Lunedì 7 Giugno ore 15:00 – 18:00

Titolo nota 07/06/2021

$$3x^2 + 2\sqrt{3}\,xy + y^2 - 4\sqrt{3}\,x - 4y = 0$$

$$A = \begin{bmatrix} 3 & \sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix}; \quad p_A(\lambda) = \det(A - \lambda I) = $$

$$= \det \begin{bmatrix} (3-\lambda) & \sqrt{3} \\ \sqrt{3} & (1-\lambda) \end{bmatrix} =$$

$$= \ldots = \lambda^2 - 4\lambda + 0 = \lambda \cdot (\lambda - 4) \begin{cases} \nearrow \lambda_1 = 4 \\ \searrow \lambda_2 = 0 \end{cases}$$

Auto<u>versori</u>

$\lambda_1 = 4$

$$\begin{bmatrix} -1 & \sqrt{3} \\ \sqrt{3} & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \begin{cases} -x + \sqrt{3}\,y = 0 \\ \sqrt{3}\,x - 3y = 0 \end{cases}$$

$x = \sqrt{3}\,y$ auto<u>vettore</u> $(\sqrt{3}y, y) = y(\sqrt{3}, 1)$ $\forall y \neq 0$

scelgo $y = \frac{1}{2}$ e ottengo l'autoversore $\vec{v} = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\lambda_2 = 0$ autoversore $\vec{u} = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$$C = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$

matrice rotazione $\quad$ $30^\circ$ antiorario

$\det C = +1$

$$\begin{bmatrix} -4\sqrt{3} & -4 \end{bmatrix} \cdot \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} (-6-2) & (2\sqrt{3}-2\sqrt{3}) \end{bmatrix} = \begin{bmatrix} -8 & 0 \end{bmatrix}$$

$$\begin{matrix} d & e \end{matrix} \qquad\qquad\qquad \begin{matrix} d' & e' \end{matrix}$$

dopo quella OPPORTUNA rotazione

$$4 \cdot (x')^2 + \cancel{0 \cdot (y')^2} + (-8)x' + \cancel{0 \cdot y'} = 0$$

$$(x')^2 - 2 \cdot x' = 0$$

$$(x'-1)^2 - 1 = 0$$

traslazione $\begin{cases} x'' = x' - 1 \\ y'' = y' \end{cases}$

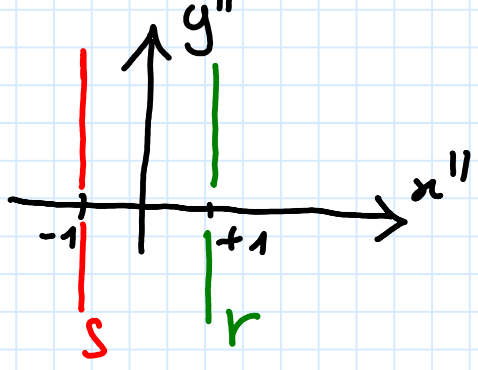
$$(x'')^2 - 1 = 0$$

$$\boxed{(x''-1)\cdot(x''+1) = 0}$$

→ retta $x''-1=0$ r

→ retta $x''+1=0$ s

equazione canonica




Quindi, la conica è l'unione di due rette reali distinte parallele

$$A = \begin{bmatrix} t & t^2 & 0 \\ 0 & 0 & \boxed{3} \\ t & -t & 0 \end{bmatrix}_{3\times 3} \quad \text{rg } A = ?$$

$\neq 0$ non dipende det

$A_{3\times3}$ in generale $\quad 0 \leq \text{rg } A \leq 3$

$\text{rg } A = 0 \iff A$ è la matrice nulla

NEL NOSTRO CASO $\quad 1 \leq \text{rg } A \leq 3$

ricordiamo che, in generale,

$A_{n\times n} \qquad \text{rg } A = n = \max \iff \det A \neq 0$

NEL NOSTRO CASO

$$\text{rg } A = 3 \iff \det \begin{bmatrix} t & t^2 & 0 \\ 0 & 0 & 3 \\ t & -t & 0 \end{bmatrix} \neq 0$$

$$(-3) \cdot \det \begin{bmatrix} t & t^2 \\ t & -t \end{bmatrix} \neq 0 \quad ; \quad -t^2 - t^3 \neq 0$$

$$t^2 \cdot (1+t) \neq 0 \iff \text{rg } A = 3 = \max$$

$$\boxed{\forall t \in \mathbb{R} - \{-1, 0\} \quad \text{rg } A = 3}$$

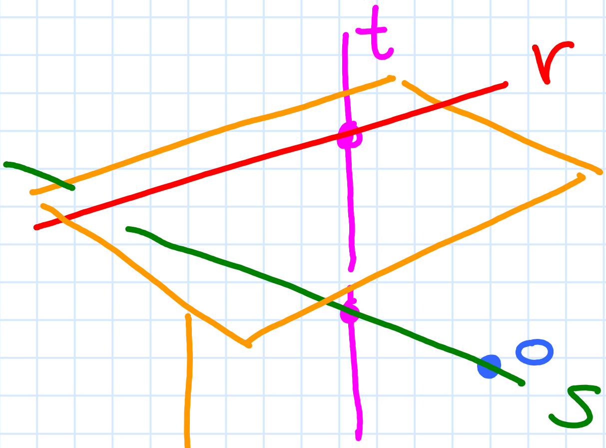
$$\boxed{t = -1}$$

$$\Rightarrow A = \begin{bmatrix} -1 & +1 & 0 \\ 0 & 0 & 3 \\ -1 & +1 & 0 \end{bmatrix} \text{ indip.}$$

$\text{rg } A = 2$

$\boxed{t=0}$

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{rg } A = 1$$

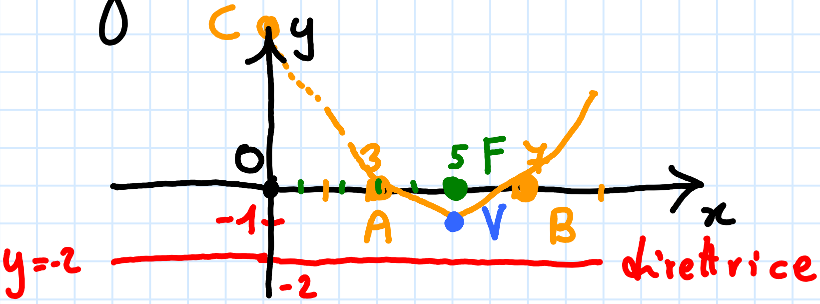



$\rightarrow \alpha : \alpha \in \mathcal{F}(r)$ et $\alpha \parallel s$

$\vec{V}_\alpha = (a,b,c)$

$t \quad (l,m,n) = (a,b,c)$

Parabola avente direttrice $y=-2$ e come fuoco $F(5,0)$.
Trovare i punti di intersezione con gli assi coordinati X e Y.




$V(5,-1)$

$A(3,0) \quad B(7,0)$

$C(0, \gamma)$ $\qquad d(C, dir) = d(C, F)$

$$|\gamma+2| = \sqrt{(0-5)^2+(\gamma-0)^2}$$

$$|\gamma+2| = \sqrt{25+\gamma^2}$$

$$(\gamma+2)^2 = 25+\gamma^2$$

$$\cancel{\gamma^2}+4+4\gamma = 25+\cancel{\gamma^2}$$

$$4\gamma = 21 \qquad \gamma = \frac{21}{4}$$

$$C\left(0, \frac{21}{4}\right)$$

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$\pi : x - y + 2z - 24 = 0$

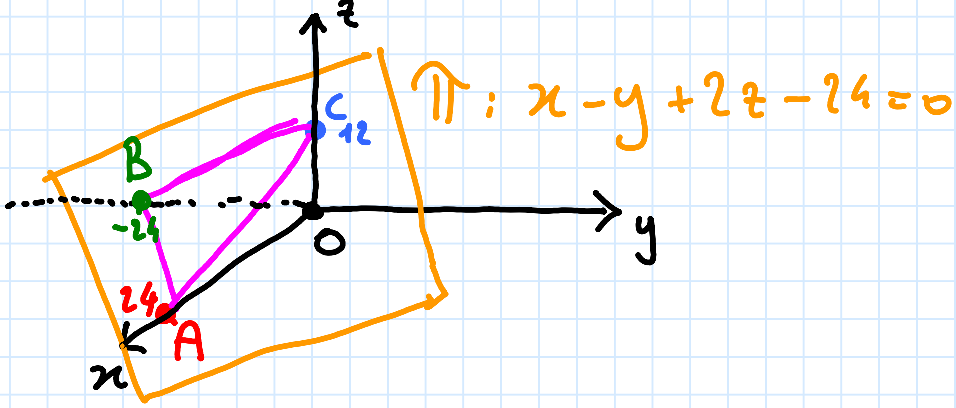
$\pi \cap$ asse $X = \{A\}$, $\pi \cap$ asse $Y = \{B\}$, $\pi \cap$ asse $z = \{C\}$

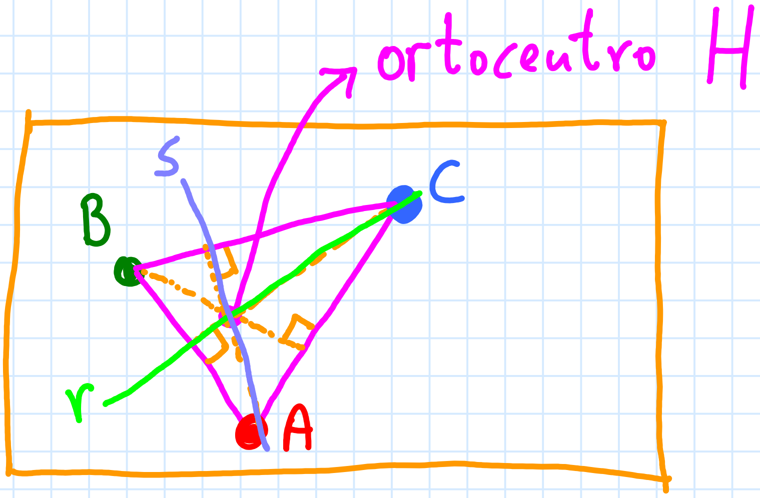
ortocentro triangolo $A\hat{B}C$

asse $X$: $y = z = 0 \Rightarrow x = 24 \Rightarrow A(24, 0, 0)$

asse $Y$: $x = z = 0 \Rightarrow y = -24 \Rightarrow B(0, -24, 0)$

asse $z$: $x = y = 0 \Rightarrow 2z = +24 \Rightarrow C(0, 0, 12)$





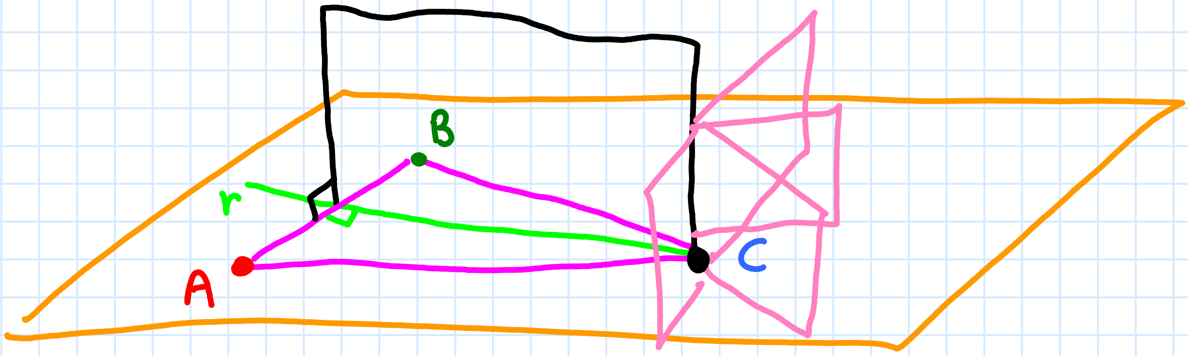

$\pi : x - y + 2z - 24 = 0$

$\{H\} = r \cap s$ 2 rette nello __SPAZIO__

$r = \pi \cap \alpha \rightarrow ?$

$s = \pi \cap \beta \rightarrow ?$

$\alpha \in S(C)$ et $\alpha \perp [\overrightarrow{AB}]$




$\beta \in S(A)$ et $\beta \perp [\overrightarrow{BC}]$

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$A(24,0,0)$ ; $B(0,-24,0)$ ; $C(0,0,12)$ ;

$\alpha \in S(C)$ et $\alpha \perp [\overrightarrow{AB}] \longrightarrow [\overrightarrow{AB}] = (-24,-24,0) =$

$= -24(1,1,0)$

$\alpha \in S(\underset{(0,0,12)}{C})$ et $\alpha \perp (\underset{a}{1}, \underset{b}{1}, \underset{c}{0})$

$$S(C): \; a(x-0)+b(y-0)+c(z-12)=0$$

$$\alpha: \; 1\cdot(x-0)+1\cdot(y-0)+0\cdot(z-12)=0$$

$$\boxed{\alpha: \; x+y=0}$$

$$r = \pi \cap \alpha \implies r: \begin{cases} x-y+2z-24=0 \\ x+y=0 \end{cases}$$

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$A(24,0,0)$ ; $B(0,-24,0)$ ; $C(0,0,12)$

$\beta \in S(A)$ et $\beta \perp [\vec{BC}]$ $\quad [\vec{BC}]=(0,+24,12)=$

$=12(0,2,1)$

$$S(A): \; a(x-24)+b(y-0)+c\cdot(z-0)=0$$

$$\beta: \; 0\cdot(x-24)+2\cdot(y-0)+1\cdot(z-0)=0$$

$$\boxed{\beta: \; 2y+z=0}$$

$$s = \pi \cap \beta \implies s: \begin{cases} x-y+2z-24=0 \\ 2y+z=0 \end{cases}$$

$$r: \begin{cases} x-y+2z-24=0 \\ x+y=0 \end{cases}$$

$$\{H\} = r \wedge s : \begin{cases} x - y + 2z - 24 = 0 \\ x + y = 0 \ \longrightarrow \ x = -y \\ 2y + z = 0 \ \longrightarrow \ z = -2y \end{cases}$$

$$(-y) - y + 2(-2y) - 24 = 0$$

$$-6y - 24 = 0 \ ; \quad y = -4 \ \begin{matrix} \nearrow x = +4 \\ \searrow z = +8 \end{matrix}$$

$$H(+4, -4, +8)$$

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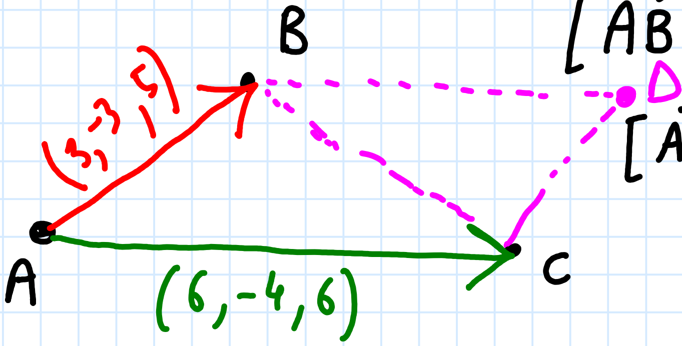
$$A(0,3,-6) \ ; \ B(3,0,-1) \ ; \ C(6,-1,0)$$




$$[\vec{AB}] = (3, -3, 5)$$

$$[\vec{AC}] = (6, -4, 6)$$

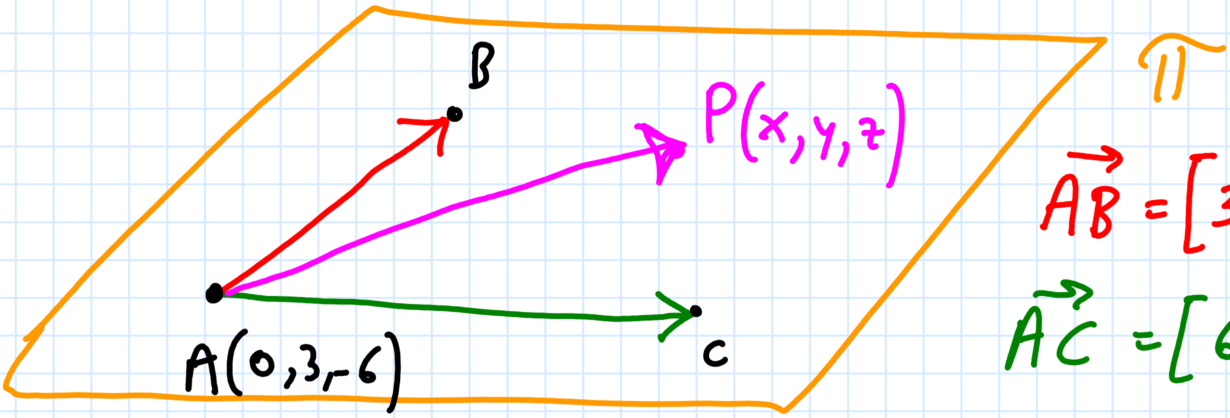
$$\text{area } \overset{\triangle}{ABC} = \frac{1}{2} \text{ area } ABDC = \frac{1}{2} \left\| [\vec{AB}] \wedge [\vec{AC}] \right\|$$

$$[\vec{AB}] \wedge [\vec{AC}] = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -3 & 5 \\ 6 & -4 & 6 \end{bmatrix} = 2\vec{i} + 12\vec{j} + 6\vec{k}$$

$$\left\| [\vec{AB}] \wedge [\vec{AC}] \right\| = \sqrt{2^2 + 12^2 + 6^2} =$$

$$= \sqrt{184}$$

$$\boxed{\text{area } \overset{\triangle}{ABC} = \frac{1}{2}\cdot\sqrt{184} = \sqrt{46}}$$



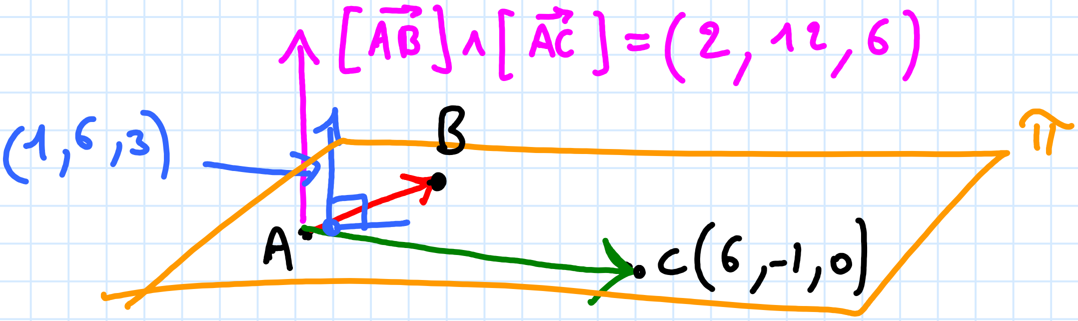

$\vec{AB} = [3, -3, 5]$

$\vec{AC} = [6, -4, 6]$

$P \in \pi \iff [\vec{AB}], [\vec{AC}], [\vec{AP}]$ sono lin. DIP.

$$[\vec{AP}] = (x,\ y-3,\ z+6)$$

$$\det\begin{bmatrix} x & (y-3) & (z+6) \\ 3 & -3 & 5 \\ 6 & -4 & 6 \end{bmatrix} = 0 \longrightarrow$$ sviluppandola ottenete l'equazione del piano




$$S(c):\quad \underset{\substack{\uparrow\\1}}{a}(x-6)+\underset{\substack{\uparrow\\6}}{b}(y-(-1))+\underset{\substack{\uparrow\\3}}{c}(z-0)=0$$

$$\pi:\ 1\cdot(x-6)+6(y+1)+3\cdot z=0$$

$$\boxed{\Pi : x + 6y + 3z = 0}$$

$r: (t-1)\cdot x + 5y + (t^2-12)\cdot z = y + 13 = 0$

retta $S$ passante per $A(2-4t, 5, -9)$ e $B(2-3t, 5, -t-8)$.

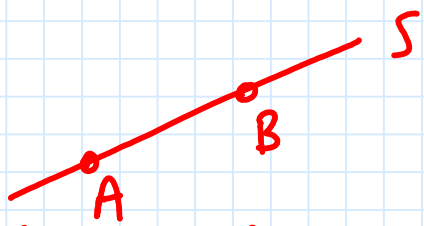
$$r: \begin{bmatrix} (t-1) & 5 & (t^2-12) \\ 0 & 1 & 0 \end{bmatrix}$$

$l = 12 - t^2$

$m = -0 = 0$

$n = t - 1$

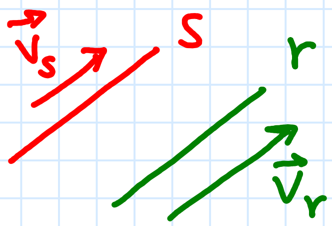
$\vec{V}_r // r \qquad \vec{V}_r = (12-t^2,\ 0,\ t-1)$




$\vec{V}_s = [\vec{AB}] = (t, 0, -t+1)$

$A(2-4t, 5, -9)$

$B(2-3t, 5, -t-8)$




$s // r \iff \vec{V}_s // \vec{V}_r \iff$

$\iff \vec{V}_s$ et $\vec{V}_r$ DIP.

$$\text{rg}\begin{bmatrix} (12-t^2) & \boxed{0} & (t-1) \\ t & \boxed{0} & (1-t) \end{bmatrix} = 1$$

$\hookrightarrow$ colonna nulla (nessun contributo al rango)

$$\text{rg}\left[\begin{array}{c|c} (12-t^2) & (t-1) \\ \hline t & (1-t) \end{array}\right]_{2\times 2} = 1 \Rightarrow \text{rg } \underline{\text{NON}} \text{ è massimo}$$

$$\det\begin{bmatrix} (12-t^2) & (t-1) \\ t & (1-t) \end{bmatrix} = 0$$

$$(12-t^2)(1-t) - t\cdot(t-1) = 0$$

$$(12-t^2)\cdot(1-t) + t\cdot(1-t) = 0$$

$$(1-t)\cdot\left[(12-t^2) + t\right] = 0$$

$$(1-t)\cdot(-t^2+t+12) = 0$$

$$(1-t)\cdot(\underbrace{t^2-t-12}_{\text{scomporre}}) = 0$$

$$(1-t)\cdot(t-4)\cdot(t+3) = 0$$

$$\boxed{t_1 = 1 \qquad t_2 = +4 \qquad t_3 = -3}$$

questi sono i valori di $t$ per i quali $r \parallel s$

Sfera passante per $O(0,0,0)$ e i punti $A$, $B$ e $C$ d'intersezione del piano

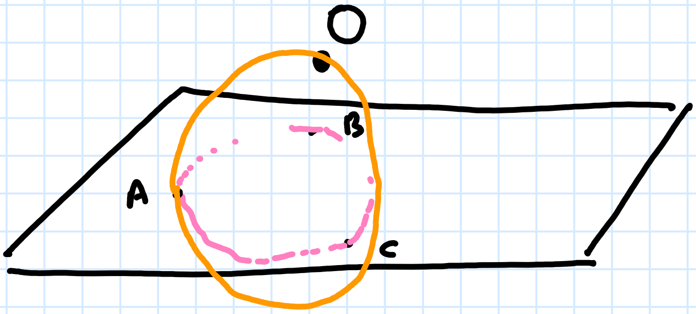
$\pi : x - 2y - z - 18 = 0$ con gli assi coordinati $X$, $Y$ e $Z$ rispettivamente

$A(18,0,0)$

$B(0,-9,0)$

$C(0,0,-18)$

$O \notin \pi$




$\pi$

$$x^2 + y^2 + z^2 + a x + b \cdot y + c \cdot z + d = 0$$
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$
$$? \quad ? \quad ? \quad ?$$

Hp $O \in$ sfera $\Rightarrow$ $d = 0$

$$x^2 + y^2 + z^2 + ax + by + cz = 0$$

$A(18,0,0) \in$ sfera $\Rightarrow$ $18^2 + 18a = 0$ $\Rightarrow$ $a = -18$

$B(0,-9,0) \in$ sfera $\Rightarrow$ $(-9)^2 - 9b = 0$ $\Rightarrow$ $b = 9$

$C(0,0,-18) \in$ sfera $\Rightarrow$ $(-18)^2 - 18c = 0$ $\Rightarrow$ $c = 18$

sfera : $x^2 + y^2 + z^2 - 18x + 9y + 18z = 0$

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$$A = \begin{bmatrix} 1 & -t & 2 \\ 0 & -5 & (t-4) \\ 0 & 0 & 1 \end{bmatrix}$$

$t \in \mathbb{R}$

$A$ diagonalizzabile

$$\det\begin{bmatrix}(1-\lambda) & -t & 2\\ 0 & (-5-\lambda) & (t-4)\\ 0 & 0 & (1-\lambda)\end{bmatrix} = (1-\lambda)^2\cdot(-5-\lambda)$$

$\hookrightarrow \lambda_1 = 1 \quad m_a(\lambda_1)=2$

$\hookrightarrow \lambda_2 = -5 \quad m_a(\lambda_2)=1$

Ovviamente $m_g(\lambda_2)=1$

Affinché A sia diagonalizzabile deve essere $m_g(\lambda_1)=2$

$$m_g(1) = 3 - rg\begin{bmatrix}0 & -t & 2\\ 0 & -6 & (t-4)\\ 0 & 0 & 0\end{bmatrix} = 2$$

$$rg\begin{bmatrix}0 & -t & 2\\ 0 & -6 & (t-4)\\ 0 & 0 & 0\end{bmatrix} = 1$$

$$rg\begin{bmatrix}-t & 2\\ -6 & (t-4)\end{bmatrix} = 1$$

$$\det\begin{bmatrix}-t & 2\\ -6 & (t-4)\end{bmatrix} = 0$$

$$(-t)(t-4)-(-6)\cdot 2=0$$

$$-t^2+4t+12=0$$

$$t^2-4t-12=0$$

$$(t-6)\cdot(t+2)=0 \quad \boxed{\begin{array}{l} t_1=+6 \\ t_2=-2 \end{array}}$$

per quei 2 valori la matrice A risulta diagonalizzabile