

Ricevimento studenti - lunedì 7 febbraio 2022

Titolo nota

07/02/2022

$$\operatorname{rg} A = 1 \quad ; \quad \operatorname{rg} B = 1 \quad ; \quad \operatorname{rg} (A+B) = 3 \Rightarrow C = A+B$$
$$A = \begin{bmatrix} A_1 \\ A_2 \\ A_3 \\ \vdots \\ A_m \end{bmatrix} \quad \begin{array}{l} A_1 \neq 0 \\ A_i = \alpha_i A_1 \\ i=2, \dots, m \end{array}$$
$$B = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ \vdots \\ B_m \end{bmatrix} \quad \begin{array}{l} B_1 \neq 0 \\ B_i = \beta_i B_1 \\ i=2, \dots, m \end{array}$$
$$C = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ \vdots \\ C_m \end{bmatrix} \quad C_i = \alpha_i A_i + \beta_i B_i$$

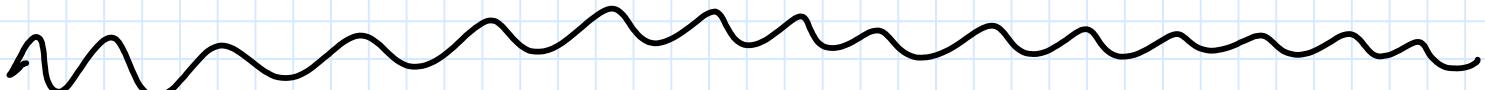
$$\operatorname{rg} C = \dim R_C \quad R_C = \langle A_1, B_1 \rangle$$

$$\dim R_C \leq 2 \neq 3 \text{ impossibile}$$



$$\operatorname{rg} A = 2 \quad ; \quad \operatorname{rg} B = 3 \quad ; \quad \operatorname{rg} (A+B) = 4$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} ; B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} ; A+B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$\operatorname{rg} A = 2 \quad \operatorname{rg} B = 3 \quad \operatorname{rg} (A+B) = 4$$



$\alpha \in F(\text{asse } Y)$ tale che α forma un angolo
 $\frac{\pi}{3}$ radianti con l'asse Z

asse Y : $x=z=0$

$$F(\text{asse } Y) : \lambda x + \mu z = 0 \quad (\lambda, \mu) \neq (0, 0)$$

$$\frac{\lambda}{a} \cdot x + \frac{0}{b} \cdot y + \frac{\mu}{c} \cdot z = 0$$

asse $\vec{z} \rightarrow$ parametri direttori $(l, m, n) = (0, 0, 1)$
angolo retta - piano

$$\sin \theta = \frac{|\lambda l + b m + c n|}{\sqrt{a^2 + b^2 + c^2} \cdot \sqrt{l^2 + m^2 + n^2}}$$

$$\frac{\pi}{3} \text{ rad} = 60^\circ$$

$$\frac{\sqrt{3}}{2} = \frac{|\lambda \cdot 0 + 0 \cdot 0 + \mu \cdot 1|}{\sqrt{\lambda^2 + 0^2 + \mu^2} \cdot \sqrt{0^2 + 0^2 + 1^2}}$$

$$\frac{\sqrt{3}}{2} = \frac{|\mu|}{\sqrt{\lambda^2 + \mu^2}} ; \quad \sqrt{3} \cdot \sqrt{\lambda^2 + \mu^2} = 2 \cdot |\mu| ;$$

$$3 \cdot (\lambda^2 + \mu^2) = 4 \mu^2 ; \quad \mu^2 = 3 \lambda^2 ; \quad \text{scelgo } \lambda = 1 \\ \mu^2 = 3 ; \quad \mu = \pm \sqrt{3}$$

$$1 \cdot x \pm \sqrt{3} z = 0 \quad \begin{array}{l} \xrightarrow{\alpha_1: x + \sqrt{3} z = 0} \\ \xrightarrow{\alpha_2: x - \sqrt{3} z = 0} \end{array}$$



$$O(0,0,0) ; \quad \Pi: x - 3y + 4z = 0 ; \quad \begin{array}{l} a' = 1 \\ b' = -3 \\ c' = 4 \end{array}$$

$$r: y = x - 3y + 4z = 0$$

α tale che $O \in \alpha$; $\alpha \perp \Pi$; $\alpha \parallel r$

$$0 \in \alpha \Leftrightarrow \alpha: ax + by + cz = 0 \quad \underline{\underline{d=0}}$$

$$\alpha \perp \pi \Leftrightarrow a \cdot a' + b \cdot b' + c \cdot c' = 0 \Rightarrow a - 3b + 4c = 0$$

$$\boxed{a = 3b - 4c} \quad (3b - 4c)x + b \cdot y + c \cdot z = 0$$

$$\alpha \parallel r \Leftrightarrow \det \begin{bmatrix} (3b - 4c) & b & c \\ 0 & 1 & 0 \\ 1 & -3 & 4 \end{bmatrix} = 0$$

$$1. \det \begin{bmatrix} (3b - 4c) & c \\ 1 & 4 \end{bmatrix} = 0$$

$$4 \cdot (3b - 4c) - c = 0 ; \quad 12b - 17c = 0 ;$$

$$(b, c) = (17, 12)$$

$$a = 3 \cdot b - 4 \cdot c = 3 \cdot 17 - 4 \cdot 12 = 51 - 48 = 3$$

$$\alpha: 3x + 17y + 12z = 0$$



$$t \in \mathbb{R} ; \quad r: (t-1)x + 5y + (t^2-12)z = y + 13 = 0$$

\vec{r} parallela alla retta passante per i punti

$$A(2-4t, 5, -9) ; \quad B(2-3t, 5, -t-8)$$



$$r \parallel s \Leftrightarrow \vec{v}_r \parallel [\vec{AB}]$$

$$[\vec{AB}] = (t, 0, -t+1)$$

$$\begin{bmatrix} (t-1) & 5 & (t^2-12) \\ 0 & 1 & 0 \end{bmatrix} \rightarrow \begin{array}{l} l = 12 - t^2 \\ m = 0 \\ n = t - 1 \end{array}$$

$$\vec{V}_r = (12 - t^2, 0, t - 1)$$

$$-\vec{V}_r = (t^2 - 12, 0, -t + 1)$$

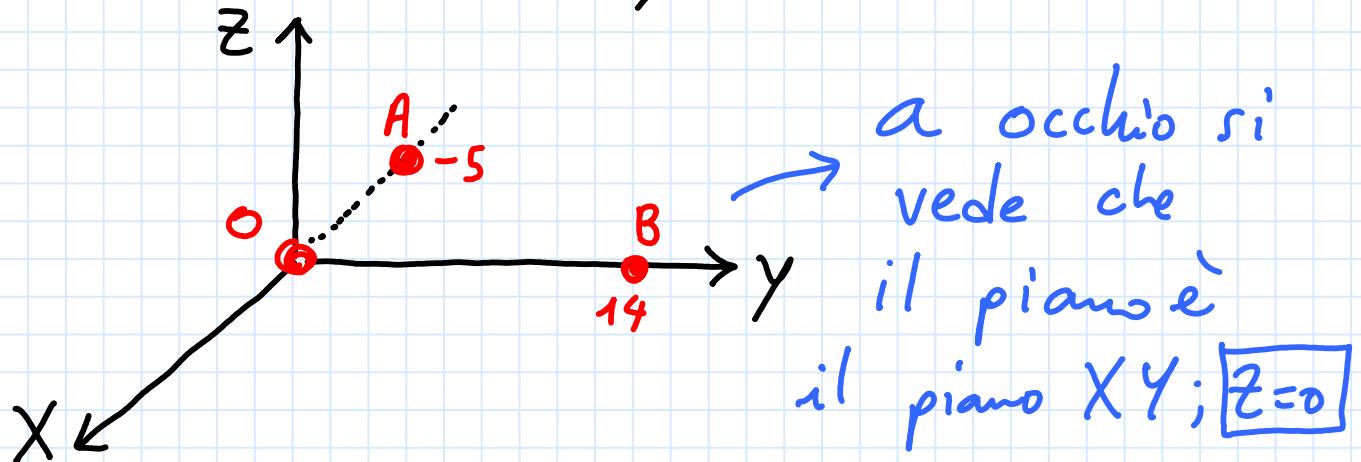
$$[\vec{AB}] = (t, 0, -t+1)$$

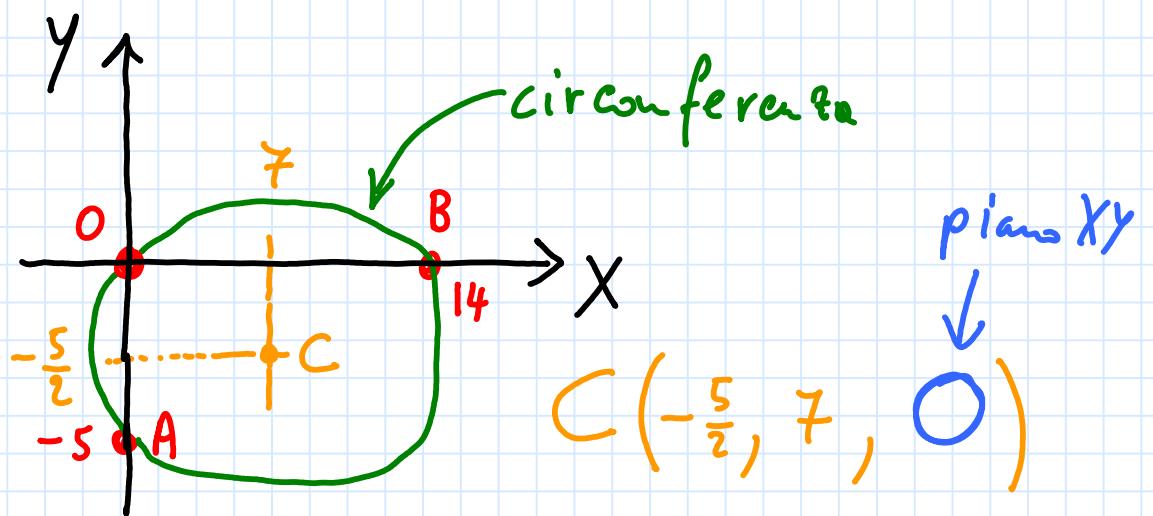
$$-\vec{V}_r \parallel [\vec{AB}] \Leftrightarrow t^2 - 12 = t \Leftrightarrow t^2 - t - 12 = 0$$

$$(t-4)(t+3) = 0 \rightarrow \begin{array}{l} t_1 = 4 \\ t_2 = -3 \end{array} \quad \text{accettabili}$$

wavy line

$O(0,0,0)$; $A(-5,0,0)$; $B(0,14,0)$





C centro sfera $(-\frac{5}{2}, 7, 0)$
 sfera per origine

$$y: x^2 + y^2 + z^2 + ax + by + cz = 0 \quad d=0$$

$$-\frac{5}{2} = x_0 = -\frac{a}{2} \Rightarrow a = 5$$

$$7 = y_0 = -\frac{b}{2} \Rightarrow b = -14$$

$$0 = z_0 = -\frac{c}{2} \Rightarrow c = 0$$

Circ: $\begin{cases} x^2 + y^2 + z^2 + 5x - 14y = 0 \\ z = 0 \end{cases}$



$r \parallel$ asse z e passante per $A(-3, 2, 5)$

Trovare i punti di r che hanno distanza 1 dal piano $\pi: 2x - 2y + z - 5 = 0$

$$r \parallel$$
 asse $z \parallel \vec{R} = (0, 0, 1) \Rightarrow (\ell, m, n) = (0, 0, 1)$

$$r: \begin{cases} x = 0 \cdot t - 3 \\ y = 0 \cdot t + 2 \\ z = 1 \cdot t + 5 \end{cases} \quad r: \begin{cases} x = -3 \\ y = +2 \\ z = t + 5 \end{cases}$$

$$P(-3, 2, t+5) \in r \quad \forall t \in \mathbb{R}$$

$$\pi: 2x - 2y + z - 5 = 0$$

$$d(P, \pi) = 1 \Rightarrow \frac{|2 \cdot (-3) - 2 \cdot (2) + (t+5) - 5|}{\sqrt{2^2 + (-2)^2 + 1^2}} = 1$$

$$\frac{|-6 - 4 + t + 5 - 5|}{3} = 1 \Rightarrow |t - 10| = 3 \Rightarrow$$

$$t - 10 = \pm 3 \Rightarrow t = 10 \pm 3 \rightarrow \begin{cases} t_1 = 13 \\ t_2 = 7 \end{cases}$$

$$t_1 = 13 \Rightarrow P_1(-3, 2, 18)$$

$$t_2 = 7 \Rightarrow P_2(-3, 2, 12)$$



dopo la rotazione (premesso che sia corretta)

$$5y^2 + 2\sqrt{5} \left(-\frac{1}{\sqrt{5}}x + \frac{2}{\sqrt{5}}y \right) + 6\sqrt{5} \left(-\frac{2}{\sqrt{5}}x + \frac{1}{\sqrt{5}}y \right) + 5 + 2\sqrt{3} = 0$$

$$5y^2 + 2x + 4y - 12x + 6y + 5 + 2\sqrt{3} = 0$$

$$5y^2 - 10x + 10y + 5 + 2\sqrt{3} = 0$$

$$5y^2 - 10y - 10x + 5 + 2\sqrt{3} = 0$$

$$5(y-1)^2 - 5 - 10x + 5 + 2\sqrt{3} = 0$$

traslo solo rispetto a y

$$5\tilde{y}^2 - 10x + 2\sqrt{3} = 0$$

$$10x - 2\sqrt{3} = 5\tilde{y}^2$$

$$10x = 5y^2 + 2\sqrt{3}$$

$$x = \frac{1}{2}\tilde{y}^2 + \frac{\sqrt{3}}{5}$$

parabola

$$\underbrace{x - \frac{\sqrt{3}}{5}}_{=} = \frac{1}{2}\tilde{y}^2$$

INCOMPLETO non e'

$$\hookrightarrow \text{traslazione } \tilde{x} = x - \frac{\sqrt{3}}{5}$$

canonica

$$\rightarrow \tilde{x} = \frac{1}{2}\tilde{y}^2 \quad \text{Eq. canonica}$$



$$r: 3x - 2y = 2z + 5 = 0$$

distanza 1 del piano $\alpha: x - 4 = 0$

$$r: \begin{cases} 3x - 2y = 0 \\ 2z + 5 = 0 \end{cases} \Rightarrow \begin{cases} 3x - 2y = 0 \\ z = -\frac{5}{2} \end{cases} \Rightarrow \begin{cases} x = 2t \\ y = 3t \\ z = -\frac{5}{2} \end{cases}$$

$$r: \begin{cases} x = 2t \\ y = 3t \\ z = -\frac{5}{2} \end{cases}$$

$$P(2t, 3t, -\frac{5}{2}) \in r \quad \forall t \in \mathbb{R}$$

$$d(P, \Pi) = 1 \Rightarrow \frac{|2t - 4|}{\sqrt{1^2 + 0^2 + 0^2}} = 1$$

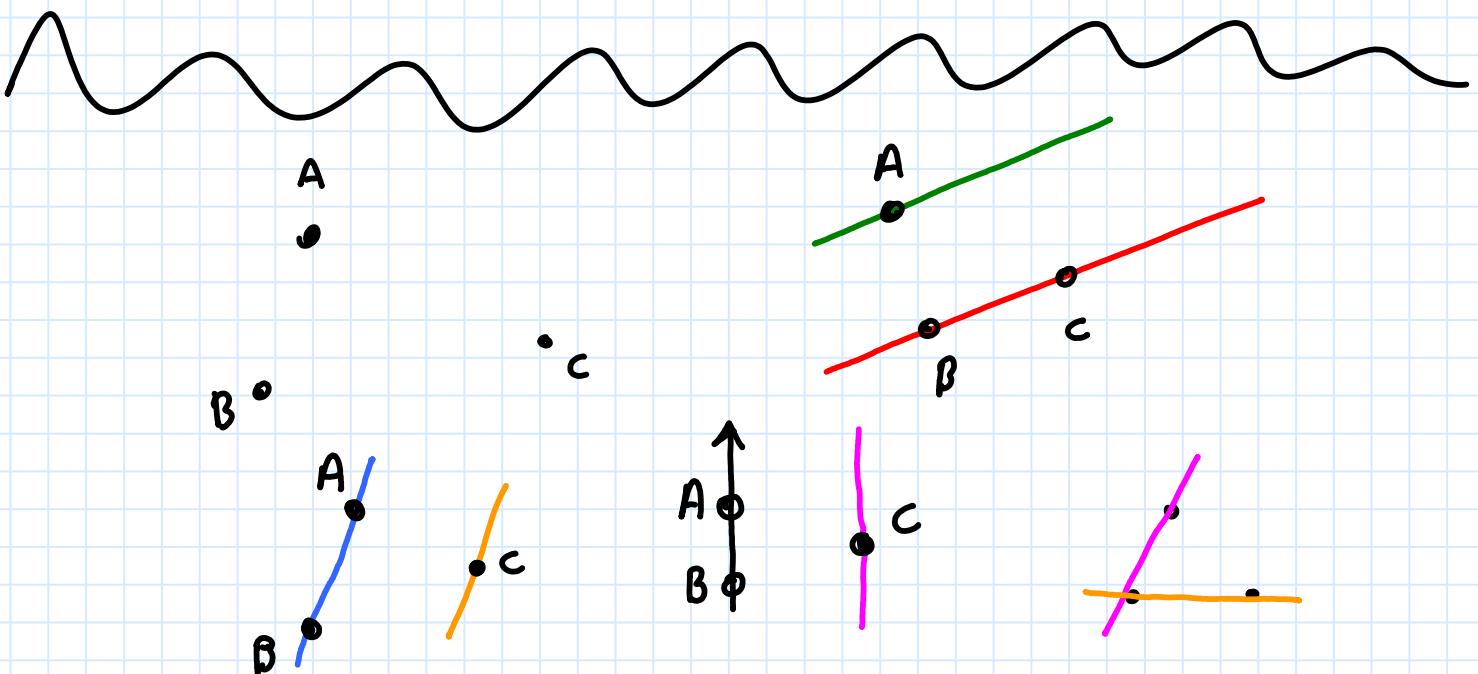
$$|2t - 4| = 1 ; \quad 2t - 4 = \pm 1 \Rightarrow 2t = 4 \pm 1 \Rightarrow$$

$$2t = \begin{matrix} \nearrow 5 \\ \searrow 3 \end{matrix} \rightarrow 2t = 5 \Rightarrow t = \frac{5}{2}$$

$$2t = 3 \rightarrow 2t = 3 \Rightarrow t = \frac{3}{2}$$

$$t_1 = \frac{5}{2} \Rightarrow P_1\left(5, \frac{15}{2}, -\frac{5}{2}\right)$$

$$t_2 = \frac{3}{2} \Rightarrow P_2\left(3, \frac{9}{2}, -\frac{5}{2}\right)$$



$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}; \quad p_A(\lambda) = \det \begin{bmatrix} (1-\lambda) & 0 & 1 \\ 0 & (1-\lambda) & 0 \\ 1 & 0 & (1-\lambda) \end{bmatrix}$$

$$\begin{aligned} p_A(\lambda) &= (1-\lambda) \cdot \det \begin{bmatrix} (1-\lambda) & 1 \\ 1 & (1-\lambda) \end{bmatrix} = (1-\lambda) \cdot [(1-\lambda)^2 - 1] = \\ &= (1-\lambda) \cdot (\lambda^2 + \cancel{-2\lambda} - \cancel{1}) = \lambda \cdot (1-\lambda)(\lambda-2), \\ &\quad \downarrow \quad \downarrow \quad \downarrow \\ \lambda_1 = 0 & \quad \lambda_2 = 1 \quad \lambda_3 = 2 \end{aligned}$$

$n=3$ e ho 3 autovalori reali e distinti.

ho sicuramente una base di autovettori
e, quindi, A è diagonalizzabile.

$$\boxed{\lambda_1 = 0} \quad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad \begin{cases} x + z = 0 \rightarrow z = -x \\ x + y = 0 \rightarrow y = -x \\ x + z = 0 \end{cases} \quad \begin{array}{l} z = -x \\ y = -x \\ x = x \end{array}$$

$$(x, y, z) = (x, 0, -x) = x \cdot (1, 0, -1) \quad \forall x \in \mathbb{R} - \{0\}$$

autovettori di $\lambda_1 = 0$

$$\boxed{\lambda_2 = 1} \quad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad \begin{cases} z = 0 \rightarrow z = 0 \\ 0 = 0 \\ x = 0 \end{cases} \quad \begin{array}{l} z = 0 \\ 0 = 0 \\ x = 0 \end{array}$$

$$(x, y, z) = (0, y, 0) = y \cdot (0, 1, 0) \quad \forall y \in \mathbb{R} - \{0\}$$

LIBERA

autovettori di $\lambda_2 = 1$

$$\boxed{\lambda_3 = 2} \quad \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} ; \quad \begin{cases} -x + z = 0 \\ -y = 0 \\ x - z = 0 \end{cases}$$

$$z = x \quad \text{et} \quad y = 0$$

$$(x, y, z) = (x, 0, x) = x(1, 0, 1) \quad \forall x \in \mathbb{R} - \{0\}$$

autovettore di $\lambda_3 = 2$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

\mathbf{A} e \mathbf{C} sono le due matrici richieste

$$\mathbf{A} = \mathbf{C} \cdot \mathbf{A} \cdot \mathbf{C}^{-1}$$



$$r: 2x + 7 = 2x + 5z - 7 = 0$$

$$s: 3y + 20 = 3y + 5z = 0$$

t retta di minima distanza

$$\{A\} = t \cap r ; \quad \{H\} = t \cap s ; \quad D = d(A, H) ;$$

B e C i punti di S aventi distanza 2D

d'A. Calcolare area $\triangle ABC$

$$r: 2x+7 = 2x+5z+7=0$$

$$r: \begin{cases} 2x+7=0 \\ 2x+5z+7=0 \end{cases}$$

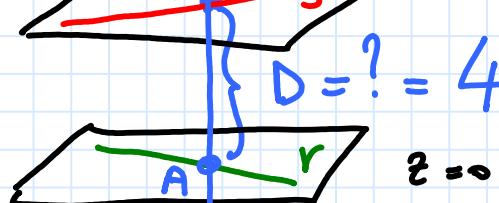
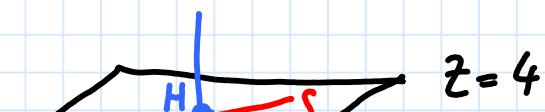
$$r: \begin{cases} 2x+7=0 \\ 5z=0 \end{cases}$$

$$r: \begin{cases} 2x+7=0 \\ z=0 \end{cases}$$

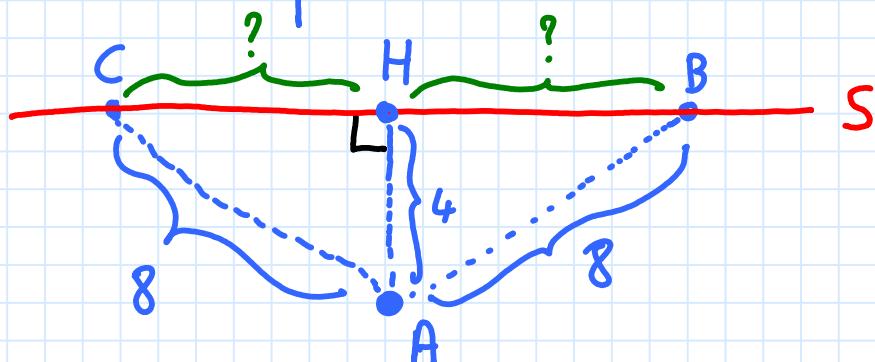
$$s: \begin{cases} 3y+20=0 \\ 3y+5z=0 \end{cases}$$

$$s: \begin{cases} 3y+20=0 \\ -20+5z=0 \end{cases}$$

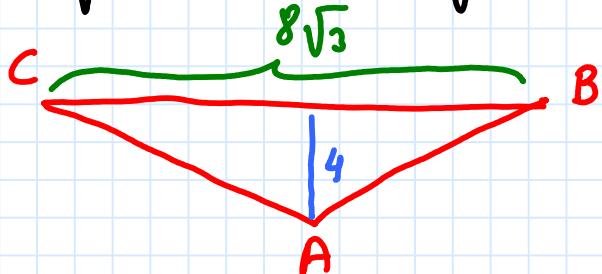
$$s: \begin{cases} 3y+20=0 \\ z=4 \end{cases}$$



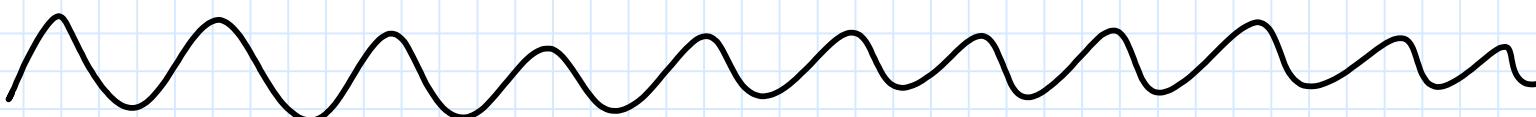
$d(s, r) = \text{distanza tra le 2 rette sghembe}$



$$d(C, H) = \sqrt{8^2 - 4^2} = \sqrt{64 - 16} = \sqrt{48} = 4\sqrt{3}$$



$$\text{area} = 8\sqrt{3} \cdot 4 \cdot \frac{1}{2} = 16\sqrt{3}$$



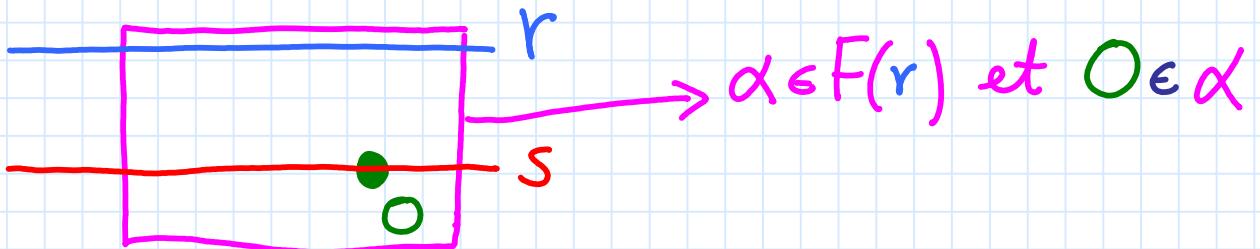
SE ESISTE, trovare il piano che contiene le seguenti due rette

$$r: 5y - z + 2 = x - 1 = 0$$

$$s: 5y - z = x = 0 \rightarrow$$

si vede subito che r e s sono parallele
e, quindi, COMPLANARI

si vede anche subito che $O(0,0,0) \in s$



$$r: 5y - z + 2 = x - 1 = 0$$

$$F(r): \lambda \cdot (5y - z + 2) + \mu \cdot (x - 1) = 0$$

$$O(0,0,0) \in \alpha \Rightarrow \lambda \cdot (5 \cdot 0 - 0 + 2) + \mu \cdot (0 - 1) = 0 \Rightarrow$$

$$\Rightarrow 2 \cdot \lambda - \mu = 0 \Rightarrow \text{sceglo } \lambda = 1 \text{ et } \mu = 2$$

$$\alpha: 1 \cdot (5y - z + 2) + 2 \cdot (x - 1) = 0$$

$$\alpha: 2 \cdot x + 5y - z = 0$$

è il
piano
richiesto



$$5x^2 - 2\sqrt{3}xy + 7y^2 - 12\sqrt{3}x + 20y + 12 = 0$$

$$A = \begin{bmatrix} 5 & -\sqrt{3} \\ -\sqrt{3} & 7 \end{bmatrix}; P_A(\lambda) = \det \begin{bmatrix} (5-\lambda) & -\sqrt{3} \\ -\sqrt{3} & (7-\lambda) \end{bmatrix}$$

$$P_A(\lambda) = \lambda^2 - 12\lambda + 32 = (\lambda - 4) \cdot (\lambda - 8)$$

$$\lambda_1 = 4 \quad \lambda_2 = 8$$

$$\boxed{\lambda_1 = 4}$$

$$\begin{bmatrix} 1 & -\sqrt{3} \\ -\sqrt{3} & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}; \quad \begin{cases} x - \sqrt{3}y = 0 \\ -\sqrt{3}x + 3y = 0 \end{cases}$$

$$x - \sqrt{3}y = 0 \rightarrow x = \sqrt{3}y$$

$$(x, y) = (\sqrt{3}y, y) = y \cdot (\sqrt{3}, 1) \quad \forall y \in \mathbb{R} - \{0\}$$

autovettori di $\lambda_1 = 4$

$$\|(\sqrt{3}, 1)\| = 2 \quad \text{scelgo } y = \frac{1}{2}$$

auto VERSORE $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ di $\lambda_1 = 4$

$$\Lambda = \begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}; C = \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \quad \det C = +1$$

(rotazione)

$$[-12\sqrt{3} \quad 20] \cdot \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = [-8 \quad 16\sqrt{3}]$$

dopo la rotazione si ha che

$$4(x')^2 + 8(y')^2 - 8x' + 16\sqrt{3}y' + 12 = 0$$

$$(x')^2 + 2 \cdot (y')^2 - 2x' + 4\sqrt{3}y' + 3 = 0$$

$$(x')^2 - 2x'$$

$$+ 2(y')^2 + 4\sqrt{3}y'$$

$$+ 3 = 0$$

$$(x'-1)^2 - 1$$

$$2 \cdot (y' + \sqrt{3})^2 - 6$$

$$+ 3 = 0$$

traslazione

$$\begin{cases} x'' = x' - 1 \\ y'' = y' + \sqrt{3} \end{cases}$$

$$(x'')^2 - 1 + 2 \cdot (y'')^2 - 6 + 3 = 0$$

$$(x'')^2 + 2 \cdot (y'')^2 = 4$$

$$\frac{(x'')^2}{4} + \frac{(y'')^2}{2} = +1$$

Equazione CANONICA di un' ELLISSE