Lunedì 14 giugno ore 15:00-18:00

Titolo nota 14/06/2021

Piano π per $O(0,0,0)$, $\perp$ piano π' di equazione $x-3y+4z=0$ e parallelo alla retta

$r: \; y = x-3y+4z=0$

$\pi \in S(O): \quad a(x-0)+b(y-0)+c(z-0)=0$

$$\underset{?}{a}x+\underset{?}{b}y+\underset{?}{c}z=0$$

$\pi \perp \pi' \qquad \pi': \underset{a'}{1}\cdot x - \underset{b'}{3}\cdot y + \underset{c'}{4}\cdot z = 0$

$\Updownarrow$

$a\cdot a'+b\cdot b'+c\cdot c'=0 \iff 1\cdot a+(-3)\cdot b+4\cdot c=0$

$$\boxed{1^a \text{ equazione} \qquad a-3b+4c=0}$$

$\pi \parallel r \qquad r: \; x-3y+4z=y=0$

$\Updownarrow$

$$\det\begin{bmatrix} a & b & c \\ 1 & -3 & 4 \\ 0 & 1 & 0 \end{bmatrix}=0 \iff (-1)\cdot(4a-c)=0$$

$$c=4a$$

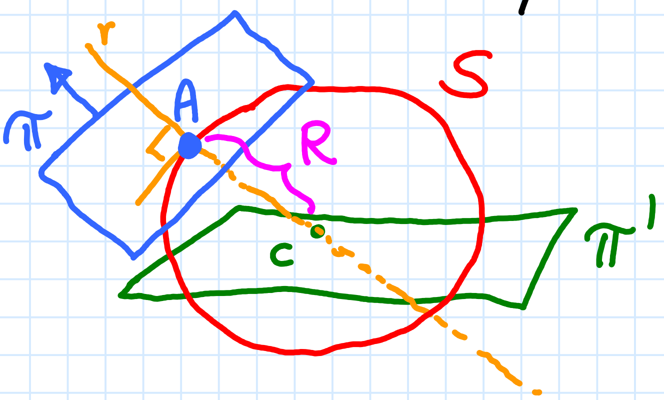
$$a-3b+4\cdot(4a)=0$$

$17a - 3b = 0$ scelgo $a = 3$ e $b = 17$

$$c = 4 \cdot a = 4 \cdot 3 = 12$$

$$\boxed{\pi : \ 3x + 17y + 12z = 0}$$

Sfera tangente al piano $\pi: x + 5y - 3z = 0$ nel punto $A(3,0,1)$ e avente il centro sul piano $\pi' : 2y + 5 = 0$.



(1°) trovare la retta r

$\vec{v}_\pi \perp \pi$; $\vec{v}_\pi = \begin{pmatrix} 1, & 5, & -3 \end{pmatrix} \parallel r$

$\qquad\qquad\qquad l \quad m \quad n$

$$\left.\begin{matrix} A(3,0,1) \in r \\ (1,5,-3) \parallel r \end{matrix}\right] \Longrightarrow r : \begin{cases} x = 1 \cdot t + 3 \\ y = 5 \cdot t + 0 \\ z = -3 \cdot t + 1 \end{cases}$$

$$r : \begin{cases} x = t + 3 \\ y = 5t \\ z = -3t + 1 \end{cases}$$

$$\{C\} = r \cap \pi'$$

$$\pi' : 2y + 5 = 0$$

$$C : \begin{cases} x = t+3 \\ y = 5t \\ z = -3t+1 \\ 2y+5=0 \end{cases} \longrightarrow 2\cdot(5t)+5=0$$

$$10t+5=0$$

$$2t+1=0$$

$$t=-\frac{1}{2}$$

$$C : \begin{cases} x = t+3 = -\frac{1}{2}+3 = \frac{5}{2} \\ y = 5t = 5\cdot\left(-\frac{1}{2}\right) = -\frac{5}{2} \\ z = -3t+1 = (-3)\left(-\frac{1}{2}\right)+1 = \frac{3}{2}+1 = \frac{5}{2} \end{cases}$$

$$C\left(\frac{5}{2}, -\frac{5}{2}, \frac{5}{2}\right)$$

$$A(3,0,1) \qquad R = d(C,A)$$

$$R^2 = \left(3-\frac{5}{2}\right)^2 + \left(0-\left(-\frac{5}{2}\right)\right)^2 + \left(1-\frac{5}{2}\right)^2$$

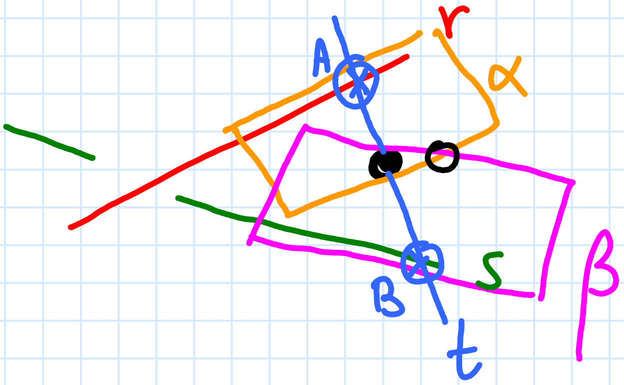
fate voi i conti

$$S : \left(x-\frac{5}{2}\right)^2 + \left(y+\frac{5}{2}\right)^2 + \left(z-\frac{5}{2}\right)^2 = R^2$$

parametri direttori retta passante per $O(0,0,0)$ e incidente alla retta

$r : y = 4x - y - 5z - 19 = 0$ e la retta

$s : y - z = x - 3y + 2z - 17 = 0$.



parametri direttori di t

$$t = \alpha \cap \beta$$

$\alpha \in \mathcal{F}(r) : \lambda(y) + \mu(4x - y - 5z - 19) = 0$

$O(0,0,0) \in \alpha \Rightarrow \lambda \cdot 0 + \mu(-19) = 0 \Rightarrow$

$\Rightarrow -19\mu = 0 \Rightarrow \mu = 0 \Rightarrow \lambda \neq 0$

scelgo $\lambda = 1$

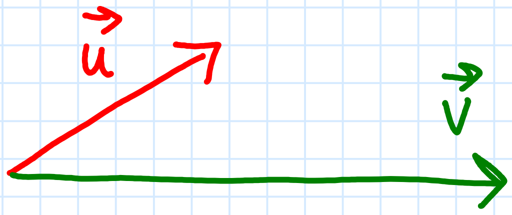
$$\boxed{\alpha : y = 0}$$

in modo analogo trovo SUBITO

il piano $\boxed{\beta : y - z = 0}$

$$t = \alpha \cap \beta : \begin{cases} y = 0 \\ y - z = 0 \end{cases} : \begin{cases} y = 0 \\ z = 0 \end{cases}$$

Quindi, t è l'asse X. Per cui i suoi parametri direttori sono $(1,0,0)$



$$|\vec{u} \bullet \vec{v}| = ?$$

Piani contenenti l'asse Y e formanti con l'asse Z un angolo di $\frac{\pi}{3}$ radianti.

asse Y : $\begin{cases} x = 0 \\ z = 0 \end{cases}$

$\mathcal{F}(\text{asse Y})$: $\lambda \cdot x + \mu \cdot z = 0 \Rightarrow (a,b,c) = (\lambda, 0, \mu)$

asse Z $\longrightarrow$ parametri direttori $(l, m, n) = (0, 0, 1)$

Formula angolo retta – piano

$\theta = \frac{\pi}{3}$ rad $= 60°$

$$\sin\theta = \frac{|a\,l + b\,m + c\,n|}{\sqrt{a^2+b^2+c^2} \cdot \sqrt{l^2+m^2+n^2}} \;;$$

$$\frac{\sqrt{3}}{2} = \frac{|\mu|}{\sqrt{\lambda^2+\mu^2} \cdot 1}$$

$$\sqrt{3} \cdot \sqrt{\lambda^2+\mu^2} = 2 \cdot |\mu|$$

$$3 \cdot (\lambda^2+\mu^2) = 4 \cdot \mu^2 \quad \Longrightarrow \quad \mu^2 = 3\lambda^2$$

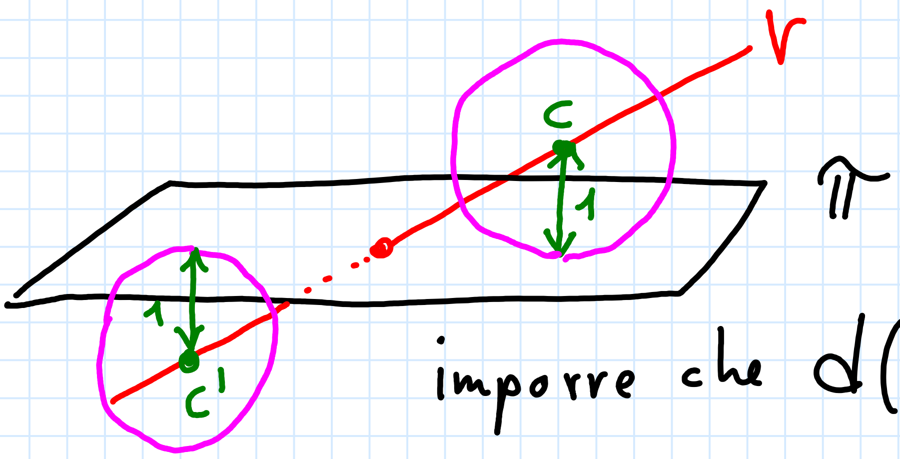
scelgo $\lambda = 1 \Rightarrow \mu^2 = 3 \cdot 1 = 3 \Rightarrow \mu = \pm\sqrt{3}$

$$\boxed{\begin{array}{ll} 1^{\circ} \text{ piano} & x + \sqrt{3}\, z = 0 \\ 2^{\circ} \text{ piano} & x - \sqrt{3}\, z = 0 \end{array}}$$

Sfere di raggio 1 tangenti al piano $\pi : 2x - 2y + z + 1 = 0$ e aventi il centro sulla retta r passante per $A(-3, 2, 5)$ e parallela all'asse z, $\rightarrow (\ell, m, n) = (0, 0, 1)$

$$r : \begin{cases} x = 0 \cdot t + (-3) \\ y = 0 \cdot t + 2 \\ z = 1 \cdot t + 5 \end{cases} \qquad r : \begin{cases} x = -3 \\ y = +2 \\ z = t + 5 \end{cases}$$

$$C \in r \Rightarrow C(-3, +2, t+5)$$



imporre che $d(C, \pi) = 1$

$\pi : 2x - 2y + z + 1 = 0 \qquad C(-3, +2, t+5)$

$$d(C, \pi) = \frac{|2 \cdot (-3) - 2 \cdot (2) + (t+5) + 1|}{\sqrt{2^2 + (-2)^2 + 1^2}} = 1$$

$$\frac{|-\cancel{6}-4+t+\cancel{5}+\cancel{1}|}{\sqrt{9}}=1$$

$$|t-4|=3$$

$$t-4=\pm 3$$

$$t=4\pm 3 \quad \begin{cases} t_1=4+3=7 \\ t_2=4-3=1 \end{cases}$$

$$t_1=7 \Rightarrow C_1(-3,\ 2,\ 12)$$

$$t_2=1 \Rightarrow C_2(-3,\ 2,\ 6)$$

Raggio $=1$

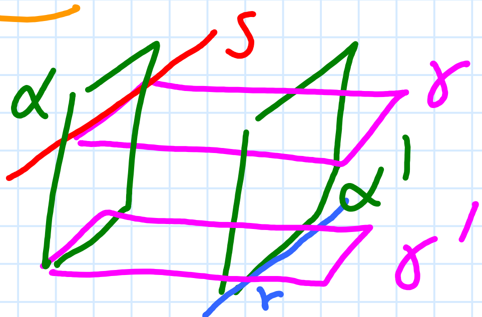
$$\boxed{\begin{aligned} S_1 &: (x+3)^2+(y-2)^2+(z-12)^2=1^2 \\ S_2 &: (x+3)^2+(y-2)^2+(z-6)^2=1^2 \end{aligned}}$$

Distanza tra le seguenti rette

$$r: \underbrace{2y-4z+2}_{\alpha'} = \underbrace{x-2}_{\gamma'} = 0 \qquad \alpha \parallel \alpha'$$

$$s: \underbrace{2y-4z}_{\alpha} = \underbrace{x+2y-4z}_{\beta} = 0 \qquad \gamma \parallel \gamma'$$

$$s: \underbrace{2y-4z}_{\alpha} = \underbrace{x}_{\gamma} = 0$$

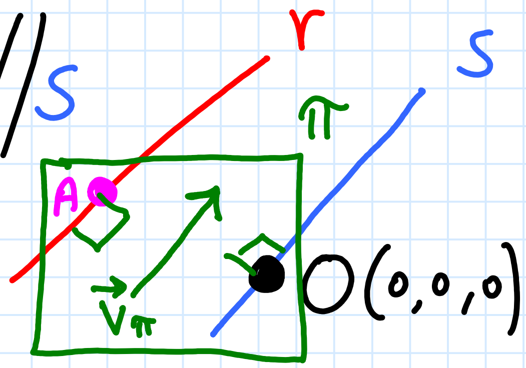


$$r : \begin{bmatrix} 0 & 2 & -4 \\ 1 & 0 & 0 \end{bmatrix} \rightarrow \left.\begin{array}{l} l = 0 \\ m = -4 \\ n = -2 \end{array}\right\}$$

$$s : \begin{bmatrix} 0 & 2 & -4 \\ 1 & 2 & -4 \end{bmatrix} \rightarrow \left.\begin{array}{l} l = 0 \\ m = -4 \\ n = -2 \end{array}\right\}$$

$$\rightarrow r \parallel s$$

$r \parallel s$



$$\vec{V}_\pi \perp \pi$$

$$\vec{V}_\pi = (0, 2, 1)$$

$$\pi : 0 \cdot x + 2 \cdot y + 1 \cdot z + 0 = 0$$

$$\boxed{\pi : 2y + z = 0}$$

$$\{A\} = \pi \cap r : \begin{cases} 2y + z = 0 \Rightarrow z = -2y \\ 2y - 4z + 2 = 0 \\ x - 2 = 0 \Rightarrow x = 2 \end{cases}$$

$$2y - 4(-2y) + 2 = 0$$

$$2y + 8y + 2 = 0$$

$$5y + 1 = 0 \Rightarrow y = -\frac{1}{5} \Rightarrow z = \frac{2}{5}$$

$$A\left(2, -\frac{1}{5}, \frac{2}{5}\right) \qquad O(0,0,0)$$

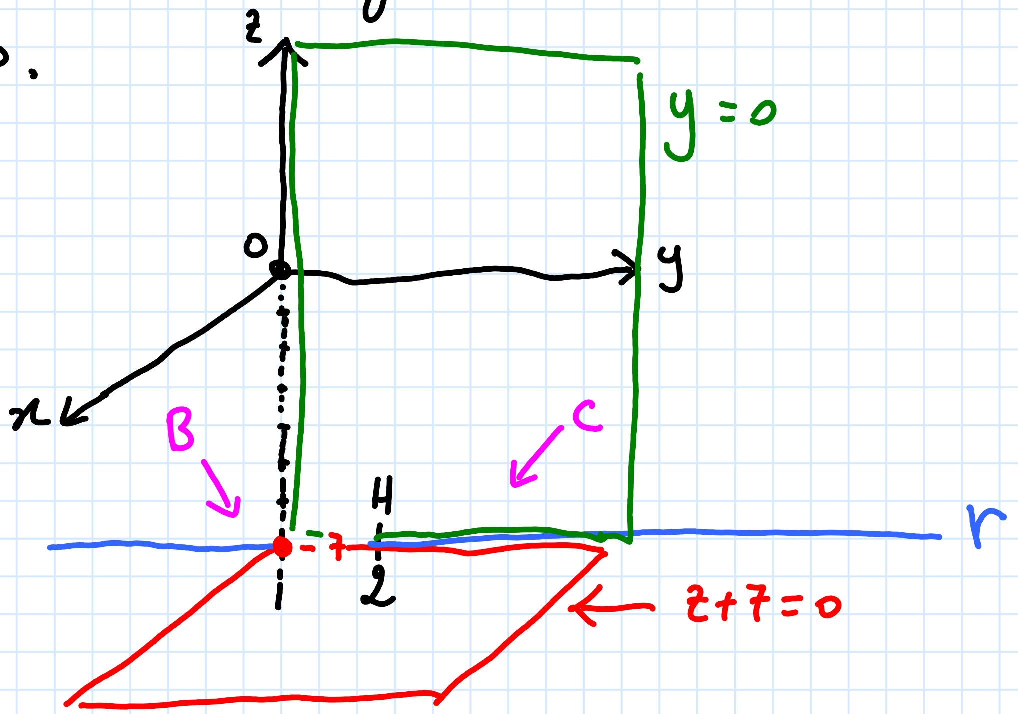
$$d(r, s) = d(O, A) = \sqrt{(2-0)^2 + \left(-\frac{1}{5} - 0\right)^2 + \left(\frac{2}{5} - 0\right)^2}$$

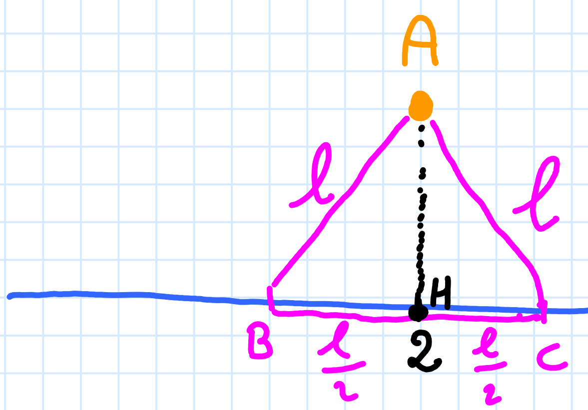
$A(2\sqrt{26}, 2, -5)$. Sulla retta r

$x = z + 7 = 0$ trovare due punti B e C

tali che il triangolo $\overset{\triangle}{ABC}$ sia

equilatero.





$B(0, ?, -7)$

$C(0, ?, -7)$

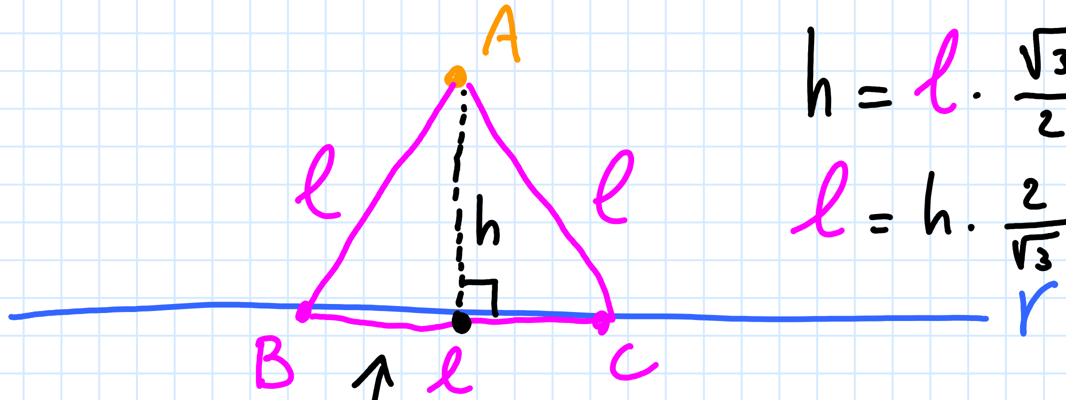
$H(0, 2, -7)$

stessa di A

$B\left(0, 2 - \frac{\ell}{2}, -7\right)$

$C\left(0, 2 + \frac{\ell}{2}, -7\right)$

Ora il problema è trovare ℓ

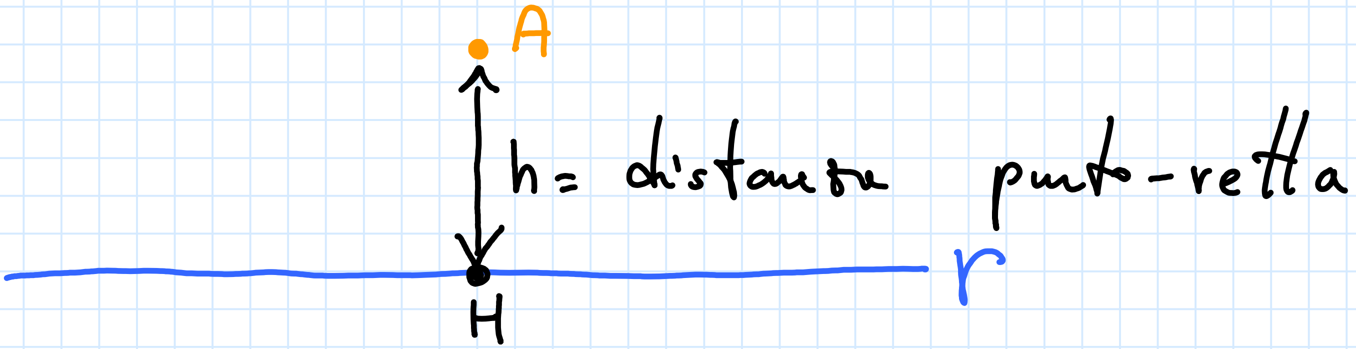


$$h = \ell \cdot \frac{\sqrt{3}}{2}$$

$$\ell = h \cdot \frac{2}{\sqrt{3}}$$

triangolo equilatero

Quindi, ora devo trovare l'altezza h



$A(2\sqrt{26}, 2, -5)$ $\quad$ $H(0, 2, -7)$

$$d(A,H) = \sqrt{(2\sqrt{26}-0)^2 + (2-2)^2 + (-5-(-7))^2} =$$

$$= \sqrt{104 + 0 + 4} = \sqrt{108} = 2\sqrt{27} = 6\sqrt{3}$$

$$h = 6\sqrt{3} \Longrightarrow \ell = 6 \cdot \sqrt{3} \cdot \frac{2}{\sqrt{3}} = 12 \Rightarrow$$

$$\Rightarrow \frac{\ell}{2} = 6$$

$$\boxed{\begin{array}{l} B(0, -4, -7) \\ C(0, +8, -7) \end{array}}$$

→ risultato finale