Lunedì 12 Luglio ore 15:30 – 18:00

Titolo nota — 12/07/2021

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Se possibile, diagonalizzarla

$$p_A(\lambda) = \det(A - \lambda I_3) = \begin{bmatrix} (1-\lambda) & 0 & 1 \\ 0 & (1-\lambda) & 0 \\ 1 & 0 & (1-\lambda) \end{bmatrix} =$$

$$= (1-\lambda) \cdot \det \begin{bmatrix} (1-\lambda) & 1 \\ 1 & (1-\lambda) \end{bmatrix} = (1-\lambda) \cdot \left[(1-\lambda)^2 - 1 \right] =$$

$$= (1-\lambda) \cdot (\lambda^2 - 2\lambda) = \lambda \cdot (1-\lambda) \cdot (\lambda - 2)$$

$$\lambda_1 = 0 \qquad \lambda_2 = 1 \qquad \lambda_3 = 2$$

Per un teorema, posso subito dire che esiste una base di $\mathbb{R}^3$ formata da autovettori di A e, quindi, A è diagonalizzabile.

$\boxed{\lambda_1 = 0}$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{cases} x + z = 0 \\ y = 0 \\ x + z = 0 \end{cases}$$

$y = 0$ et $z = -x$

$$(x,y,z)=(x,0,-x)=x(1,0,-1) \quad \forall x\in\mathbb{R}, x\neq 0$$

$\boxed{\lambda_2=1}$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{cases} z=0 \\ 0=0 \\ x=0 \end{cases}$$

y è libera

$$(x,y,z)=(0,y,0)=y(0,1,0) \quad \forall y\in\mathbb{R}, y\neq 0$$

$\boxed{\lambda_3=2}$

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{cases} -x + z=0 \\ -y=0 \\ x - z=0 \end{cases}$$

$y=0$ et $z=x$

$$(x,y,z)=(x,0,x)=x(1,0,1) \quad \forall x\in\mathbb{R}, x\neq 0$$

$\lambda_1=0$ autovettore $(1,0,-1)$

$\lambda_2=1$ autovettore $(0,1,0)$

$\lambda_3=2$ autovettore $(1,0,1)$

sono una base di $\mathbb{R}^3$ quindi sono indip.

$$\Lambda=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}; \qquad P=\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

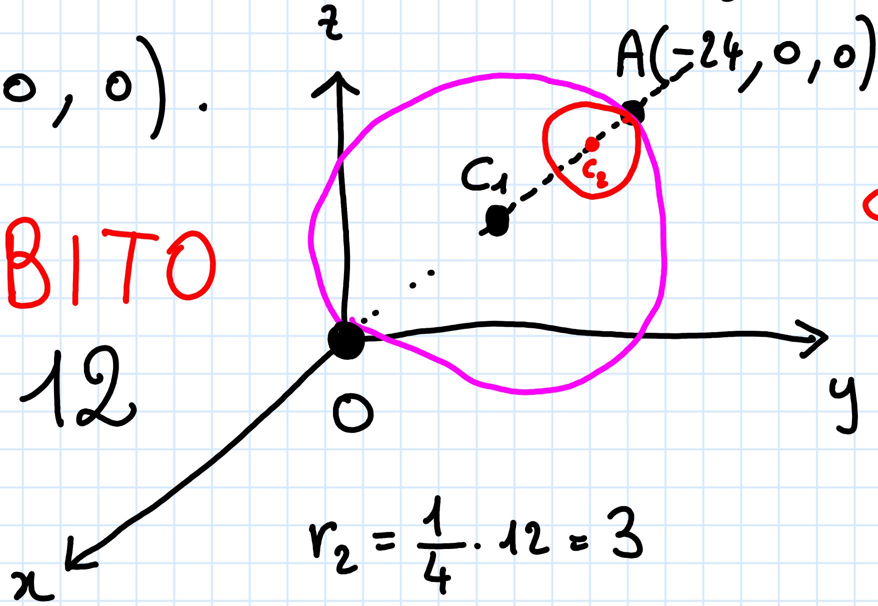
quindi $\operatorname{rg} P=3 \Rightarrow \det P\neq 0 \Rightarrow \exists P^{-1}$

Sfera ① $x^2+y^2+z^2+24x=0$ → vedo subito che $O \in S_1$

Sfera ② tang. int. a S_1 nel punto $A(-24,0,0)$ e avente il raggio $r_2 = \frac{1}{4} r_1$.

$C_1(-12,0,0)$.

vedo SUBITO che $r_1 = 12$



$C_2(-21,0,0)$

$r_2 = \frac{1}{4} \cdot 12 = 3$

$$S_2 : \left(x-(-21)\right)^2 + (y-0)^2 + (z-0)^2 = 3^2$$

$$S_2 : x^2+y^2+z^2+42x+432=0$$

$$A = \begin{bmatrix} 0 & 0 & 6 \\ 0 & -6 & 0 \\ 6 & 0 & 0 \end{bmatrix}$$

$$p_A(\lambda) = \det(A - \lambda \cdot I_3) = \det \begin{bmatrix} -\lambda & 0 & 6 \\ 0 & (-6-\lambda) & 0 \\ 6 & 0 & -\lambda \end{bmatrix} =$$

$$= (-6-\lambda)\cdot \det\begin{bmatrix} -\lambda & 6 \\ 6 & -\lambda \end{bmatrix} = -(\lambda+6)\cdot(\lambda^2-36) =$$

$$= -(\lambda+6)\cdot(\lambda+6)(\lambda-6) = (6-\lambda)^1\cdot(\lambda+6)^2$$

$\lambda_1 = 6 \qquad m_a(6) = 1$

$\lambda_2 = -6 \qquad m_a(-6) = 2$

$\boxed{\lambda_1 = 6}$
$$\begin{bmatrix} -6 & 0 & 6 \\ 0 & -12 & 0 \\ 6 & 0 & -6 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{cases} -6x + 6z = 0 \\ -12y = 0 \\ 6x - 6z = 0 \end{cases}$$

$y = 0 \;;\; z = x$

$(x,y,z) = (x,0,x) = x(1,0,1) \quad \forall x \in \mathbb{R},\ x \neq 0$

Base autospazio relativa a $\lambda_1 = 6$

$B_1 = \{(1,0,1)\}$

$\boxed{\lambda_2 = -6}$
$$\begin{bmatrix} 6 & 0 & 6 \\ 0 & 0 & 0 \\ 6 & 0 & 6 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{cases} 6x + 6z = 0 \\ 0 = 0 \\ 6x + 6z = 0 \end{cases}$$

$z = -x$ et y è libera

$(x,y,z) = (x,y,-x) = x(1,0,-1) + y(0,1,0)$

Base per autospazio relativo a autovalore

$\lambda_2 = -6$

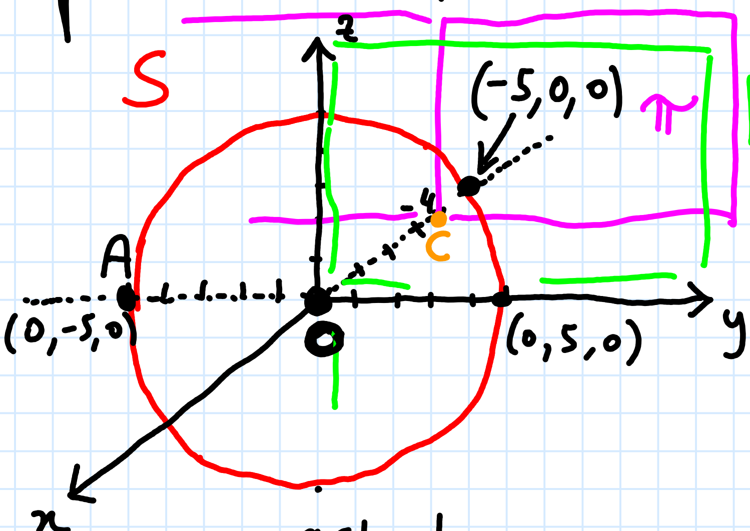
$$B_2 = \big((1,0,-1),(0,1,0)\big)$$

$$\dim E_{\lambda_2} = 2$$

DEF $\|$

$$m_g(-6) = 2$$

S sfera di centro $O(0,0,0)$ e passante per $A(0,-5,0)$. π piano // piano yz e passante per $B(-4,-3,2)$.
Trovare centro e raggio della circonferenza ottenuta secando S con π.



piano yz : $X=0$

$C(-4,0,0)$ centro della circonferenza

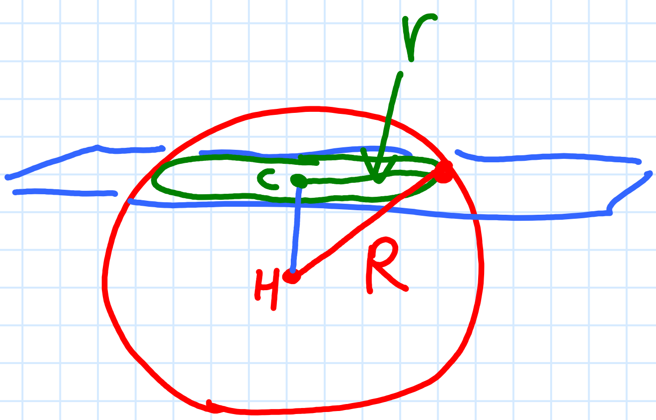
piano yz : $\overbrace{x=0}^{a=1 \quad b=0 \quad c=0}$

π // piano yz $\qquad \pi : x + d = 0 \qquad d = ?$

$B(-4,-3,2) \in \pi \Rightarrow -4+d=0 \Rightarrow d=4$

piano π : $x+4=0$

RAGGIO SFERA $= 5$



$r = \sqrt{R^2 - [d(C,H)]^2}$

$r = \sqrt{5^2 - 4^2} = \sqrt{9} = 3$