Lunedì 5 Luglio ore 15:00-18:00

Titolo nota 05/07/2021

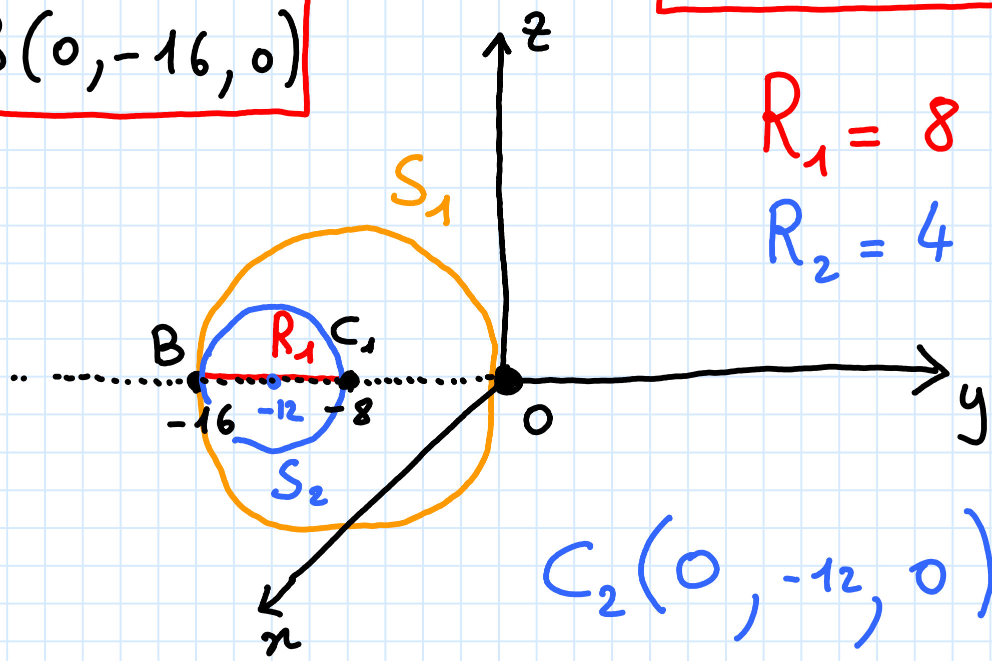
6) Sfera $S_1: x^2+y^2+z^2+16y=0$
sfera S_2 tang. internamente a S_1 nel punto $B(0,-16,0)$ e avente il raggio $R_2 = \frac{1}{2} R_1$.

$B \in S_1$ infatti $0^2+(-16)^2+0^2+16(-16)=0$

$\boxed{O \in S_1}$

C_1 centro sfera S_1 $\boxed{C_1(0,-8,0)}$

$\boxed{B(0,-16,0)}$



$R_1 = 8$

$R_2 = 4$

$C_2(0,-12,0)$

$S_2: (x-0)^2+(y-(-12))^2+(z-0)^2 = 4^2$

$x^2+y^2+z^2+24y+144=16$

$S_2: x^2+y^2+z^2+24y+128=0$

(1) $A = \begin{bmatrix} 0 & -4 & 0 \\ 0 & 4 & 0 \\ 0 & -4 & 0 \end{bmatrix}$; $A - \lambda I = \begin{bmatrix} -\lambda & -4 & 0 \\ 0 & (4-\lambda) & 0 \\ 0 & -4 & -\lambda \end{bmatrix}$

$$\det(A-\lambda I) = (4-\lambda)\cdot \det\begin{bmatrix} -\lambda & 0 \\ 0 & -\lambda \end{bmatrix} = \lambda^2 \cdot (4-\lambda)^1$$

$\boxed{\lambda_1 = 0}$ et $m_a(0) = 2$

$\boxed{\lambda_2 = 4}$ et $m_a(4) = 1$

$\boxed{\lambda_1 = 0}$ $\begin{bmatrix} 0 & -4 & 0 \\ 0 & 4 & 0 \\ 0 & -4 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$; $\begin{cases} -4y = 0 \\ 4y = 0 \\ -4y = 0 \end{cases}$ $\boxed{y = 0}$

x e z sono LIBERE

$$(x,y,z) = (x, 0, z) = x(1,0,0) + z(0,0,1)$$

$$E(0) = \{ x(1,0,0) + z(0,0,1) \in \mathbb{R}^3 \mid x, z \in \mathbb{R} \}$$

generatori e inoltre sono indipendenti

$B_0 = ((1,0,0),(0,0,1))$ base autospazio $E(0)$

$\boxed{\lambda_2 = 4}$ $\begin{bmatrix} -4 & -4 & 0 \\ 0 & 0 & 0 \\ 0 & -4 & -4 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$; $\begin{cases} -4x - 4y = 0 \\ 0 = 0 \\ -4y - 4z = 0 \end{cases}$

$\begin{cases} x + y = 0 \\ y + z = 0 \end{cases}$; $\begin{cases} y = -z \\ x - z = 0 \end{cases}$; $\begin{cases} y = -z \\ x = z \end{cases}$

$(x, y, z) = (z, -z, z) = z\,(1, -1, 1)$

$E(4) = \{ z(1,-1,1) \in \mathbb{R}^3 \mid z \in \mathbb{R} \}$

gen. lin. ind.

$B_4 = ((1,-1,1))$ base dell'autospazio $E(4)$

(5) $9x^2 - 10xy + 9y^2 + 44x - 12y + 32 = 0$

$A = \begin{bmatrix} 9 & -5 \\ -5 & 9 \end{bmatrix}$; $p_A(\lambda) = \lambda^2 - 18\lambda + 56 = (\lambda - 4)\cdot(\lambda - 14)$

$\lambda_1 = 4$; $\lambda_2 = 14$

$\boxed{\lambda_1 = 4}$ $\begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$; $\begin{cases} 5x - 5y = 0 \\ -5x + 5y = 0 \end{cases}$

$x - y = 0$; $y = x$

$(x, y) = (x, x) = x \cdot \underset{\text{autovettore}}{(1, 1)}$ $\forall x \in \mathbb{R},\ x \neq 0$

auto <u>VERSORE</u> $\frac{1}{\sqrt{2}} \cdot (1, 1) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

auto<u>versore</u> relativo a $\lambda_1 = +4$

$\vec{u}_1 = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$$C = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} \qquad \det C = +1$$

matrice associata ad una rotazione di $\frac{\pi}{4}$ rad in senso antiorario

$$\begin{bmatrix} 44 & -12 \end{bmatrix} \cdot \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} 16\sqrt{2} & -28\sqrt{2} \end{bmatrix}$$

$$4 \cdot (x')^2 + 14 \cdot (y')^2 + 16\sqrt{2} \cdot x' + (-28\sqrt{2}) \cdot y' + 32 = 0$$

$$\boxed{4(x')^2 + 16\sqrt{2}\, x'} \quad \boxed{+14(y')^2 - 28\sqrt{2}\, y'} \quad + 32 = 0$$

$$\boxed{4 \cdot \left(x' + 2\sqrt{2}\right)^2 - 32} \quad \boxed{+14 \cdot \left(y' - \sqrt{2}\right)^2 - 28} \quad + 32 = 0$$

traslazione $\begin{cases} x'' = x' + 2\sqrt{2} \\ y'' = y' - \sqrt{2} \end{cases}$

$$4 \cdot (x'')^2 - \cancel{32} + 14 \cdot (y'')^2 - 28 + \cancel{32} = 0$$

$$2 \cdot (x'')^2 + 7 \cdot (y'')^2 = +14$$

eq. canonica $\longrightarrow \dfrac{(x'')^2}{7} + \dfrac{(y'')^2}{2} = +1$ 99% (ellisse) 1%

(2) piano α passante per $A(-34, 9, \sqrt{7})$

parallelo alla retta $r: y+3z-7 = x-5z+10 = 0$

e perpendicolare al piano $\pi: 5x-3y+3=0$

$$\alpha: \underset{?}{a}\cdot(x-(-34)) + \underset{?}{b}\cdot(y-9) + \underset{?}{c}\cdot(z-\sqrt{7}) = 0$$

$$r: \begin{bmatrix} 0 & 1 & 3 \\ 1 & 0 & -5 \end{bmatrix} \begin{array}{l} \nearrow \ell = -5 \\ \rightarrow m = -(-3) = +3 \\ \searrow n = -1 \end{array}$$

$$(\ell_r, m_r, n_r) = (-5, 3, -1)$$

$$\alpha // r \iff a\cdot\ell_r + b\cdot m_r + c\cdot n_r = 0 \iff$$

$$\iff -5a + 3b - c = 0 \iff$$

$$\iff \boxed{c = 3b - 5a}$$

$$\vec{u}_\alpha = (a, b, 3b-5a) \perp \alpha$$

$\alpha \perp \pi: 5x-3y+3=0 \qquad (a', b', c') = (5, -3, 0)$

$$\alpha \perp \pi \iff a\cdot a' + b\cdot b' + c\cdot c' = 0 \iff$$

$$\iff 5a - 3b + 0(3b-5a) = 0 \iff$$

$\Leftrightarrow \quad 5a - 3b = 0$

scelgo $a = 3$ e $b = 5$

ottengo $c = 3b - 5a = 15 - 15 = 0$

$$\alpha : 3 \cdot (x + 34) + 5 \cdot (y - 9) + 0 \cdot (z - \sqrt{7}) = 0$$

$$\boxed{\alpha : 3x + 5y + 57 = 0}$$

(4) $r \parallel$ asse X passante per $A(\sqrt{2}, 4\sqrt{3}, -4\sqrt{3})$.

Piani α contenenti r e formano angolo $\frac{\pi}{4}$ rad con il piano XY

asse $X \parallel \vec{i} = (1, 0, 0)$

$\vec{i} \parallel r \Rightarrow (\ell_r, m_r, n_r) = (1, 0, 0)$

$A \in r \Rightarrow \quad r : \begin{cases} x = 1 \cdot t + \sqrt{2} \\ y = 0 \cdot t + 4\sqrt{3} \\ z = 0 \cdot t - 4\sqrt{3} \end{cases} ; \quad r : \begin{cases} x = t + \sqrt{2} \\ y = 4\sqrt{3} \\ z = -4\sqrt{3} \end{cases}$

$r : \begin{cases} y - 4\sqrt{3} = 0 \\ z + 4\sqrt{3} = 0 \end{cases}$ eq. cartesiane di r

$\alpha \in \mathcal{F}(r)$: $\lambda \cdot (y - 4\sqrt{3}) + \mu \cdot (z + 4\sqrt{3}) = 0$

piano XY : $z = 0$

α : $\underbrace{0}_{a} \cdot x + \underbrace{\lambda}_{b} \cdot y + \underbrace{\mu}_{c} \cdot z + 4\sqrt{3}(\mu - \lambda) = 0$

piano XY : $\underbrace{0}_{a'} \cdot x + \underbrace{0}_{b'} \cdot y + \underbrace{1}_{c'} \cdot z = 0$

Formula angolo θ tra 2 piani

$$\cos\theta = \pm \frac{a \cdot a' + b \cdot b' + c \cdot c'}{\sqrt{a^2 + b^2 + c^2} \cdot \sqrt{(a')^2 + (b')^2 + (c')^2}}$$

θ per noi è $\frac{\pi}{4}$ rad $= 45°$

$$\frac{\sqrt{2}}{2} = \pm \frac{\mu}{\sqrt{\lambda^2 + \mu^2} \cdot \sqrt{1^2}}$$

$$\sqrt{2} \cdot \sqrt{\lambda^2 + \mu^2} = \pm 2\mu$$

$$2 \cdot (\lambda^2 + \mu^2) = 4\mu^2 \quad ; \quad \mu^2 = \lambda^2$$

scelgo $\lambda = +1$; $\mu^2 = +1$; $\mu = \pm 1$

piano α_1 con $\lambda = 1$ e $\mu = 1$

piano α_2 con $\lambda = 1$ e $\mu = -1$

$$\lambda\cdot(y-4\sqrt{3})+\mu(z+4\sqrt{3})=0$$

$\lambda=1$ et $\mu=1 \Rightarrow \boxed{y+z=0}$ piano α_1

$\lambda=1$ et $\mu=-1 \Rightarrow \boxed{y-z-8\sqrt{3}}$ piano α_2

(3) $r: 5x+y-36=3x+2z=0$

$s: y+7z=2x+y-2z=0$ $\quad O\in s$

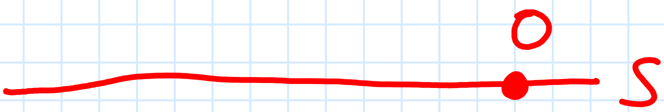
$\parallel \Rightarrow$ trovare piano che le contiene

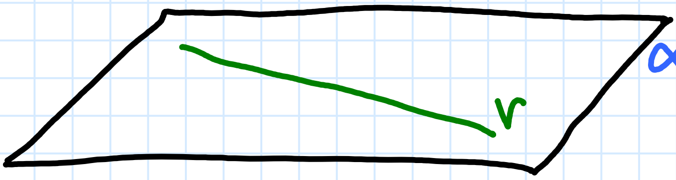
$\nparallel \Rightarrow$ trovare la loro distanza

$$r:\begin{bmatrix}5&1&0\\3&0&2\end{bmatrix}\begin{matrix}\nearrow l=2\\ \rightarrow m=-10\\ \searrow n=-3\end{matrix}\quad ; \quad s:\begin{bmatrix}0&1&7\\2&1&-2\end{bmatrix}\begin{matrix}\nearrow l'=-9\\ \rightarrow m'=+14\\ \searrow n'=-2\end{matrix}$$

$\vec{u}_r=(2,-10,-3)$

$\vec{u}_s=(-9,14,-2)$ $\quad \vec{u}_r \nparallel \vec{u}_s$





$\alpha: \alpha\in\mathcal{F}(r)$ et $\alpha\parallel s$

Trovato α

$$d(r,s)=d(O,\alpha)$$ distanza piano-punto

$r: 5x+y-36=3x+2z=0$

$\mathcal{F}(r): \quad \lambda(5x+y-36)+\mu(3x+2z)=0$

$$\underbrace{(5\lambda+3\mu)}_{a}\cdot x+\underbrace{\lambda}_{b}\cdot y+\underbrace{(2\mu)}_{c}\cdot z\ \underbrace{-36\lambda}_{d}=0$$

$$(\ell_s, m_s, n_s) \xrightarrow[\text{trovati}]{\text{già}} (\underbrace{-9}_{\ell}, \underbrace{14}_{m}, \underbrace{-2}_{n})$$

parallelismo retta - piano $\quad a\ell+bm+cn=0$

$$-9\cdot(5\lambda+3\mu)+14\cdot\lambda-2\cdot2\cdot\mu=0$$

$$-45\lambda-27\mu+14\lambda-4\mu=0$$

$$-31\lambda-31\mu=0 \quad ; \quad \boxed{\lambda+\mu=0}$$

scelgo $\lambda=1$ e ottengo $\mu=-1$

$$1\cdot(5x+y-36)-1\cdot(3x+2z)=0$$

piano α: $2x+y-2z-36=0$

$$d(O,\alpha)=\frac{|2\cdot0+0-2\cdot0-36|}{\sqrt{2^2+1^2+(-2)^2}}=\frac{|-36|}{\sqrt{9}}=\frac{36}{3}=12$$

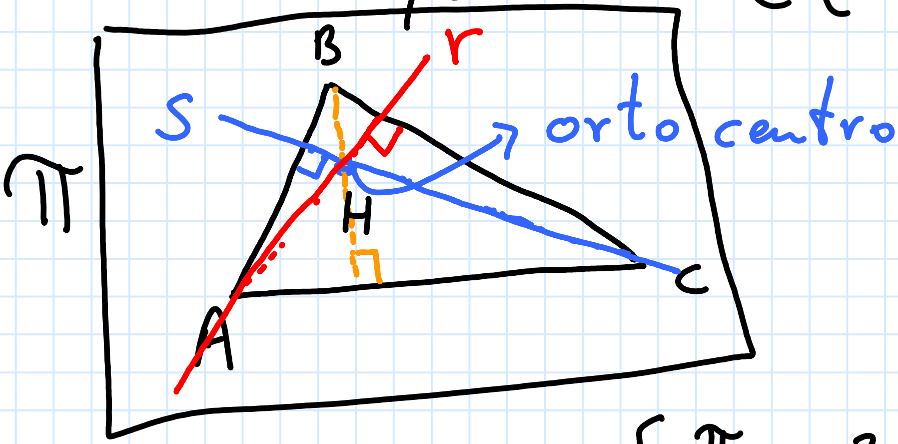
$$d(r,s)=12$$

$$\pi : 2x + y - z + 12 = 0$$

asse X : $y = z = 0$ $\qquad A(-6, 0, 0)$

asse Y : $x = z = 0$ $\qquad B(0, -12, 0)$

asse Z : $x = y = 0$ $\qquad C(0, 0, 12)$



$r = \pi \cap \alpha \rightarrow ?$

$s = \pi \cap \beta \rightarrow ?$

$\{H\} = r \cap s : \left\{ \begin{array}{l} \pi \\ \alpha \\ \beta \end{array} \right.$ 3 eq in 3 incognite

$$\boxed{\alpha = ? \quad \alpha \perp [\overrightarrow{BC}] \text{ et } A \in \alpha} \quad 90\%$$

$\beta = ?$ $\quad \beta \perp [\overrightarrow{AB}]$ et $C \in \beta$ $\quad 1\%$

$A(-6,0,0)$; $B(0,-12,0)$; $C(0,0,12)$

$$[\overrightarrow{BC}] = (0, 12, 12) = 12 \cdot (\underset{a}{0}, \underset{b}{1}, \underset{c}{1})$$

$$\alpha : 0 \cdot (x - (-6)) + 1 \cdot (y - 0) + 1 \cdot (z - 0) = 0$$

$$\boxed{\alpha : y + z = 0} \quad 2\%$$

$$[\overrightarrow{AB}] = (6, -12, 0) = 6(\underset{a}{1}, \underset{b}{-2}, \underset{c}{0})$$

$$\beta: 1\cdot(x-0)+(-2)\cdot(y-0)+0(z-12)=0$$

$$\boxed{\beta:\ x-2y=0}$$

$$\{H\}:\begin{cases} x-2y=0 \\ y+z=0 \\ 2x+y-z+12=0 \end{cases} \qquad \begin{cases} x=2y \\ z=-y \\ 2\cdot(2y)+y-(-y)+12=0 \end{cases}$$

$$6y+12=0 \Rightarrow y=-2 \Rightarrow x=-4$$

$$y=-2 \Downarrow z=+2$$

$H(-4,-2,+2)$ è l'ortocentro

$$a\cdot x^2+b\cdot x \qquad a\neq 0$$

$$\boxed{a\cdot x^2+bx} = a\cdot\left(x^2+\frac{b}{a}\cdot x\right) =$$

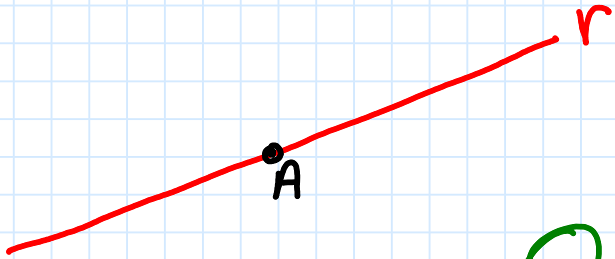
$$= a\cdot\left[\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a^2}\right] =$$

$$= \boxed{a\cdot\left(x+\frac{b}{2a}\right)^2-\frac{b^2}{4a}}$$

traslazione $\quad x' = x+\dfrac{b}{2a}$

Conica unione 2 rette reali distinte <u>NON</u> parallele che contenga la retta $r: 2x-y+1=0$ e passi per $A(0,1)$.

$A \in r$



$\mathscr{C} = r \cup s$ → una QUALSIASI retta del piano <u>NON</u> parallela ad s

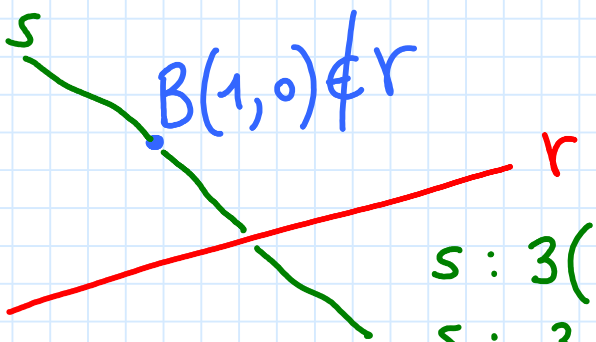
$r: 2x-y+1=0$

$t // r \qquad t: 2x-y+k=0$ → $\forall k \in \mathbb{R}$

$s: 3x+y+5=0$

$r: 2x-y+1=0$

$\mathscr{C} = r \cup s : (3x+y+5)\cdot(2x-y+1)=0$ <u>FINE</u>



$r: 2x-y+1=0$

$s: 3(x-1)+1(y-0)=0$

$s: 3x+y-3=0$

$$(2x - y + 1)\cdot(3x + y - 3) = 0$$

$$\underbrace{5x^2 + 2xy + 5y^2} - 2x - 10y + 7 = 0$$

$$A = \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix} ; \quad P_A(\lambda) = \det(A - \lambda I) = \dots =$$

$$= \lambda^2 - 10\lambda + 24 =$$

$$= (\lambda - 6)\cdot(\lambda - 4) ;$$

$$\lambda_1 = 6 \qquad \lambda_2 = 4$$

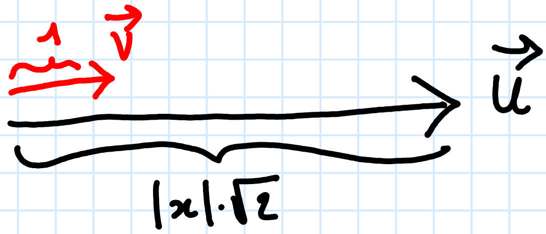
$\boxed{\lambda_1 = 6}$
$$\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} ; \quad \begin{cases} -x + y = 0 \\ x - y = 0 \end{cases}$$

$\boxed{y = x}$
$$(x, y) = (x, x) = x\,(1, 1) \quad \forall x \neq 0$$

autovettore

$$\vec{u} = x \cdot (1, 1) \qquad \forall x \neq 0$$

$$\|\vec{u}\| = ? = |x| \cdot \sqrt{2}$$



$$\vec{V}_1 = \frac{1}{\sqrt{2}} \cdot (1, 1) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = \left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$$

$\boxed{\lambda_2 = 4}$
$$\vec{V}_2 = \left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$$

$$\vec{V}_1 \bullet \vec{V}_2 = 0$$

ovvero

$$\vec{V}_1 \perp \vec{V}_2$$

$$\begin{array}{c c} C = & \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} \\ & \begin{array}{cc} \uparrow \vec{v}_1 & \uparrow \vec{v}_2 \end{array} \end{array}$$

$\det C = -1$

(no rotazione)

$$\begin{bmatrix} -2 & -10 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix} = \begin{bmatrix} -6\sqrt{2} & 4\sqrt{2} \end{bmatrix}$$

$$6 \cdot (x')^2 + 4 \cdot (y')^2 - 6\sqrt{2} \cdot x' + 4\sqrt{2}\, y' + 7 = 0$$

$$\boxed{6 \cdot (x')^2 - 6\sqrt{2} \cdot x'} \quad \boxed{+4 \cdot (y')^2 + 4\sqrt{2}\, y'} \quad +7 = 0$$

|| è uguale ||

$$\boxed{6 \cdot \left(x' - \frac{\sqrt{2}}{2}\right)^2 - 3} + \boxed{4 \cdot \left(y' + \frac{\sqrt{2}}{2}\right)^2 - 2} \quad +7 = 0$$

traslazione $\begin{cases} x'' = x' - \frac{\sqrt{2}}{2} \\ y'' = y' + \frac{\sqrt{2}}{2} \end{cases}$

$$\boxed{6 \cdot (x'')^2 - 3} + \boxed{4 \cdot (y'')^2 - 2} + 7 = 0$$

$$6 \cdot (x'')^2 + 4 \cdot (y'')^2 + 2 = 0$$

$$6(x'')^2 + 4\cdot(y'')^2 = -2$$

$$3\cdot(x'')^2 + 2\cdot(y'')^2 = -1$$

attenzione!
MENO UNO

$$\frac{(x'')^2}{\underbrace{\left(\frac{1}{3}\right)}_{a^2}} + \frac{(y'')^2}{\underbrace{\left(\frac{1}{2}\right)}_{b^2}} = -1$$

ELLISSE IMMAGINARIA