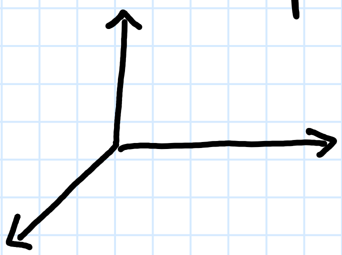
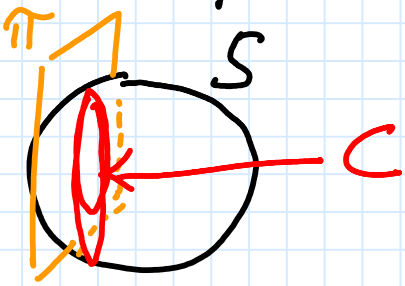
Lunedì 31 Maggio ore 15:00 – 18:00

Titolo nota 31/05/2021

C circonferenza ottenuta intersecando la sfera $S: x^2+y^2+z^2+8y=0$ con il piano $\pi: y+3=0$. Scrivere l'equazione del cilindro avente C come direttrice e generatrici parallele asse y





$y=-3$ costante

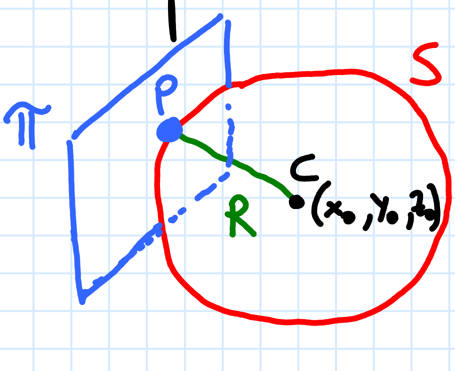
$x^2+(-3)^2+z^2+8(-3)=0$ $\qquad +9-24=-15$

$\boxed{x^2+z^2-15=0}$ circonferenza nel piano $y=-3$

$x^2+z^2-15=0$ eq. cilindro

Sfera centro $C(2, 13, -14)$ ed è tangente al piano $\pi: 3y+8z=0$.

$x_0 \quad y_0 \quad z_0$



$S \cap \pi = \{P\}$

$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2$

$R = ?$

S tangente a π $\Rightarrow$ $R = d(C,\pi)$

↳ distanza punto-piano

$C(2, 13, -14)$

$\pi: 3y + 8z = 0$

$$R = d(C,\pi) = \frac{|3\cdot 13 + 8\cdot(-14)|}{\sqrt{0^2+3^2+8^2}} = \frac{|-73|}{\sqrt{73}}$$

$$R = \sqrt{73} \Rightarrow R^2 = +73$$

$$\boxed{(x-2)^2 + (y-13)^2 + [z-(-14)]^2 = 73}$$

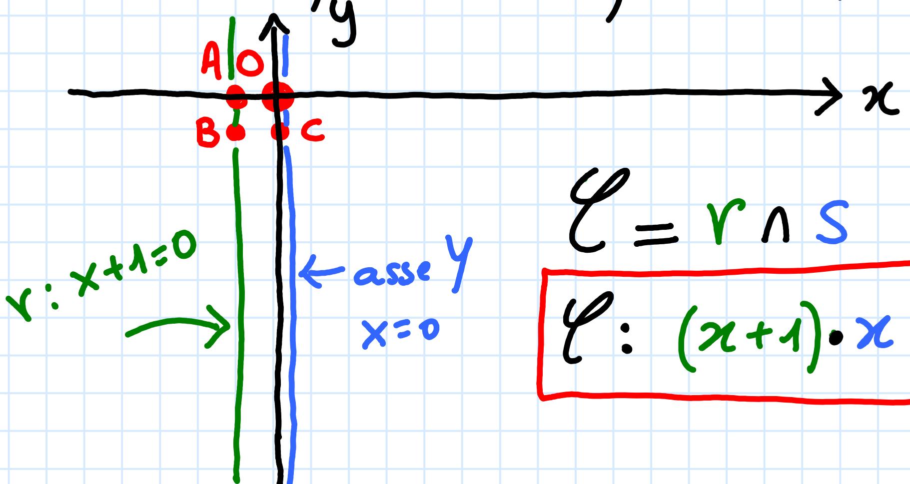
$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

di fatto devo trovare 5 numeri

con 5 punti ottengo un sistema

di 5 eq in 5 incognite. E poi lo risolvo.

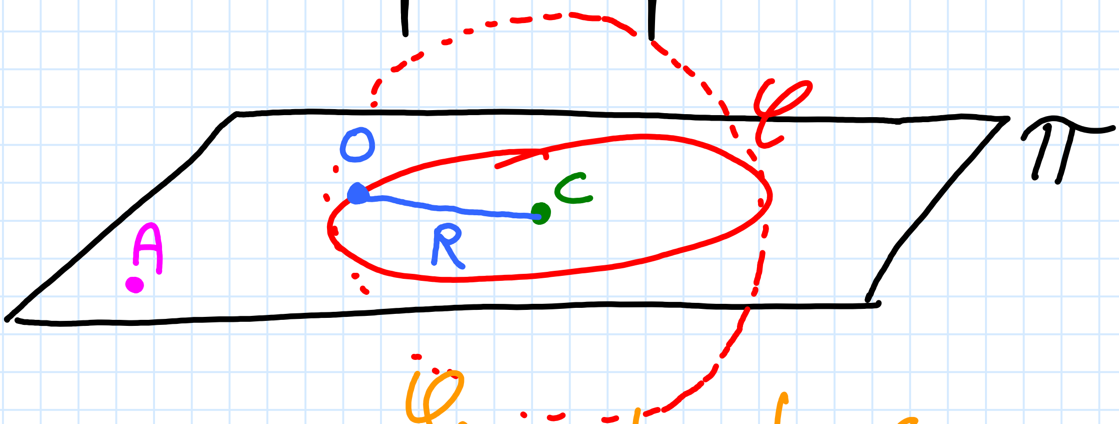
$O(0,0)$, $A(-1,0)$, $B(-1,-1)$, $C(0,-1)$, $D(-1,-12)$



$$\mathcal{C} = r \cap S$$

$$\boxed{\mathcal{C}: (x+1)\cdot x = 0}$$ eq. richiesta

π piano per i punti $O(0,0,0)$, $A(2,0,-3)$ e $C(0,4,0)$. Trovare equazioni circonferenza che giace su π e ha centro in C e passa per O.



$\mathscr{S}$ Sfera di centro C e raggio R

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2$$

$$x_0 \downarrow 0 \qquad y_0 \downarrow 4 \qquad z_0 \downarrow 0$$

$$\to R = ? = d(O,C) = \sqrt{(0-0)^2+(4-0)^2+(0-0)^2} = 4$$

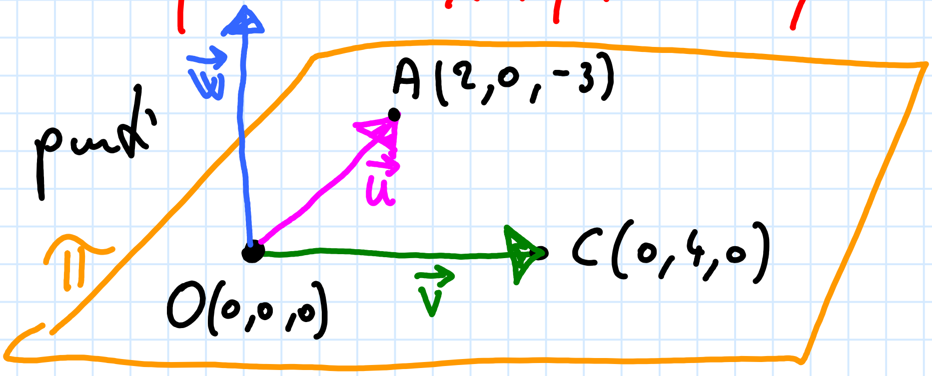
$$x^2+(y-4)^2+z^2=16$$

$$x^2+y^2+\cancel{16}-8y+z^2=\cancel{16}$$

Sfera ha equazione $x^2+y^2+z^2-8y=0$

piano per 3 punti

$\vec{w}=\vec{u}\wedge\vec{v}$



$\pi : a \cdot x + b \cdot y + c \cdot z + 0 = 0$

$\downarrow$? $\quad \downarrow$? $\quad \downarrow$? $\quad \hookrightarrow$ poiché $O \in \pi$

$\vec{V}_\pi = (a, b, c) \perp \pi$

$$\vec{w} = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 0 & -3 \\ 0 & 4 & 0 \end{bmatrix} = \underset{a}{12}\,\vec{i} - \underset{b}{0}\,\vec{j} + \underset{c}{8}\,\vec{k}$$

$$12x + 0y + 8z = 0$$

piano π $\quad 3x + 2z = 0$

$$\boxed{\text{Circonferenza}: \begin{cases} 3x + 2z = 0 \\ x^2 + y^2 + z^2 - 8y = 0 \end{cases}}$$

$r : 3x - 4y - 5 = 0$ direttrice parabola
parabola passante per l'origine e avente i fuochi sull'asse X delle ascisse.
Trovare le coordinate dei fuochi.

DEF. (parabola) luogo dei punti del piano

$$d(P, F) = d(P, \text{direttrice})$$

Hp. Fuoco $\in$ asse X $\qquad F(\alpha, 0)$

$$\alpha \to ?$$

Hp. l'origine O è un punto della parabola

$$d(O,F) = d(O, r) \qquad r: 3x-4y-5=0$$

$$\sqrt{(0-\alpha)^2+(0-0)^2} = \frac{|3\cdot 0-4\cdot 0-5|}{\sqrt{3^2+(-4)^2}}$$

$$\sqrt{\alpha^2} = \frac{|-5|}{5} = 1$$

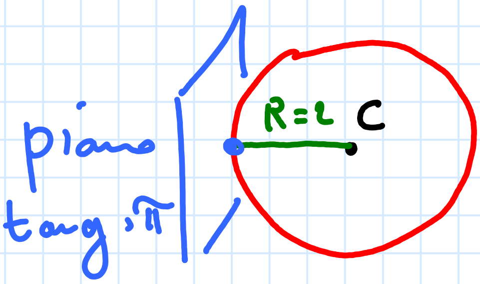
$$|\alpha| = 1 \Rightarrow \alpha = \pm 1 \quad \begin{cases} F_1(1,0) \\ F_2(-1,0) \end{cases}$$

Equazioni sfere raggio 2 tangenti al piano $\pi: x-2y+2z-5=0$ e aventi il centro sulla retta r passante per $A(\underset{x_0}{1}, \underset{y_0}{-2}, \underset{z_0}{5})$ e parallela all'asse X

$$r \parallel \text{asse } X \parallel \vec{i} = (\underset{l}{1}, \underset{m}{0}, \underset{n}{0})$$

$$r: \begin{cases} x = l\cdot t + x_0 \\ y = m\cdot t + y_0 \\ z = n\cdot t + z_0 \end{cases} \qquad r: \begin{cases} x = t+1 \\ y = -2 \\ z = +5 \end{cases}$$

$\underline{Hp.}$ $C \in r \Rightarrow C(t+1, -2, +5)$



$$d(C, \pi) = R = 2$$

$$\pi: x - 2y + 2z - 5 = 0$$

$$\frac{|(t+1) - 2\cdot(-2) + 2\cdot(5) - 5|}{\sqrt{1^2 + (-2)^2 + 2^2}} = 2$$

$$\frac{|t + \cancel{1} + \cancel{4} + 10 - \cancel{5}|}{\sqrt{9}} = 2$$

$$|t + 10| = 6 \Rightarrow t + 10 = \pm 6 \quad \begin{array}{l} \nearrow t_1 = -16 \\ \searrow t_2 = -4 \end{array}$$

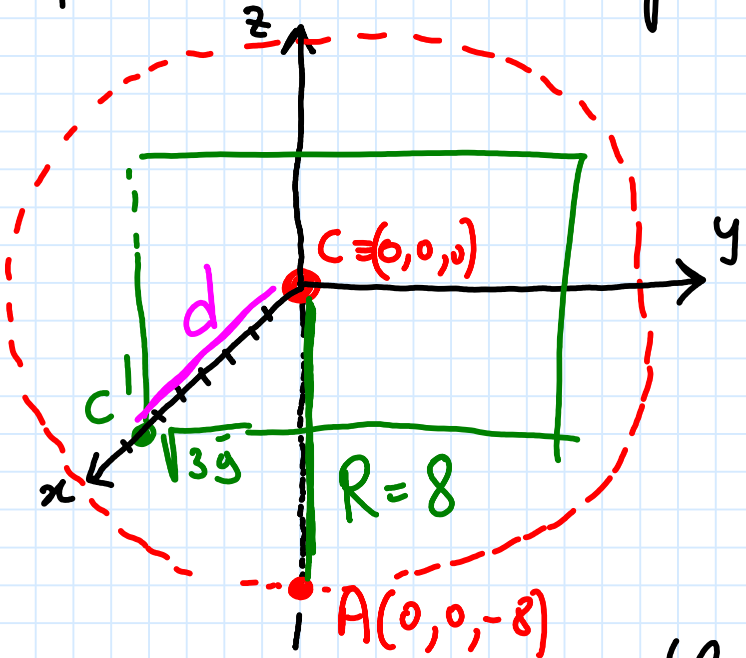
$$C(t+1, -2, +5)$$

$$t_1 = -16 \Rightarrow C_1(-15, -2, +5)$$

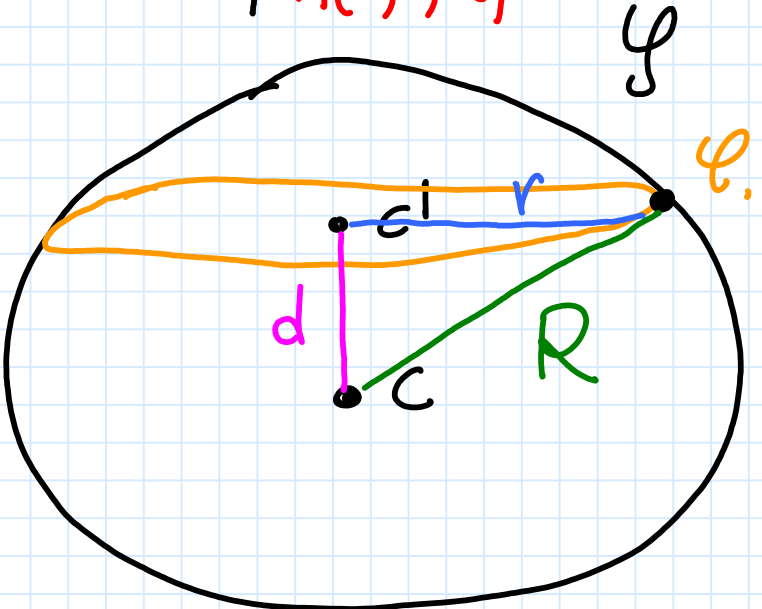
$$t_2 = -4 \Rightarrow C_2(-3, -2, +5)$$

Sfera di centro l'origine e passante per $A(0,0,-8)$. Sia π piano parallelo al piano yz e passante per il punto $B(\sqrt{39}, -\sqrt{20}, \sqrt{7})$.

Trovare il centro e raggio della circonferenza ottenuta intersecando la sfera con il piano.



$C'(\sqrt{39},0,0)$ centro della circonferenza



$r=\sqrt{R^2-d^2}$

$R=8$

$d=\sqrt{39}$

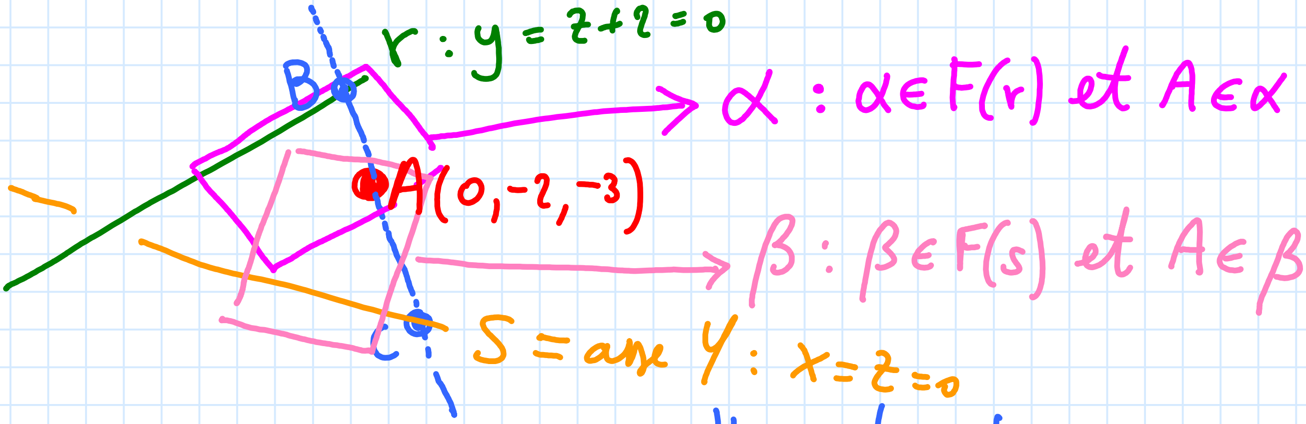
$r=\sqrt{8^2-(\sqrt{39})^2}$

$r=\sqrt{64-39}=\sqrt{25}=5$

$$\boxed{\begin{gathered}C'(\sqrt{39},0,0)\\ r=5\end{gathered}}$$

$A(0,-2,-3)$. Tra le rette passanti

per A trovare (se esistono) quelle che si appoggia all'asse Y e alla retta $r: y = z+2 = 0$.



$t = \alpha \cap \beta$

$\alpha \in F(r): \quad \lambda(y) + \mu(z+2) = 0 \qquad (\lambda, \mu) \neq (0,0)$

$A(0,-2,-3) \in \alpha \Rightarrow \lambda(-2) + \mu(-3+2) = 0 \Rightarrow$

$\Rightarrow -2\lambda + \mu = 0 \Rightarrow$ scelgo $(\lambda, \mu) = (1, 2)$

$$\boxed{\alpha : y + 4z = 0}$$

$\beta \in F(s): \quad \lambda x + \mu z = 0$

$A(0,-2,-3) \in \beta \Rightarrow \lambda \cdot 0 + (-3) \cdot \mu = 0 \Rightarrow$

$\Rightarrow -3\mu = 0 \Rightarrow \mu = 0$

$(\lambda, \mu) \neq (0,0) \Rightarrow$ scelgo $\lambda = 1$

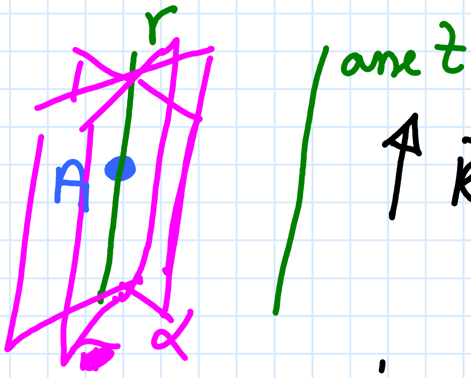
$\beta : x = 0$

$t = \alpha \cap \beta : \begin{cases} y+4z=0 \\ x=0 \end{cases}$

$$\boxed{t: y+4z=x=0}$$ retta richiesta

$r \parallel$ asse z e passante per $A(\sqrt{13}, \sqrt{13}, -2)$

Trovare i piani che contengono r e formano un angolo di $\frac{\pi}{4}$ radianti con asse X



$\vec{k} = (\underset{l}{0}, \underset{m}{0}, \underset{n}{1})$

$A \in r \subseteq$ piano richiesto $\overset{\text{ovvio}}{\Longrightarrow}$ $A \in$ piano richiesto

$\alpha \in F(r) \Rightarrow A \in \alpha \Rightarrow$

$$\Rightarrow \underset{\overset{\downarrow}{?}}{a} \cdot (x - \underset{\overset{\downarrow}{\sqrt{13}}}{x_A}) + \underset{\overset{\downarrow}{?}}{b} \cdot (y - \underset{\overset{\downarrow}{\sqrt{13}}}{y_A}) + \underset{\overset{\downarrow}{?}}{c} \cdot (z - \underset{\overset{\downarrow}{-2}}{z_A}) = 0$$

Quindi, devo trovare (a, b, c)

$\vec{V}_\alpha = (a, b, c) \perp \alpha$

$$r \parallel \alpha \Rightarrow \underset{\downarrow \atop 0}{a\ell} + \underset{\downarrow \atop 0}{bm} + \underset{\downarrow \atop 1}{cn} = 0 \Rightarrow c = 0$$

$$\vec{V}_\alpha = (\underset{\downarrow \atop ?}{a}, \underset{\downarrow \atop ?}{b}, 0) \quad \hookrightarrow \text{zero}$$

il piano α deve formare un angolo di $\frac{\pi}{4}$ radianti con l'asse X

$\hookrightarrow (\ell', m', n') = (1, 0, 0)$

Problema angolo retta - piano

$\downarrow$ $\frac{\pi}{4}$ rad $\quad$ $\downarrow$ asse X $\quad$ $\downarrow$ piano α

$$\sin\left(\underset{\downarrow \atop 45°}{\frac{\pi}{4}} \text{rad}\right) = \frac{|a \cdot 1 + b \cdot 0 + 0 \cdot 0|}{\sqrt{a^2+b^2+0^2} \cdot \sqrt{1^2+0^2+0^2}}$$

$$\frac{\sqrt{2}}{2} = \frac{|a|}{\sqrt{a^2+b^2}}$$

$$\sqrt{2} \cdot \sqrt{a^2+b^2} = 2 \cdot |a|$$

$$2 \cdot (a^2+b^2) = 4 \cdot a^2$$

$$a^2 = b^2 \qquad b = \pm a$$

Scelgo $a=1$, quindi $b=\pm 1$

$\vec{V}_\alpha = (1, \pm 1, 0)$ $\quad A(\sqrt{13}, \sqrt{13}, -2)$

$b=+1 \Rightarrow 1\cdot(x-\sqrt{13})+1\cdot(y-\sqrt{13})+0\cdot(z-(-2))=0$

$\longrightarrow \alpha_1: x+y-2\sqrt{13}=0$

$b=-1 \Rightarrow 1\cdot(x-\sqrt{13})+(-1)\cdot(y-\sqrt{13})+0\cdot(z-(-2))=0$

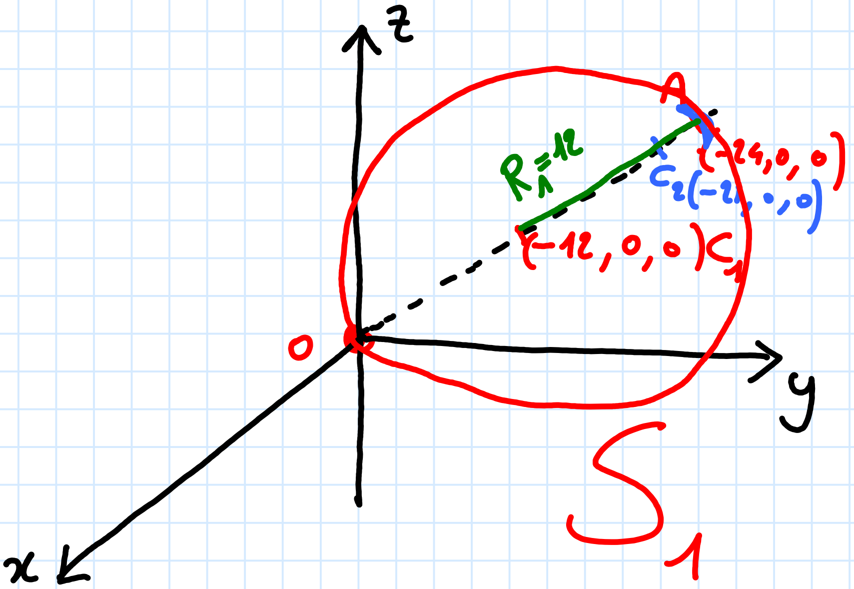
$\longrightarrow \alpha_2: x-y=0$

Sfera S_1 di equazione $x^2+y^2+z^2+24x=0$.
Sfera S_2 tangente internamente a S_1 nel punto A di coordinate $A(-24,0,0)$ e avente il raggio uguale a $\frac{1}{4}$ del raggio di S_1.

$S_1: x^2+y^2+z^2+24x=0$ $\leftarrow$ S_1 passa per l'origine

$A \in S_1$? $(-24)^2+0^2+0^2+24(-24)=0$ SI

$C_1 =$ centro di $S_1 (-12, 0, 0)$



$r = \frac{1}{4} \cdot 12 = 3$

$C_2(-21, 0, 0)$

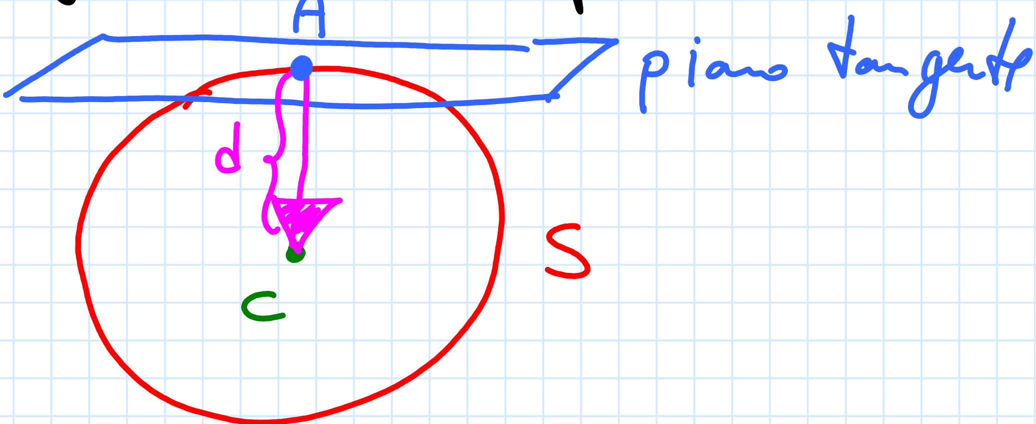
passaggio chiave

$C_2(-21,0,0) \qquad r=3$

$$(x-(-21))^2+(y-0)^2+(z-0)^2=3^2$$

$$(x+21)^2+y^2+z^2-9=0$$

Sfera di equazione $x^2+y^2+z^2+20x-6y-30z+269=0$
piano tangente a S nel punto $A(-14,3,8)$.



$C(-10, 3, 15) \qquad A(\underset{x_0}{-14}, \underset{y_0}{3}, \underset{z_0}{8})$

$$[\overrightarrow{AC}] = (\underset{a}{4}, \underset{b}{0}, \underset{c}{7})$$

$$\pi : a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

$$\pi : 4(x-(-14))+0\cdot(y-3)+7(z-8)=0$$

$$\pi : 4x+56+7\cdot z-56=0$$

piano richiesto $\boxed{\pi : 4x+7z=0}$