

SFERA

$$C(x_c, y_c, z_c)$$

$$R > 0$$

$$(x - x_c)^2 + (y - y_c)^2 + (z - z_c)^2 = R^2$$

$$x^2 + y^2 + z^2 - \underbrace{2x_c \cdot x}_a - \underbrace{2y_c \cdot y}_b - \underbrace{2z_c \cdot z}_c + x_c^2 + y_c^2 + z_c^2 - R^2 = 0$$

$$x^2 + y^2 + z^2 + a \cdot x + b \cdot y + c \cdot z + d = 0$$

$$C = \left(-\frac{a}{2}, -\frac{b}{2}, -\frac{c}{2} \right) \quad R = \frac{1}{2} \sqrt{a^2 + b^2 + c^2 - 4d}$$

$$(8.1) \quad (a) \quad x^2 + y^2 + z^2 - 12x + 4y - 1 = 0 \quad \begin{array}{l} a = -12 \\ b = +4 \\ c = 0 \end{array}$$

$$(b) \quad x^2 + y^2 + z^2 - 6y + 8z - 3 = 0$$

$$(c) \quad x^2 + y^2 + z^2 - 8x + 3y - 7z + \frac{61}{2} = 0$$

$$(a) \quad C(+6, -2, 0) \quad R = \frac{1}{2} \sqrt{(-12)^2 + 4^2 + 0^2 - 4(-1)} =$$

$$= \frac{1}{2} \sqrt{144 + 16 + 4} = \frac{1}{2} \sqrt{164} =$$

$$= \sqrt{41}$$

$$(b) \quad C(0, +3, -4) \quad R = \frac{1}{2} \sqrt{0 + 36 + 64 + 12} = \frac{1}{2} \sqrt{112} =$$

$$= \sqrt{28} = 2\sqrt{7}$$

$$(c) \quad C\left(4, -\frac{3}{2}, \frac{7}{2}\right) \quad R = \frac{1}{2} \sqrt{64 + 9 + 49 - 122} =$$

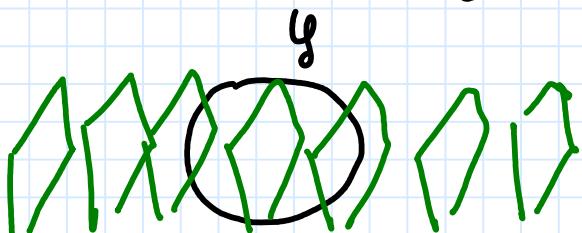
↪ è il punto C $= \frac{1}{2} \sqrt{122 - 122} = 0$

$$a^2 + b^2 + c^2 - 4d > 0$$

$d < 0 \Rightarrow a^2 + b^2 + c^2 - 4d$ è sicuramente > 0

(8.2) $\pi: 2x - z + R = 0 \quad R \in \mathbb{R} \rightarrow$ fascio
improprio
di piani

 $y: x^2 + y^2 + z^2 - 2x + 6z - 3 = 0$



mutua posizione

PIANO - SFERA

$$C(1, 0, -3) \quad R = \frac{1}{2} \sqrt{4+0+36+12} = \frac{1}{2} \sqrt{52}$$

$$\pi: 2x - z + R = 0 \quad = \sqrt{13}$$

$$d(C, \pi) = \frac{|2 \cdot 1 - (-3) + R|}{\sqrt{2^2 + 0^2 + (-1)^2}} = \frac{|5 + R|}{\sqrt{5}}$$

$d(C, \pi)$

- $\nearrow > R \rightarrow$ piano EST
- $= R \rightarrow$ piano TANG
- $\searrow < R \rightarrow$ piano SEC (circonferenza)

$$\pi \text{ è secente} \Leftrightarrow d(C, \pi) = \frac{|5 + R|}{\sqrt{5}} < \sqrt{13}$$

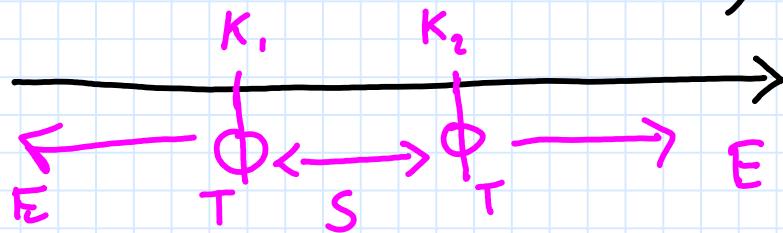
$$|5 + R| < \sqrt{65}$$

$$-\sqrt{65} < (5 + R) < \sqrt{65}$$

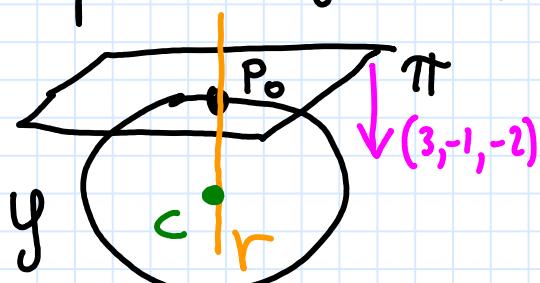
$$\pi \text{ è secente} \Leftrightarrow -\underbrace{\sqrt{65} - 5}_{K_1} < R < \underbrace{\sqrt{65} - 5}_{K_2}$$

$$\pi \text{ è tangente} \Leftrightarrow R = -\sqrt{65} - 5 ; R = \sqrt{65} - 5$$

Π è esterno $\Leftrightarrow k < -\sqrt{65} - 5$; $k > \sqrt{65} - 5$



(8.3) $\Pi : 3x - y - 2z + 5 = 0$ è tangente alla sfera $S : x^2 + y^2 + z^2 - 6x - 5 = 0$. Poi trovare il punto P_0 di contatto.



$$C(+3, 0, 0)$$

$$R = \frac{1}{2} \sqrt{36+0+0+20} = \frac{1}{2} \sqrt{56} = \sqrt{14}$$

verificare

$$d(C, \Pi) \stackrel{\downarrow}{=} R \quad \Pi : 3x - y - 2z + 5 = 0$$

$$d(C, \Pi) = \frac{|3 \cdot 3 - 0 - 2 \cdot 0 + 5|}{\sqrt{9+1+4}} = \frac{|14|}{\sqrt{14}} = \frac{14}{\sqrt{14}} = \sqrt{14} = R \quad \text{OK}$$

Come si trova il punto P_0 di contatto?

le sue coordinate

$$\vec{u}_\Pi = (3, -1, -2) \perp \Pi \Rightarrow \vec{u}_\Pi \parallel r$$

$$C(3, 0, 0) \in r$$

$$r : \begin{cases} x = 3 \cdot t + 3 \\ y = -1 \cdot t + 0 \\ z = -2 \cdot t + 0 \end{cases}$$

$$\Pi : 3x - y - 2z + 5 = 0$$

$$\{P_0\} = \widetilde{\Pi} \cap r : \left\{ \begin{array}{l} x = 3t + 3 \\ y = -t \\ z = -2t \\ 3x - y - 2z + 5 = 0 \end{array} \right.$$

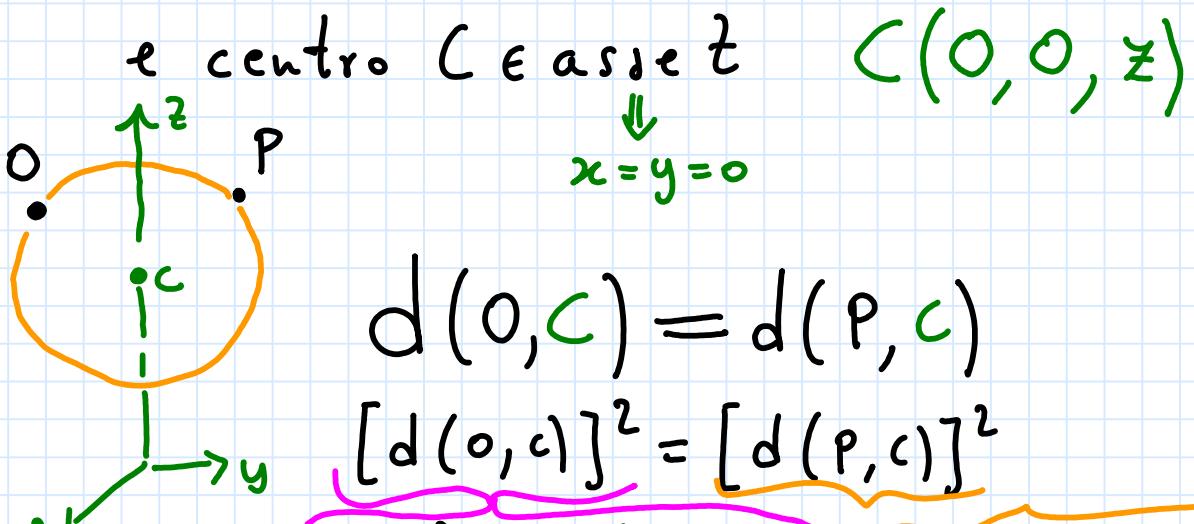
$$3(3t+3) - (-t) - 2(-2t) + 5 = 0 \quad 1^{\circ} \text{ in } t$$

$$\begin{array}{ll} \vdots & x = 3t + 3 = 0 \\ \Downarrow & y = -t = +1 \\ t = -1 \text{ (sol)} & z = -2t = +2 \end{array}$$

$P_0(0, +1, +2)$

ESATTO (ho verificato)

$$(8.8) \quad \mathcal{Y} : \quad O(0,0,0) \in \mathcal{Y} \quad \text{et} \quad P(2, -3, -1) \in \mathcal{Y}$$



$$z^2 = 4 + 9 + 1 + z^2 + 2z$$

$$2z + 14 = 0$$

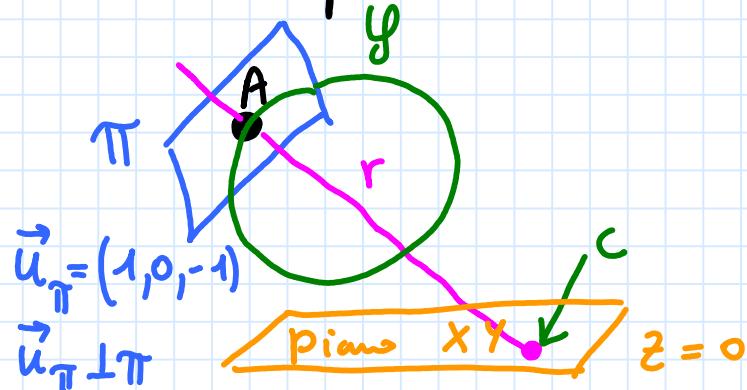
$$z = -7$$

$$C(0,0,-7)$$

$$y: x^2 + y^2 + z^2 + 14z = 0$$

vi ricordo
che $O \in \mathcal{Y}$

(8.11) γ tangente al piano π : $x - z - 1 = 0$
nel punto $A(0, 1, -1)$ e avente il centro C
nel piano $z = 0$ (piano XY)



$$r : \begin{cases} x = 1 \cdot t + 0 \\ y = 0 \cdot t + 1 \\ z = -1 \cdot t - 1 \end{cases}$$

$r \cap (\text{piano XY})$

$$\begin{aligned} z &= -t - 1 = 0 \\ t &= -1 \rightarrow x = t = -1 \\ &\quad \downarrow y = 1 \\ &\quad \downarrow z = -t - 1 = 0 \end{aligned}$$

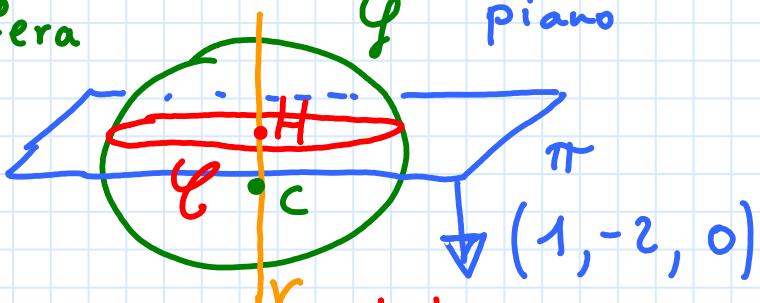
$$R = d(A, C) = ? = \sqrt{2}$$

$$(x - (-1))^2 + (y - 1)^2 + (z - 0)^2 = (\sqrt{2})^2$$

$$x^2 + y^2 + z^2 + 2x - 2y = 0$$

$$(8.18) \quad x^2 + y^2 + z^2 - 2x - 4y - 1 = \underline{x - 2y} = 0$$

equazione
della circonferenza



Trovare centro e raggio della circonferenza

C centro della sfera $C(1, 2, 0)$

$$R = \frac{1}{2} \sqrt{4+16+0+4} = \frac{1}{2} \sqrt{24} = \sqrt{6} \quad \text{raggio sfera}$$

H centro circonferenza
 $(l, m, n) = (1, -2, 0)$

$$C(1, 2, 0)$$

$$r : \begin{cases} x = 1 \cdot t + 1 \\ y = -2 \cdot t + 2 \\ z = 0 \cdot t + 0 \end{cases}$$

$$r : \begin{cases} x = t + 1 \\ y = -2t + 2 \\ z = 0 \end{cases}$$

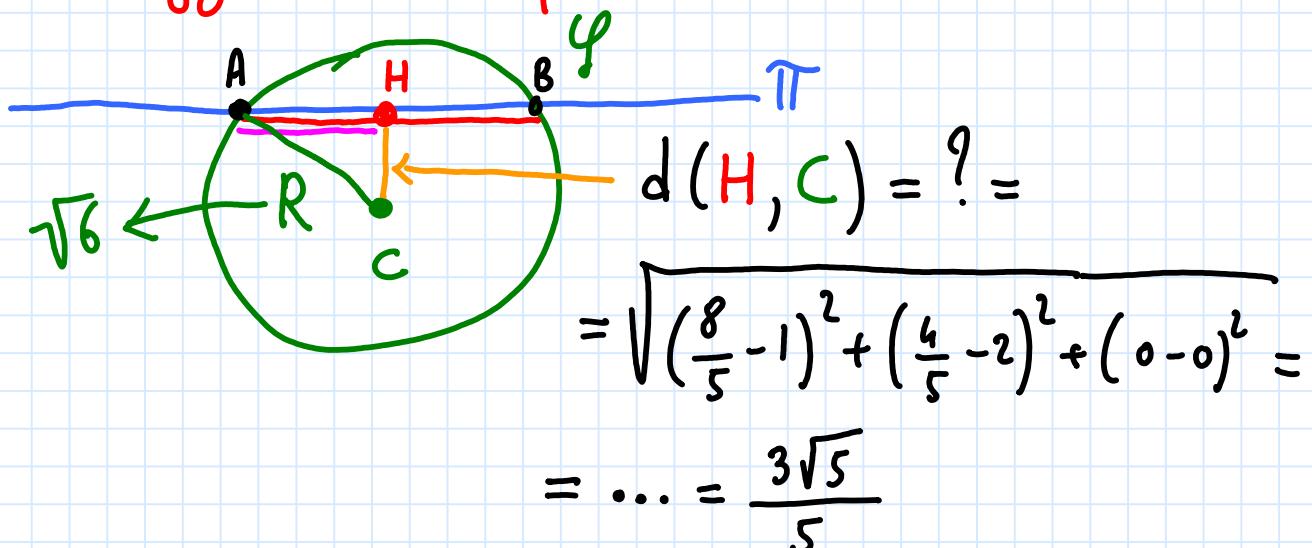
$$\pi : x - 2y = 0$$

$$r \cap \pi \Rightarrow (t+1) - 2(-2t+2) = 0 \Rightarrow$$

$$\Rightarrow t = \frac{3}{5} \quad \begin{aligned} x &= t + 1 = \frac{8}{5} \\ y &= -2t + 2 = \frac{4}{5} \\ z &= 0 \end{aligned}$$

$$H\left(\frac{8}{5}, \frac{4}{5}, 0\right) \quad ; \quad C(1, 2, 0)$$

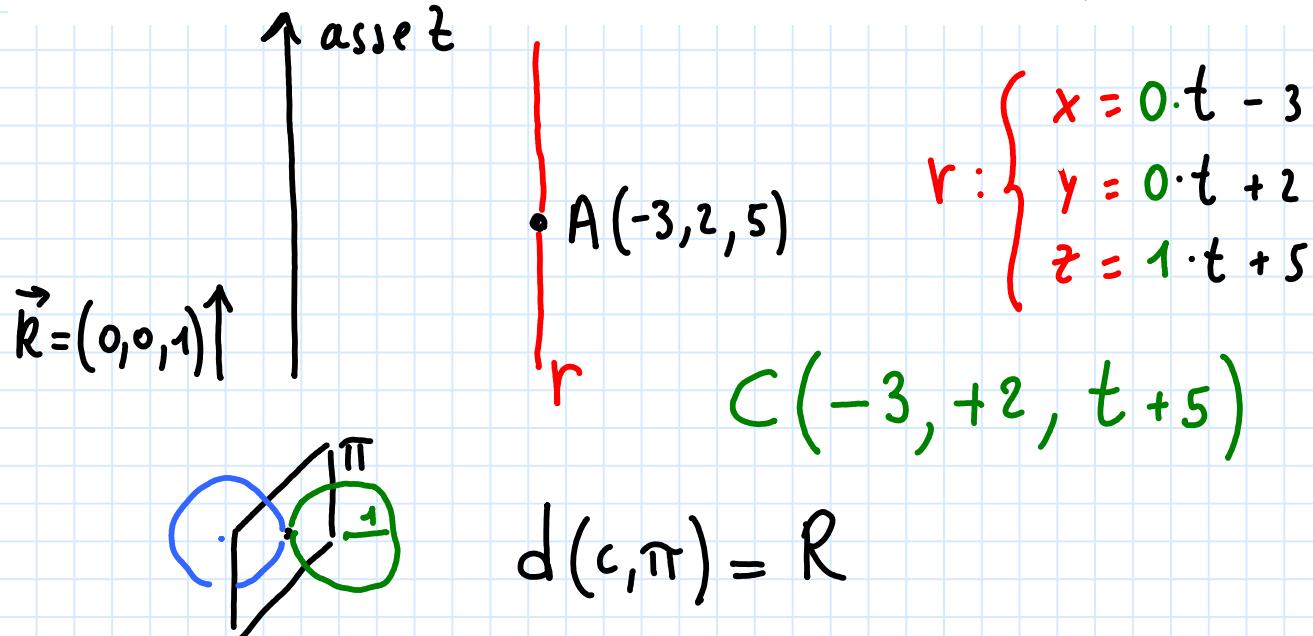
R = raggio circonferenza



$$r = \sqrt{R^2 - [d(H, c)]^2} = \dots = \frac{\sqrt{105}}{5}$$

6) Trovare le equazioni delle sfere di raggio 1, tangenti al piano $\pi : 2x - 2y + z + 1 = 0$ e aventi il centro sulla retta passante per $A(-3, 2, 5)$ e parallela all'asse Z.

2014



$$\pi : 2x - 2y + z + 1 = 0$$

$$d(c, \pi) = \frac{|-6 - 4 + t + 5 + 1|}{\sqrt{2^2 + (-2)^2 + 1^2}} = 1$$

$$|t - 4| = 3 \quad t - 4 = \pm 3 \quad t = 4 \pm 3$$

$t_1 = 7$
 $t_2 = 1$

$$C_1(-3, 2, 12)$$

$$C_2(-3, 2, 6)$$

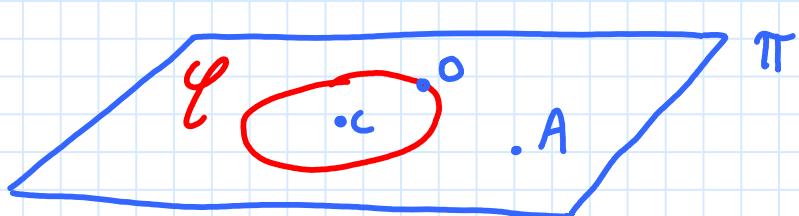
$$R = 1$$

$$R = 1$$

$$S_1 : (x+3)^2 + (y-2)^2 + (z-12)^2 = 1$$

$$S_2 : (x+3)^2 + (y-2)^2 + (z-6)^2 = 1$$

6) Sia π il piano per i punti $O(0, 0, 0)$, $A(2, 0, -3)$ e $C(0, 4, 0)$. Trovare delle equazioni per la circonferenza che giace su π , ha il centro in C e passa per O .



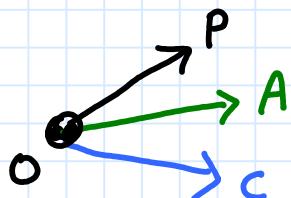
NELLO SPAZIO

$$\mathcal{C} = \pi \cap \mathcal{S} \rightarrow \text{Centro sfera } C(0,4,0)$$

$$O(0,0,0); A(2,0,-3); C(0,4,0) R=d(O,C)$$

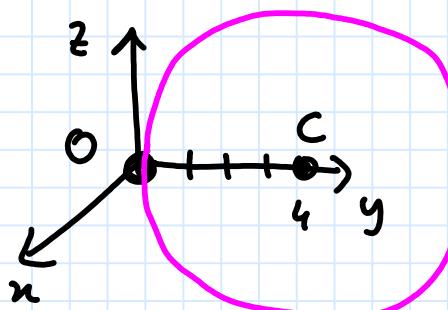
piano passante per 3 punti:

$$P(x, y, z)$$



$$\det \begin{bmatrix} x & y & z \\ 2 & 0 & -3 \\ 0 & 4 & 0 \end{bmatrix} \leftarrow [\vec{OP}] \quad \leftarrow [\vec{OA}] \quad \leftarrow [\vec{OC}] = 0$$

$$\boxed{\pi: 3x + 2z = 0}$$



$$C(0,4,0) \quad R=4$$

$$x^2 + y^2 + z^2 - 8y = 0$$

$$(x-0)^2 + (y-4)^2 + (z-0)^2 = 4^2$$

$$x^2 + y^2 + z^2 - 8y = 0$$

$$\boxed{\mathcal{C}: x^2 + y^2 + z^2 - 8y = 3x + 2z = 0}$$

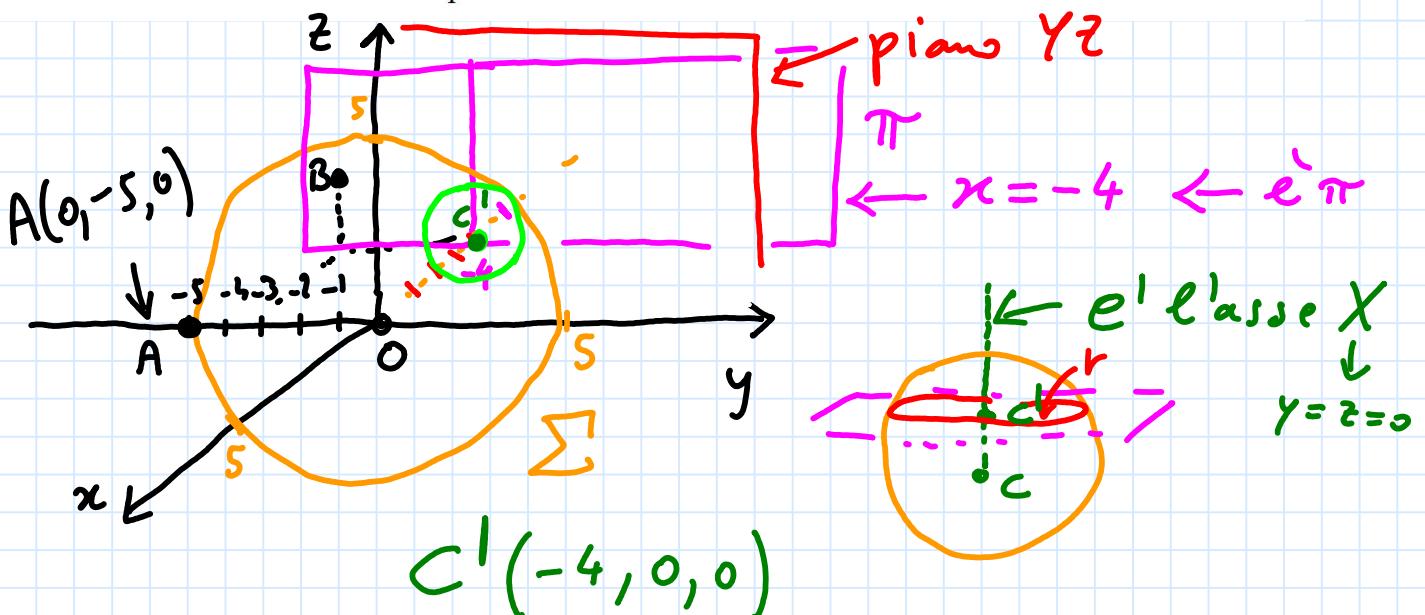
- 6) Scrivere l'equazione della sfera Σ che ha il centro in $C(2, 13, -14)$ ed è tangente al piano $\pi: 3y + 8z = 0$.

$$R = d(C, \pi) = \frac{|3 \cdot 13 + 8 \cdot (-14)|}{\sqrt{0^2 + 3^2 + 8^2}} = \dots = \sqrt{73}$$

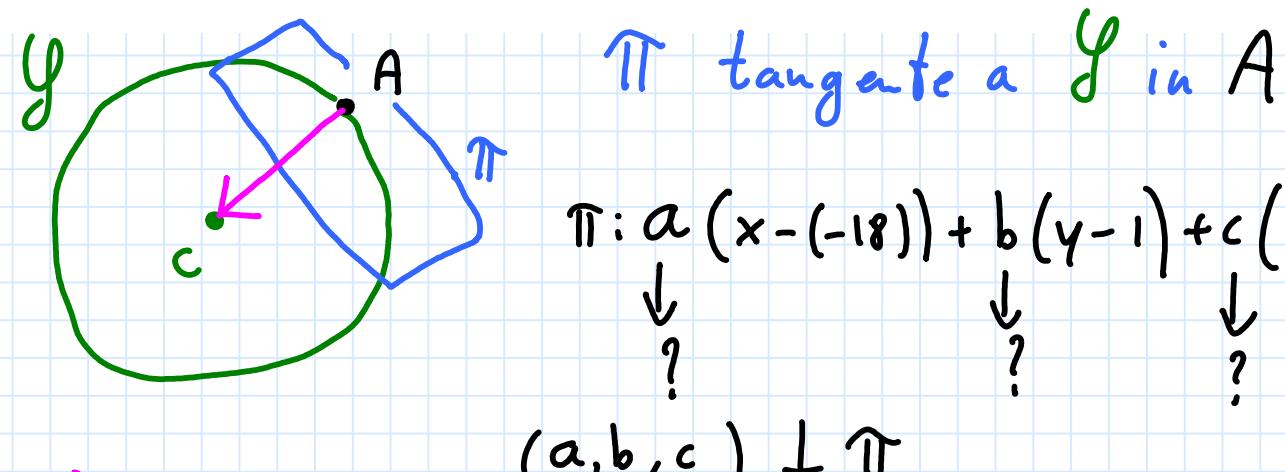
$$(x-2)^2 + (y-13)^2 + (z-(-14))^2 = R^2 = \cancel{73}$$

$$x^2 + y^2 + z^2 - 4x - 26y + 28z + 4 + \underbrace{169 + 14^2 - 73 = 0}_{\text{calcolate}}$$

- 6) Sia Σ la sfera di centro l'origine $O(0, 0, 0)$ e passante per $A(0, -5, 0)$. Sia π il piano parallelo al piano YZ e passante per il punto $B(-4, -3, 2)$. Trovare il centro e il raggio della circonferenza C ottenuta secando la sfera Σ col piano π .



- 6) Sia S la sfera di centro $C(-20, 1, 5)$ e passante per il punto $A(-18, 1, -4)$. Scrivere l'equazione del piano π tangente ad S nel punto A .



$$[\vec{AC}] = (-20 - (-18), 1 - 1, 5 - (-4)) = (-2, 0, +9)$$

$a \quad b \quad c$

$$-2(x + 18) + 0(y - 1) + 9(z + 4) = 0$$

$$-2x - 3y + 9z + 36 = 0$$

$$\boxed{11: 2x - 9z = 0}$$

