Lunedì 21 Giugno ore 15:00-18:00

Titolo nota 21/06/2021

$$r: \overbrace{4y+2z+1}^{\alpha' /\!/ \alpha} = \overbrace{x-1}^{\gamma' /\!/ \gamma} = 0$$

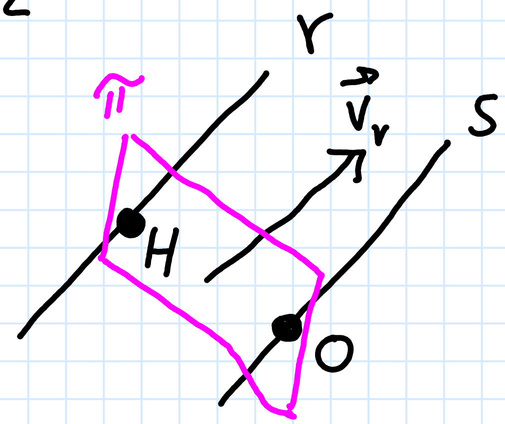
$$s: \underbrace{4y+2z}_{\alpha} = \underbrace{x+4y+2z}_{\beta} = 0$$ (si vede subito che $O \in s$)

$$\gamma: (x+4y+2z)-(4y+2z)=0 \qquad \gamma: x=0$$

$$s: \underbrace{4y+2z}_{\alpha} = \underbrace{x}_{\gamma} = 0$$

$$\begin{bmatrix} 0 & 4 & 2 \\ 1 & 0 & 0 \end{bmatrix} \begin{array}{l} \nearrow \; l = 0 \\ \rightarrow \; m = -(-2) = +2 \\ \searrow \; n = -4 \end{array}$$

$$\vec{V}_r = (l, m, n) = (0, 1, -2)$$



$$\pi: 0 \cdot x + 1 \cdot y + (-2)z = 0$$

$$\pi: y - 2z = 0$$

$$\{H\} = \pi \cap r: \begin{cases} y-2z=0 & \longrightarrow y = 2z \\ x-1=0 & \longrightarrow x = 1 \\ 4y+2z+1=0 & \longrightarrow 8z+2z+1=0 \end{cases}$$

$$10z+1=0 \rightarrow z = -\frac{1}{10} \rightarrow y = -\frac{2}{10} = -\frac{1}{5}$$

$$H\left(1, -\frac{1}{5}, -\frac{1}{10}\right); \quad O(0,0,0)$$

$$d(r,s) = d(H,O) =$$

$$= \sqrt{(1-0)^2 + \left(-\frac{1}{5} - 0\right)^2 + \left(-\frac{1}{10} - 0\right)^2} =$$

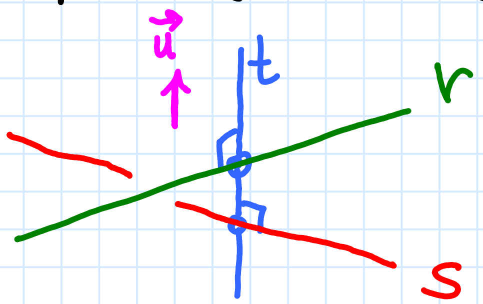
$$= \sqrt{1 + \frac{4}{100} + \frac{1}{100}} = \sqrt{\frac{105}{100}} =$$

$$\boxed{= \sqrt{\frac{21}{20}} = \frac{\sqrt{105}}{10} \;;}$$

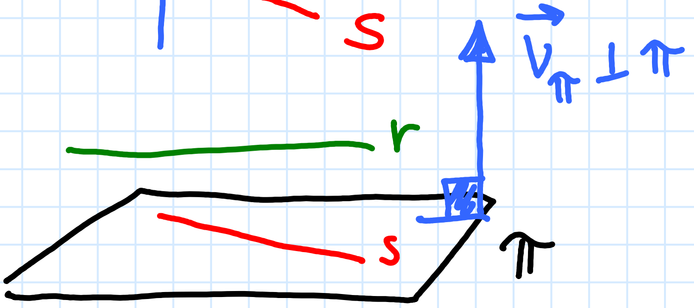
$$r : \; 4x - y + 15 = x + z = 0$$

$$s : \; 7y + z = 2x - y - 2z = 0$$

VERSORE retta minima distanza



$$\|\vec{u}\| = 1$$



$$\vec{u} = \frac{1}{\|\vec{v}_\pi\|} \cdot \vec{v}_\pi$$

$$\pi \in \mathcal{F}(s) : \; \lambda(7y + z) + \mu(2x - y - 2z) = 0$$

$$\underbrace{(2\mu)}_{a} \cdot x + \underbrace{(7\lambda - \mu)}_{b} \cdot y + \underbrace{(\lambda - 2\mu)}_{c} \cdot z = 0$$

$$\pi \parallel r \iff \det \begin{bmatrix} 2\mu & (7\lambda-\mu) & (\lambda-2\mu) \\ 4 & -1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = 0$$

$$-2\mu + (\lambda - 2\mu) - 4(7\lambda - \mu) = 0$$

$$-2\mu + \lambda - 2\mu - 28\lambda + 4\mu = 0$$

$$-27\lambda = 0 \Rightarrow \boxed{\lambda = 0} \Rightarrow \mu \neq 0 \Rightarrow \text{scelgo } \mu = 1$$

$$\pi : 2x - y - 2z = 0$$

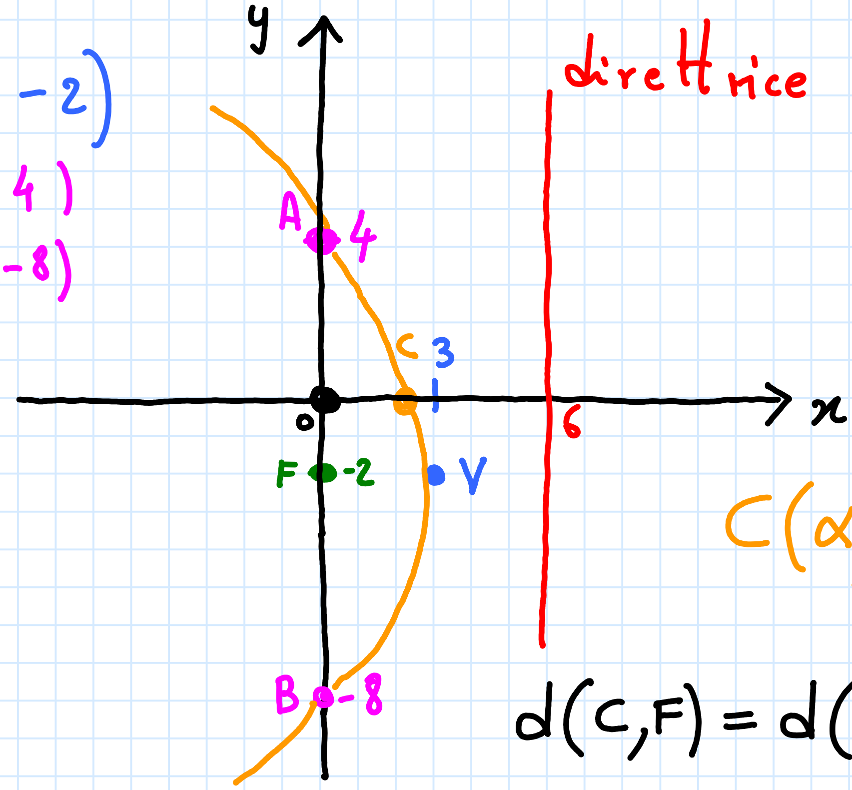
$$\vec{V}_\pi = (2, -1, -2) \parallel \text{retta } t$$

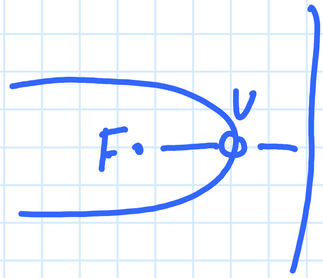
$$\| \vec{V}_\pi \| = \sqrt{2^2 + (-1)^2 + (-2)^2} = 3$$

VERSORE $\vec{u} = \frac{1}{3}(2, -1, -2) = \left(\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)$

Ω parabola fuoco $F(0,-2)$ e con direttrice $x - 6 = 0$. Trovare punti di intersezione con gli assi.

$V(3,-2)$
$A(0,4)$
$B(0,-8)$





$C(\alpha, 0)$

$d(C,F) = d(C, \text{direttrice})$

$C(\alpha,0)$; $F(0,-2)$; direttrice : $x=6$

$$|6-\alpha| = \sqrt{(\alpha-0)^2+(0-(-2))^2}$$

$$|6-\alpha| = \sqrt{\alpha^2+4}$$

$$36+\cancel{\alpha^2}-12\alpha = \cancel{\alpha^2}+4$$

$$12\alpha = 32$$

$$3\alpha = 8 \Rightarrow \alpha = \frac{8}{3}$$

$$C\left(\frac{8}{3}, 0\right)$$

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piano π contenente asse z e passante per $A(0,-3,5)$. Poi, sul piano π sia γ la circonferenza tangente all'asse z nel punto $B(0,0,4)$ e passante per $D(0,-8,0)$.

Trovare CENTRO e RAGGIO della circonferenza

$\pi \in \mathcal{F}(\text{asse } z)$ $\qquad$ asse z: $x = y = 0$

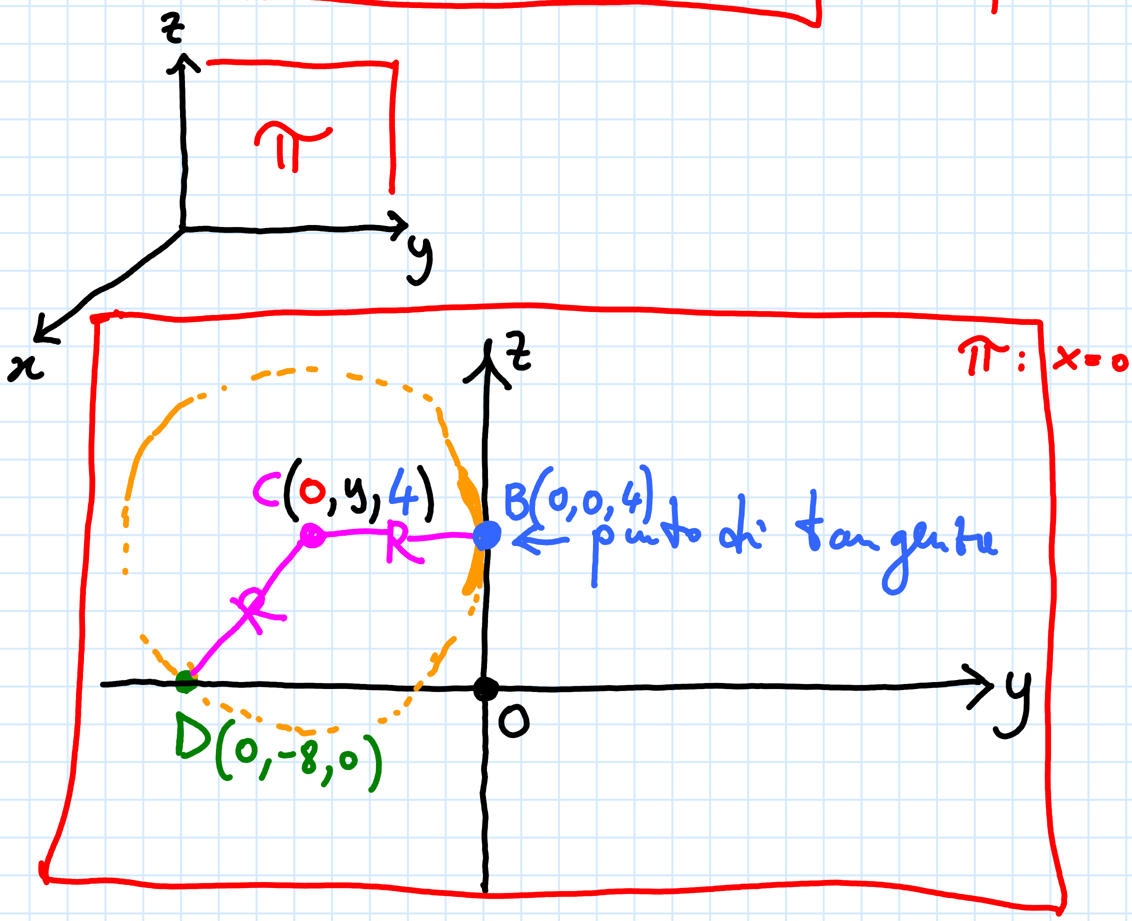
$\pi : \lambda \cdot x + \mu \cdot y = 0$

$A(0,-3,5) \in \pi \Rightarrow \lambda \cdot 0 + \mu \cdot (-3) = 0 \Rightarrow$

$\Rightarrow -3\mu = 0 \Rightarrow \mu = 0 \Rightarrow \lambda \neq 0$

scelgo $\lambda = 1$

$\boxed{\pi : x = 0}$ piano YZ



$$d(C,D) = d(C,B)$$

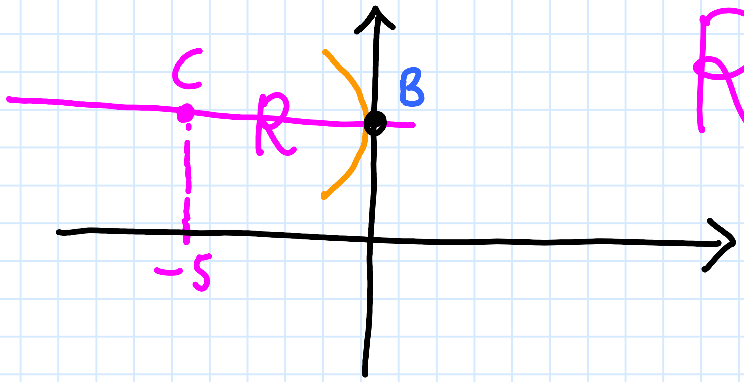
$$\sqrt{(0-0)^2+(y-(-8))^2+(4-0)^2} = \sqrt{(0-0)^2+(y-0)^2+(4-4)^2}$$

$$\sqrt{y^2+64+16y+16} = \sqrt{y^2}$$

$$\cancel{y^2}+64+16y+16 = \cancel{y^2}$$

$$4+y+1=0 \Rightarrow y=-5$$

$$C(0,-5,4)$$



$$R=5$$

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r retta // asse X e passante per $A(1, 9, -3\sqrt{3})$

Trovare i <u>PIANI</u> che contengono r e formano un angolo di $\frac{\pi}{6}$ rad col <u>PIANO</u> XY $(z=0)$

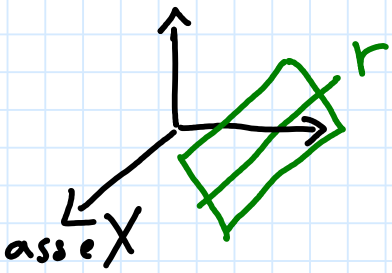
angolo tra 2 piani

$$\cos\theta = \pm \frac{a\cdot a' + b\cdot b' + c\cdot c'}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{(a')^2+(b')^2+(c')^2}}$$

$$\theta = \frac{\pi}{6}\,\text{rad} = 30^\circ \Rightarrow \cos\theta = \frac{\sqrt{3}}{2}$$

(a,b,c) del piano contenente r

$r \parallel$ asse X



$r \subseteq \text{piano}_{(a,b,c)}$ (caso di parallelismo)

$$(l,m,n) = (1,0,0)$$

$r \parallel$ piano $\Leftrightarrow$ $al + bm + cn = 0$

$$a\cdot 1 + b\cdot 0 + c\cdot 0 = 0$$

$$a = 0$$

piano nostro $(0, b, c)$

secondo piano (piano XY) $z = 0$ $\rightarrow (a', b', c') = (0,0,1)$

$$\frac{\sqrt{3}}{2} = \pm \frac{0\cdot 0 + b\cdot 0 + c\cdot 1}{\sqrt{0^2+b^2+c^2}\cdot\sqrt{0^2+0^2+1^2}}$$

$$\frac{\sqrt{3}}{2} = \pm \frac{c}{\sqrt{b^2+c^2}}$$

$$\sqrt{3}\cdot\sqrt{b^2+c^2} = \pm 2\cdot c$$

$$3\cdot(b^2+c^2) = 4\cdot c^2$$

$$3b^2 = c^2$$

ricordo che $a=0$

scelgo, a piacere, $\boxed{b=1} \Rightarrow c^2=3 \Rightarrow c=\pm\sqrt{3}$

nostro piano passa per $A(1, 9, -3\sqrt{3})$

$$a(x-1)+b(y-9)+c(z-(-3\sqrt{3}))=0$$

$$0\cdot(x-1)+1\cdot(y-9) \pm \sqrt{3}(z+3\sqrt{3})=0$$

(1) $y-9+\sqrt{3}z+9=0$

(2) $y-9-\sqrt{3}z-9=0$

$$\boxed{\alpha_1 : \; y+\sqrt{3}z=0}$$

$$\boxed{\alpha_2 : \; y-\sqrt{3}z-18=0}$$

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retta r // asse X $\qquad A(1,9,-3\sqrt{3})$

$$r: \begin{cases} x = 1\cdot t+1 = t+1 \\ y = 0\cdot t+9 = 9 \\ z = 0\cdot t-3\sqrt{3} = -3\sqrt{3} \end{cases}$$

$$\boxed{r:\quad y-9 = z+3\sqrt{3} = 0}$$

$$\pi \in \mathcal{F}(r):\ \lambda(y-9)+\mu\left(z+3\sqrt{3}\right)=0$$

e così via...

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2 sfere

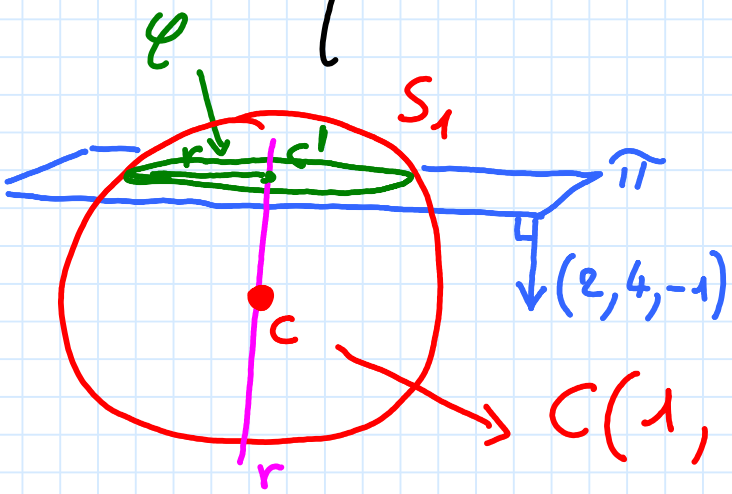
$$S_1:\ x^2+y^2+z^2-2x-1=0$$

$$S_2:\ x^2+y^2+z^2+4y-z-6=0$$

$$\pi:\ 4y-z-6-(-2x-1)=0$$

$$\pi:\ 2x+4y-z-5=0$$

$$\mathcal{C}=\pi\cap S_1:\ \begin{cases} x^2+y^2+z^2-2x-1=0 \\ 2x+4y-z-5=0 \end{cases}$$



$C(1,0,0)$

r retta passante per centro C della sfera e ortogonale al piano π

$$r:\ \begin{cases} x = 2\cdot t + 1 \\ y = 4\cdot t + 0 \\ z = -1\cdot t + 0 \end{cases} \qquad r:\ \begin{cases} x = 2t+1 \\ y = 4t \\ z = -t \end{cases}$$

$$\{C'\} = r \cap \pi : \begin{cases} x = 2t+1 \\ y = 4t \\ z = -t \\ 2x+4y-z-5=0 \end{cases}$$

$$2\cdot(2t+1)+4\cdot(4t)-(-t)-5=0$$

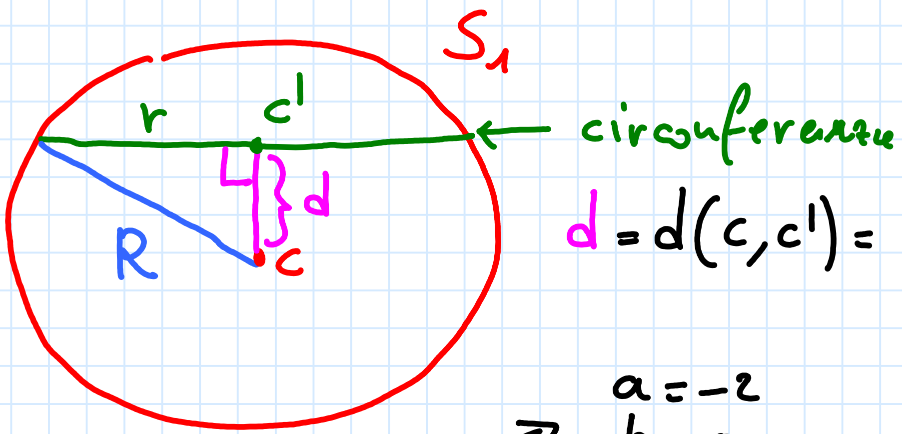
$$4t+2+16t+t-5=0$$

$$21\cdot t-3=0$$

$$7\cdot t-1=0 \Rightarrow t=\frac{1}{7} \quad x=\frac{9}{7},\ y=\frac{4}{7},\ z=-\frac{1}{7}$$

$$C'\left(\frac{9}{7}, \frac{4}{7}, -\frac{1}{7}\right); \quad C(1,0,0)$$

centro della circonferenza



$$d = d(c,c') =$$

$$S_1: x^2+y^2+z^2-2x-1=0 \quad a=-2,\ b=0,\ c=0,\ d=-1$$

$$R = \frac{1}{2}\cdot\sqrt{a^2+b^2+c^2-4d}$$

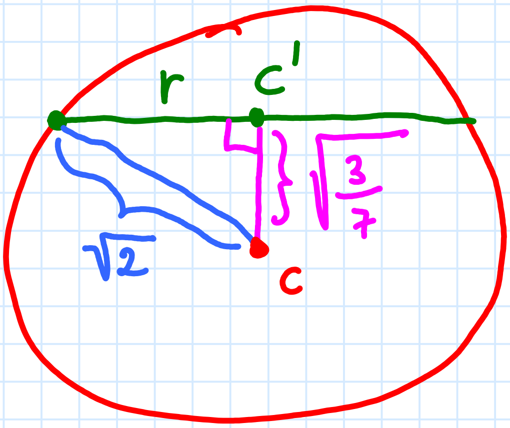
$$R = \frac{1}{2}\cdot\sqrt{(-2)^2+0^2+0^2-4(-1)} = \frac{1}{2}\cdot\sqrt{8} = \sqrt{2}$$

$$R = \sqrt{2}$$

$C'\left(\frac{9}{7}, \frac{4}{7}, -\frac{1}{7}\right)$; $C(1,0,0)$

$$d(C,C') = \sqrt{\left(\frac{9}{7}-1\right)^2+\left(\frac{4}{7}-0\right)^2+\left(-\frac{1}{7}-0\right)^2} =$$

$$= \sqrt{\frac{4}{49}+\frac{16}{49}+\frac{1}{49}} = \sqrt{\frac{21}{49}} = \sqrt{\frac{3}{7}}$$



$$r = \sqrt{2-\frac{3}{7}} =$$

$$= \sqrt{\frac{11}{7}}$$

$r = \sqrt{\frac{11}{7}}$ = raggio circonferenza
