Lunedì 28 Giugno ore 15:00 - 17:00

Titolo nota 28/06/2021

$r: 2x+7z-81 = y+3z = 0$

$s: 2x-7y = 2x-2y+z=0 \Rightarrow O \in s$

t retta di minima distanza

$D = d(r,s)$

$\vec{v} // t$ tale che $\|\vec{v}\| = D$

$\vec{u} = -\vec{v} // t$ ovviamente $\|\vec{u}\| = \|\vec{v}\| = D$

$\alpha \in \mathcal{F}(r) : \alpha // s$

$\mathcal{F}(r): \lambda(2x+7z-81)+\mu(y+3z)=0$

$$\alpha: \underbrace{(2\lambda)}_{a}\cdot x + \underbrace{\mu}_{b}\cdot y + \underbrace{(7\lambda+3\mu)}_{c}\cdot z - 81\lambda = 0$$

$s: 2x-7y = 2x-2y+z=0$

$$\alpha // s \iff \det\begin{bmatrix} 2\lambda & \mu & (7\lambda+3\mu) \\ 2 & -7 & 0 \\ 2 & -2 & 1 \end{bmatrix} = 0$$

$l = -7$

$m = -2$

$n = +10$

$$(2\lambda)\cdot(-7) + \mu\cdot(-2) + (7\lambda+3\mu)\cdot(10) = 0$$

$$-14\lambda - 2\mu + 70\lambda + 30\mu = 0$$

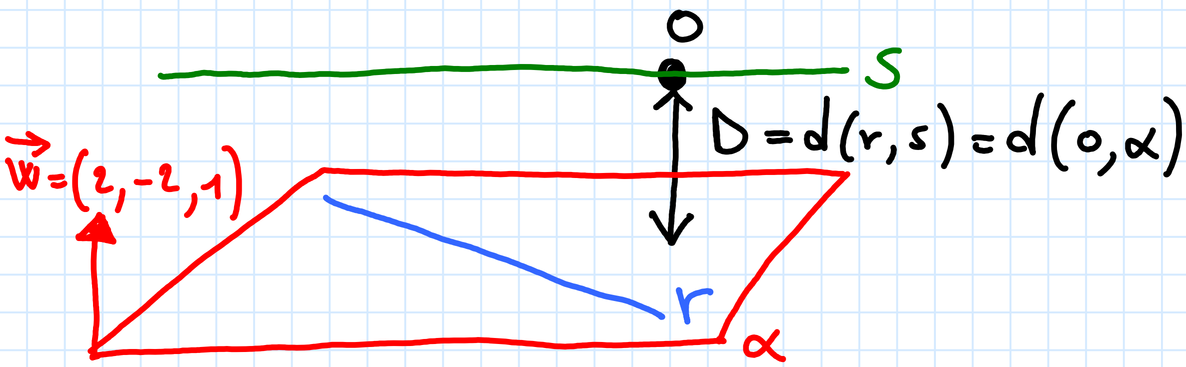
$56\lambda + 28\mu = 0$

$\boxed{2\lambda + \mu = 0}$ una sua autosoluzione è

$(\lambda, \mu) = (1, -2)$

$\alpha : 1\cdot(2x + 7z - 81) - 2\cdot(y + 3z) = 0$

$\boxed{\alpha : 2x - 2y + z - 81 = 0}$



$$D = \frac{|2\cdot 0 - 2\cdot 0 + 0 - 81|}{\sqrt{2^2 + (-2)^2 + 1^2}} = \frac{|-81|}{\sqrt{9}} = \frac{81}{3} = 27$$

$\vec{v} = (?, ?, ?) = 9\,\vec{w}$

$\vec{w} = (2,-2,1)$ $\}$ $\|\vec{w}\| = 3$

$\|\vec{v}\| = 27$

$\vec{v} = 9\cdot(2,-2,1) = (18, -18, 9)$

$$\boxed{\vec{v} = \pm(18, -18, 9)}$$

Sfera tangente alla retta r in $O(0,0,0)$

$r: x-y = 4x-z = 0$

contenente i punti $P(0,1,-1)$ e $Q(1,1,1)$

$\alpha : \alpha \perp r$ et $O \in \alpha$ 1° piano

M punto medio PQ

$\beta : \beta \perp PQ$ et $M \in \beta$

$S = \alpha \cap \beta$; $C \in S$

$r : x-y = 4x-z=0$; $\begin{bmatrix} 1 & -1 & 0 \\ 4 & 0 & -1 \end{bmatrix}$ $\rightarrow l=1$, $\rightarrow m=1$, $\rightarrow n=4$

$$\boxed{\alpha : x+y+4z=0}$$

$P(0,1,-1)$; $Q(1,1,1)$ $\Rightarrow M\left(\frac{1}{2},1,0\right)$ punto medio

$\overrightarrow{PQ} = (1,0,-2)$

$$\beta : 1\cdot\left(x-\frac{1}{2}\right)+0\cdot(y-1)+(-2)\cdot(z-0)=0$$

$$\beta : x-\frac{1}{2}-2z=0$$

$$\boxed{\beta : 2x-4z-1=0}$$

$$\boxed{\alpha : x+y+4z=0}$$

$$S=\alpha\cap\beta : 2x-4z-1 = x+y+4z=0$$

$x = 2z + \frac{1}{2}$

$\left(2z + \frac{1}{2}\right) + y + 4z = 0 \Rightarrow y = -6z - \frac{1}{2}$

chiamo $t = z$

$$s: \begin{cases} x = 2t + \frac{1}{2} \\ y = -6t - \frac{1}{2} \\ z = t \end{cases} \qquad C \in s$$

$$C\left(2t + \frac{1}{2}, -6t - \frac{1}{2}, t\right) \to t = -\frac{3}{14}$$

$O(0,0,0)$; $P(0,1,-1)$

$d(O,C) = d(O,P)$

$[d(O,C)]^2 = [d(O,P)]^2$

$$\cancel{\left(2t + \frac{1}{2}\right)^2} + \left(-6t - \frac{1}{2}\right)^2 + t^2 = \cancel{\left(2t + \frac{1}{2}\right)^2} + \left(-6t - \frac{3}{2}\right)^2 + (t+1)^2$$

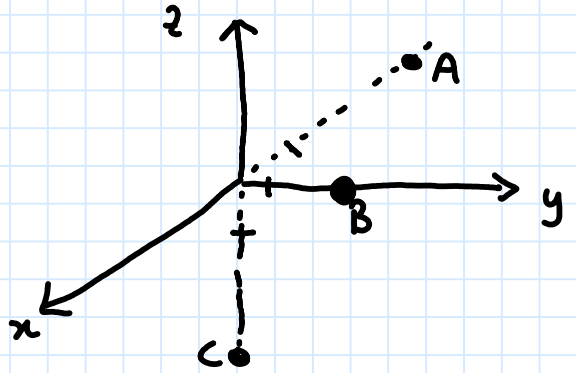
$$\cancel{36t^2} + \frac{1}{4} + 6t + \cancel{t^2} = \cancel{36t^2} + \frac{9}{4} + 18t + \cancel{t^2} + 1 + 2t$$

$$14t + 3 = 0 \Rightarrow t = -\frac{3}{14}$$

ORTOCENTRO

piano π : $x - 2y + z + 18 = 0$

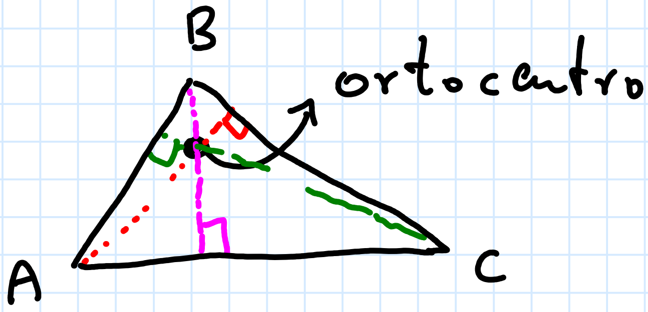
$\{A\} = \pi \cap$ asse X $\Rightarrow$ $A(-18, 0, 0)$
$\{B\} = \pi \cap$ asse Y $\Rightarrow$ $B(0, 9, 0)$
$\{C\} = \pi \cap$ asse Z $\Rightarrow$ $C(0, 0, -18)$

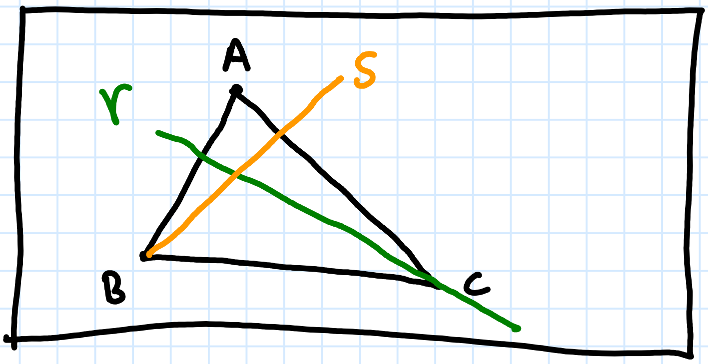


$\vec{v} = (18, -18, 9)$

$\vec{u} = (2, -2, 1) \parallel \vec{v}$

$A(-18,$





$\pi: x - 2y + z + 18 = 0$

$r = \pi \cap \alpha \to ?$

$s = \pi \cap \beta \to ?$

α : $\alpha \perp \vec{AB}$ et $C \in \alpha$

$A(-18, 0, 0)$; $B(0, 9, 0)$; $\Rightarrow$ $\vec{AB} = (18, 9, 0)$

$C(0, 0, -18)$ $\qquad \frac{1}{9}\vec{AB} = (2, 1, 0)$

$\alpha : 2 \cdot (x - 0) + 1 \cdot (y - 0) + 0 \cdot (z - (-18)) = 0$

$$\boxed{\alpha : 2x + y = 0}$$

β : $\beta \perp \vec{AC}$ et $B \in \beta$

$A(-18,0,0)$; $C(0,0,-18)$; $\Rightarrow$ $\vec{AC} = (18, 0, -18)$

$\frac{1}{18}\vec{AC} = (1, 0, -1)$

$B(0,9,0)$

$\beta : 1 \cdot (x-0) + 0 \cdot (y-9) + (-1) \cdot (z-0) = 0$

$$\boxed{\beta : x - z = 0}$$

$$\{H\} = \pi \cap \alpha \cap \beta : \begin{cases} x - 2y + z + 18 = 0 \\ 2x + y = 0 & \Rightarrow y = -2x \\ x - z = 0 & \Rightarrow z = x \end{cases}$$

$x - 2(-2x) + x + 18 = 0$

$6x + 18 = 0$; $x + 3 = 0$; $x = -3$

$H(-3, +6, -3)$ ortocentro

$$\underbrace{5(x')^2} + \underbrace{20(y')^2} \underbrace{- 38\sqrt{5}\, x'} \underbrace{- 16\sqrt{5}\, y'} + 105 = 0$$

$$\boxed{5x^2 - 38\sqrt{5}\,x} \quad \boxed{+ 20y^2 - 16\sqrt{5}\,y} \quad + 105 = 0$$

$$\boxed{5\cdot\left(x-\frac{19\sqrt{5}}{5}\right)^2-19^2}$$

$$\boxed{20\cdot\left(y-\frac{2\sqrt{5}}{5}\right)^2-16}$$

$$20\cdot\left(-\frac{2\sqrt{5}}{5}\right)^2$$

$$\cancel{20}^{4}\cdot\frac{4\cdot\cancel{5}}{\cancel{5}^{\cancel{2}}}=16$$

$$\underbrace{5\cdot\left(x-\frac{19\sqrt{5}}{5}\right)^2}_{\text{nuova ascissa}}+\underbrace{20\cdot\left(y-\frac{2\sqrt{5}}{5}\right)^2}_{\text{nuova ordinata}}-361-16+105=0$$

$$5\cdot(x'')^2+20\cdot(y'')^2-272=0$$

$$\frac{5}{272}\cdot(x'')^2+\frac{20}{272}\cdot(y'')^2=+1$$

$$\frac{(x'')^2}{\left(\frac{272}{5}\right)}+\frac{(y'')^2}{\left(\frac{272}{20}\right)}=+1$$

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

equazione canonica ellisse

$(1,0,1) \quad (1,0,2) \quad (1,0,3) \qquad \mathbb{R}^3$

$$\det \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 2 \\ 1 & 0 & 3 \end{bmatrix} = 0 \Rightarrow \text{lin. } \underline{\underline{\text{DIP.}}}$$

NO BASE

$\mathbb{R}^4$

$u_1 = 1\ 0\ 1\ 0$
$u_2 = 1\ 0\ 2\ 0$
$u_3 = 0\ 1\ 0\ 0$

$0\ 0\ 0\ 0$

u_1, u_2, u_3 sono DIP o INDIP?
- DIP $\rightarrow$ NO BASE
- INDIP $\rightarrow$ $\exists$ base

$$A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Lei vuole che A abbia rango 3

$$B = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}; \quad \det B = ? = -1 \neq 0$$

quindi $rg\, B = 3$ e dunque $rg\, A = 3$

$u_1 = (1, 0, 1, \boxed{0})$
$u_2 = (1, 0, 2, \boxed{0})$
$u_3 = (0, 1, 0, \boxed{0})$
$u_4 = (0, 0, 0, 1)$

$r: x-y-5 = 5y-z-1 = 0$

$s: 5x-z = x-y = 0$

$$r: \begin{bmatrix} 1 & -1 & 0 \\ 0 & 5 & -1 \end{bmatrix} \rightarrow \begin{cases} \ell = 1 \\ m = -(-1) = 1 \\ n = 5 \end{cases} \qquad \vec{V}_r = (1,1,5)$$

$$s: \begin{bmatrix} 5 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \rightarrow \begin{cases} \ell' = -1 \\ \boxed{m' = -(1) = -1} \\ n' = -5 \end{cases} \qquad \vec{V}_s = (-1,-1,-5)$$

$$m' = -\det\begin{bmatrix} 5 & -1 \\ 1 & 0 \end{bmatrix} = -(5\cdot 0 - 1\cdot(-1)) = -(0+1) = -1$$

$$\vec{V}_s = -\vec{V}_r \Rightarrow s \parallel r$$

$|\vec{u} \cdot \vec{v}|$ con $\vec{u} \neq \vec{0}$

$$|\vec{u} \cdot \vec{v}| = \underbrace{\|\vec{u}\|}_{OA} \cdot \underbrace{\underbrace{\|\vec{v}\|}_{OB} \cdot |\cos\theta|}_{OC}$$

$\alpha = \pi - \theta$

$\cos\alpha = -\cos\theta$

$|\cos\alpha| = |\cos\theta|$

$$|\vec{u} \cdot \vec{v}| = OA \cdot OC$$

$$x'' = x' + \delta$$

$$5\cdot\underbrace{(x'+\delta)}_{x''=x'+\delta}{}^2 + 7\cdot\underbrace{(y'+\gamma)}_{y''=y'+\gamma}{}^2 + 36 = 0$$

$$5\cdot(x'')^2 + 7\cdot(y'')^2 \pm 36 = 0$$

$$\frac{(x'')^2}{\underbrace{\left(\frac{36}{5}\right)}_{=a^2}} + \frac{(y'')^2}{\underbrace{\left(\frac{36}{7}\right)}_{=b^2}} = \mp 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \pm 1$$