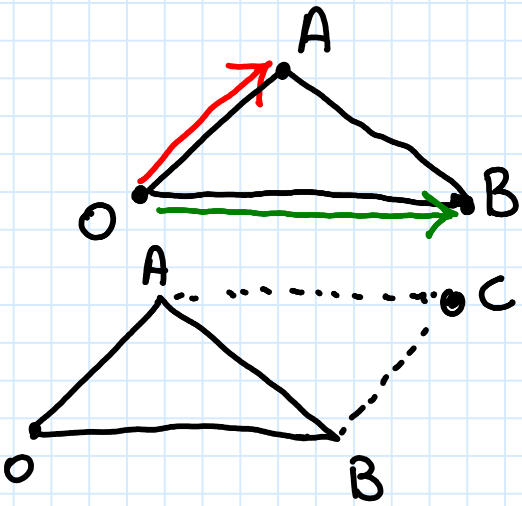
Lunedì 24 maggio ore 14:45 – 15:45

Titolo nota 24/05/2021

Trovare per quali valori del parametro reale t l'area del triangolo di vertici $O(0,0,0)$, $A(t,-t,t)$ e $B(0,-11,11)$ vale 154.

$[\overrightarrow{OA}] = (t, -t, t)$

$[\overrightarrow{OB}] = (0, -11, 11)$



area $O\hat{A}B = \frac{1}{2}$ area $OACB$

area $OACB = \| [\overrightarrow{OA}] \wedge [\overrightarrow{OB}] \|$

$$\vec{v} = [\overrightarrow{OA}] \wedge [\overrightarrow{OB}] = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ t & -t & t \\ 0 & -11 & 11 \end{bmatrix} = 0\vec{i} - 11t\vec{j} - 11t\vec{k}$$

$$\vec{v} = (0, -11t, -11t)$$

$$\|\vec{v}\| = \sqrt{0^2 + (-11t)^2 + (-11t)^2} = \sqrt{2 \cdot 121 \cdot t^2} = 11|t|\sqrt{2}$$

area $OACB = 11|t|\sqrt{2}$

area $O\overrightarrow{AB} = \frac{1}{2} \cdot 11 \cdot |t| \cdot \sqrt{2} = 154$

$$11\cdot\sqrt{2}\cdot|t| = 308$$

$$\sqrt{2}\cdot|t| = 28$$

$$|t| = \frac{28}{\sqrt{2}} = 14\sqrt{2}$$

$$\boxed{t = \pm 14\cdot\sqrt{2}}$$ risultato finale

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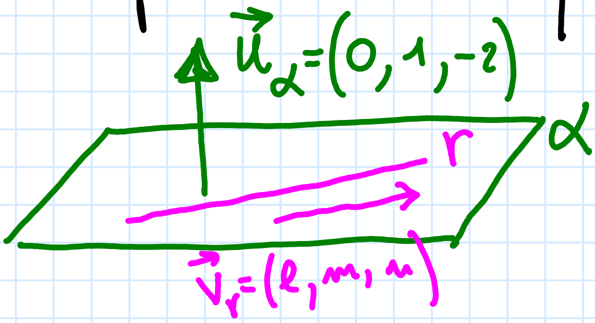
parametri direttori rette che si trova sul piano di eq. $\alpha: y-2z+6=0$ e formano un angolo di $\frac{\pi}{6}$ rad. con il piano di equazione $\beta: x+11=0$.



$$\vec{v}_r \perp \vec{u}_\alpha$$

$$\vec{v}_r \bullet \vec{u}_\alpha = 0$$

$$l\cdot 0 + m\cdot 1 + n\cdot(-2) = 0$$

$$\boxed{m = 2n}$$

$$\vec{v}_r = (l, 2n, n) \quad ; \quad \beta: x+11=0$$

$$\hookrightarrow (a,b,c) = (1,0,0)$$

angolo tra $\underbrace{r \text{ e } \beta}_{\text{retta-piano}}$ sia di $\frac{\pi}{6}$ rad $= 30°$

$$\sin\left(\frac{\pi}{6}\text{ rad}\right)=\frac{|al+bm+cn|}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{l^2+m^2+n^2}}$$

$$\frac{1}{2}=\frac{|l+0+0|}{\sqrt{1^2+0^2+0^2}\cdot\sqrt{l^2+4n^2+n^2}}$$

$$\sqrt{l^2+5n^2}=2|l|$$

$$l^2+5n^2=4l^2$$

$$5n^2=3l^2 \qquad n^2=\frac{3}{5}l^2$$

scelgo $\boxed{\text{a piacere}}$ $l=5 \Rightarrow n^2=15 \Rightarrow n=\pm\sqrt{15}$

$$\vec{V}_1=(5,\ 2\sqrt{15},\ \sqrt{15}) \qquad \vec{V}_2=(5,\ -2\sqrt{15},\ -\sqrt{15})$$

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s la retta $\perp$ alla retta $r: y+z-3=x-4z+19=0$ parallela al piano $\pi: 4x-y+5=0$ e passante per $A(11,8,0)$.

Trovare coordinate del punto B di intersezione tra la retta s e il piano Yz.

$$r:\begin{bmatrix}0&1&1\\1&0&-4\end{bmatrix} \begin{array}{l} \nearrow l=-4 \\ \rightarrow m=-(-1)=+1 \\ \searrow n=-1 \end{array} \qquad (-4,1,-1)$$

$s \perp r \iff (\ell, m, n) \bullet (-4, 1, -1) = 0 \iff$

$\hookrightarrow (\ell, m, n) \iff \boxed{-4\ell + m - n = 0}$

$\pi : 4x - y + 5 = 0 \Rightarrow (a, b, c) = (4, -1, 0)$

$s \parallel \pi \iff (\ell, m, n) \bullet (4, -1, 0) = 0 \iff$

$\iff \boxed{4\ell - m + 0 = 0}$

$s \perp r$ et $s \parallel \pi \iff \begin{cases} -4\ell + m - n = 0 \\ 4\ell - m = 0 \Rightarrow m = 4\ell \end{cases}$

$-4\ell + 4\ell - n = 0 \Rightarrow n = 0$

$(\ell, m, n) = (\ell, 4\ell, 0)$

scelgo $\ell = 1$ e ottengo $(1, 4, 0)$

$A(11, 8, 0)$

$s : \begin{cases} x = 1 \cdot t + 11 \\ y = 4 \cdot t + 8 \\ z = 0 \cdot t + 0 \end{cases}$

$s : \begin{cases} x = t + 11 \\ y = 4t + 8 \\ z = 0 \end{cases}$

$\{B\} = s \cap$ piano yz

equazione $x = 0$

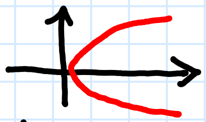
$s \cap$ piano yz : $\begin{cases} x = t + 11 \\ y = 4t + 8 \\ z = 0 \\ x = 0 \end{cases} \longrightarrow t = -11 \rightarrow y = -36$

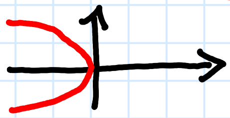
$B(0, -36, 0)$ è il punto richiesto

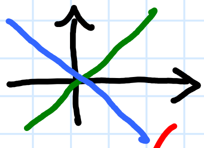
$y^2=0 \rightarrow y \cdot y = 0 \rightarrow$ la retta $y=0$ 2 volte — 2 rette reali coincidenti

$y^2=1 \rightarrow y^2-1=0 \rightarrow (y-1)\cdot(y+1)=0$ → retta $y-1=0$, → retta $y+1=0$ — 2 rette reali parallele in senso stretto

$y^2=-1 \rightarrow y^2 \geq 0$ et $-1<0 \rightarrow$ senza punti reali

$y^2=x \rightarrow$ 

$y^2=-x \rightarrow$ 

$y^2=x^2 \rightarrow y^2-x^2=0 \rightarrow (y-x)\cdot(y+x)=0 \rightarrow$  2 rette reali incidenti (non parallele)

$y^2=-x^2 \rightarrow \underbrace{y^2+x^2}_{\geq 0}=0$ solo $(x,y)=(0,0)$

conica avente l'origine come unico punto reale

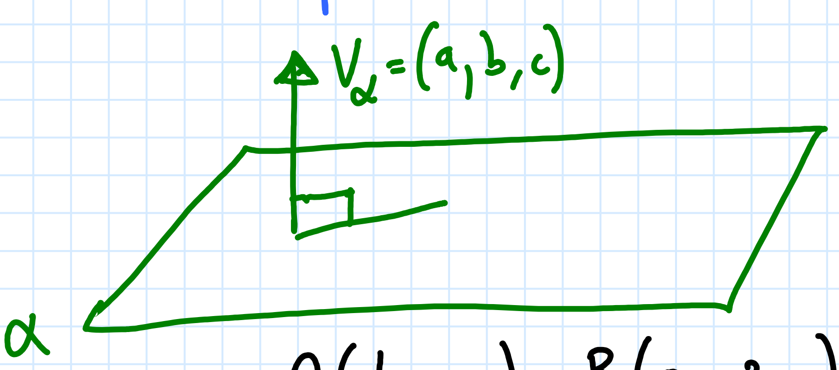
$A(t,0,0)$, $B(0,-2,0)$, $C(0,0,\sqrt{23})$. Trovare valori parametro reale t per i quali il piano α passante per A, B e C forma col piano$_\beta$ Yz un angolo di $\frac{2}{3}\pi$ radianti.

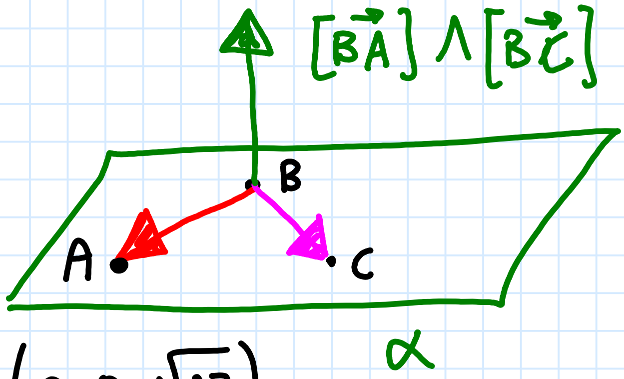
Problema $\underbrace{\text{angolo}}_{\theta}$ tra 2 piani (α e β)

$$\cos\theta = \pm \frac{a\cdot a' + b\cdot b' + c\cdot c'}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{(a')^2+(b')^2+(c')^2}}$$

$\theta = \frac{2}{3}\pi$ rad. $\quad \vec{V}_\alpha = (a, b, c) \quad \vec{V}_\beta = (1, 0, 0)$

β = piano yz : $x = 0$





$A(t,0,0)$; $B(0,-2,0)$; $(0,0,\sqrt{23})$

$$[\vec{BA}] = (t-0,\ 0-(-2),\ 0-0) = (t, 2, 0)$$

$$[\vec{BC}] = (0-0,\ 0-(-2),\ \sqrt{23}-0) = (0, 2, \sqrt{23})$$

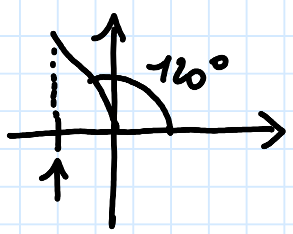
$$[\vec{BA}] \wedge [\vec{BC}] = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ t & 2 & 0 \\ 0 & 2 & \sqrt{23} \end{bmatrix} =$$

$$= 2\cdot\sqrt{23}\cdot\vec{i} - t\cdot\sqrt{23}\cdot\vec{j} + 2t\cdot\vec{k}$$

$\vec{V}_\alpha \perp \alpha$; $\vec{V}_\alpha = (a,b,c) = (2\cdot\sqrt{23},\ -t\cdot\sqrt{23},\ 2t)$

$\vec{V}_\beta \perp \beta$; $\vec{V}_\beta = (a', b', c') = (1, 0, 0)$

$\theta = \frac{2}{3}\pi$ rad $= 120°$



$\cos\theta = \ ? = -\frac{1}{2}$

$$-\frac{1}{2} = \pm \frac{2\sqrt{23}}{\sqrt{4\cdot 23 + 23t^2 + 4t^2}\cdot\sqrt{1^2+0^2+0^2}}$$

$$-\sqrt{4\cdot 23 + 27t^2} = \pm 4\sqrt{23}$$

$$4\cdot 23 + 27\cdot t^2 = 16\cdot 23$$

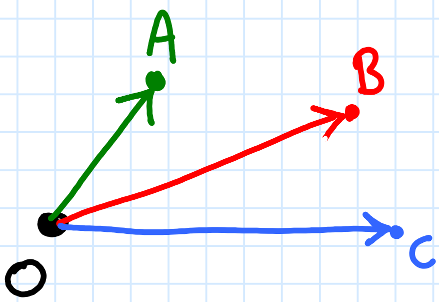
$$27\cdot t^2 = 16\cdot 23 - 4\cdot 23 = 12\cdot 23$$

$$9\cdot t^2 = 4\cdot 23$$

$$t^2 = \frac{4}{9}\cdot 23 \Longrightarrow \boxed{t = \pm\frac{2}{3}\cdot\sqrt{23}}$$

questi sono i valori richiesti

$O(0,0,0)$, $A(0,7,0)$, $B(-3,4,1)$, $C(t^2-2t, -5, 2-t)$



4 punti complanari

$\Updownarrow$

3 vettori complanari

$[\vec{OA}] = (0, 7, 0)$

$[\vec{OB}] = (-3, 4, 1)$

$[\vec{OC}] = (t^2-2t, -5, 2-t)$

$$\operatorname{rg}\begin{bmatrix} 0 & 7 & 0 \\ -3 & 4 & 1 \\ (t^2-2t) & -5 & (2-t) \end{bmatrix} \leq 2 \iff \det\begin{bmatrix} 0 & 7 & 0 \\ -3 & 4 & 1 \\ (t^2-2t) & -5 & (2-t) \end{bmatrix} = 0$$

$$-7 \cdot \det \begin{bmatrix} -3 & 1 \\ (t^2-2t) & (2-t) \end{bmatrix} = 0$$

$$(-3)(2-t) - (t^2-2t) \cdot 1 = 0$$

$$3 \cdot (t-2) - t \cdot (t-2) = 0$$

$$(t-2) \cdot (3-t) = 0$$

$$\boxed{\begin{array}{l} t_1 = +2 \\ t_2 = +3 \end{array}}$$

i valori richiesti