rightarrow \mathrm{rg}\, A = 2 \Rightarrow$ parallele

Sia A il punto di intersezione tra il piano $3x+3y+7z-14=0$ e l'asse z. Sia r la retta per l'origine e perpendicolare al piano $11x+2y+3=0$. Sia h la distanza di A da r.
Siano B e C i punti di r aventi distanza $4h$ da A. Calcolare l'area del triangolo $A\hat{B}C$.

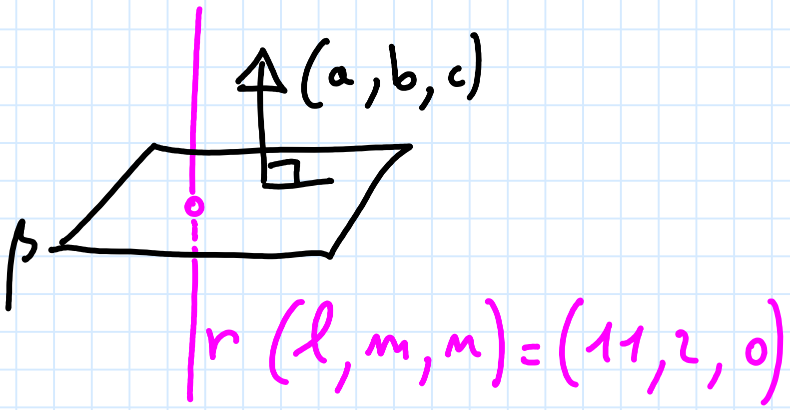
$\alpha : 3x+3y+7z-14=0$ asse z: $x=y=0$

$\alpha \cap$ asse z $\begin{cases} 3x+3y+7z-14=0 \\ x=0 \\ y=0 \end{cases} \Rightarrow \boxed{A(0,0,2)}$

$\beta : \underset{a=11}{11}x + \underset{b=2}{2}y + 3 = 0 \quad c=0$

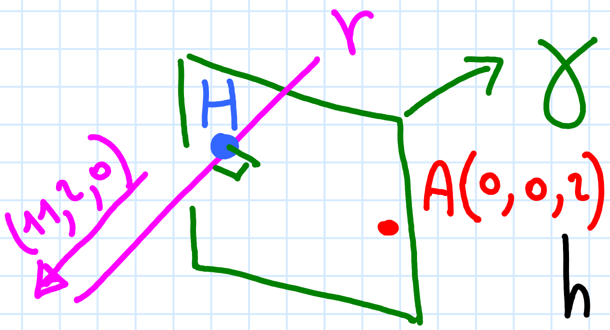
$O(0,0,0)$

$(11,2,0) \perp \beta$



$$r:\begin{cases} x = 11t + 0 \\ y = 2t + 0 \\ z = 0\cdot t + 0 \end{cases} \qquad r:\begin{cases} x = 11\cdot t \\ y = 2\cdot t \\ z = 0 \end{cases}$$

h = distanza del punto A dalla retta r
↳ NELLO SPAZIO



γ passante per A e perpendicolare alla retta r

$$h = d(A,H)$$

$$\gamma \perp r \qquad 11\cdot(x-0) + 2\cdot(y-0) + 0\cdot(z-2) = 0$$

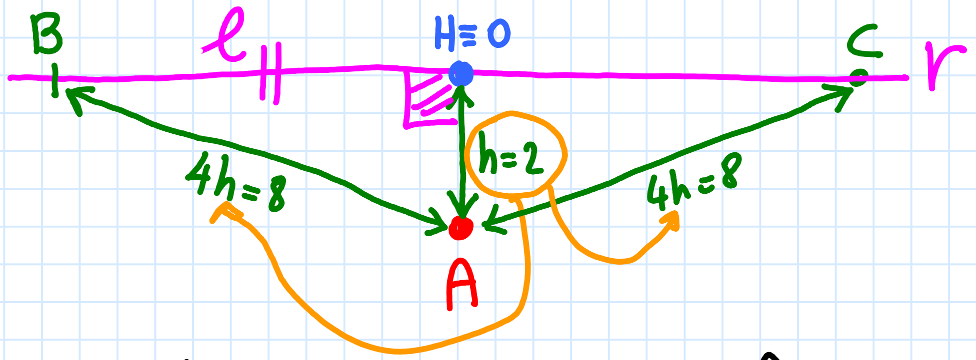
$$\boxed{\gamma : 11x + 2y = 0}$$

$$H:\begin{cases} 11x + 2y = 0 \\ x = 11t \\ y = 2t \\ z = 0 \end{cases} \quad \begin{cases} z = 0 \\ 121t + 4t = 0 \\ x = 11t \\ y = 2t \end{cases} \quad \begin{cases} 125t = 0 \Rightarrow t = 0 \\ z = 0 \\ x = 11\cdot 0 = 0 \\ y = 2\cdot 0 = 0 \end{cases}$$

$$H = O(0,0,0) \qquad A(0,0,2)$$

$$h = d(A,H) = 2$$

$$\boxed{h = 2}$$



area del triangolo $A\hat{B}C$ = doppio $A\hat{H}B$

$$\ell = \sqrt{64-4} = \sqrt{60} = 2\sqrt{15}$$

area $A\hat{B}C = 2 \cdot \left(\frac{\cancel{2} \cdot 2 \cdot \sqrt{15}}{\cancel{2}} \right) = 4\sqrt{15}$