Ricevimento studenti - lunedì 13 giugno 2022

Titolo nota 13/06/2022

$t \in \mathbb{R}$ $\begin{cases} 12x - tx + ty - t = 0 \\ tx - 12x - 2y + t = 0 \\ -tx + 12x + 2y - t = 0 \end{cases}$ equivalenti

$$\begin{cases} (12-t)\cdot x + t \cdot y = t \\ (12-t)\cdot x + 2 \cdot y = t \end{cases}$$

$$A = \begin{bmatrix} (12-t) & t \\ (12-t) & 2 \end{bmatrix}$$ matrice dei coefficienti

$t_1 = 12$ $t_2 = 2$

$$\det A = (12-t)\cdot 2 - (12-t)\cdot t = (12-t)\cdot(2-t)$$

$\forall t \in \mathbb{R} - \{2, 12\}$ $\det A \neq 0$

$\forall t \in \mathbb{R} - \{2, 12\}$ $\operatorname{rg} A = \text{massimo} = 2$

$\forall t \in \mathbb{R} - \{2, 12\}$ ho un sistema di CRAMER

$\forall t \in \mathbb{R} - \{2, 12\}$ ho un' UNICA soluzione che ovviamente è una coppia (x, y) dipendente da t

regola di Cramer $$x = \frac{\det \begin{bmatrix} t & t \\ t & 2 \end{bmatrix}}{\det A}$$

$$x = \frac{2\cdot t - t^2}{(12-t)\cdot(2-t)} = \frac{t\cdot(2-t)}{(12-t)\cdot(2-t)} = \frac{t}{12-t}$$

$$y = \frac{\det\begin{bmatrix}(12-t) & t\\ (12-t) & t\end{bmatrix}}{\det A} = \frac{0}{\det A} = 0$$

$\forall t \in \mathbb{R} - \{2, 12\}$ ho un' <u>UNICA</u> soluzione data dalla coppia ordinata

$$(x, y) = \left(\frac{t}{12-t}, 0\right)$$

cosa accade per $t=2$ e per $t=12$?

$$\begin{cases}(12-t)x + ty = t\\ (12-t)x + 2y = t\end{cases}$$

$\boxed{t=2}$ $\begin{cases}10x + 2y = 2\\ 10x + 2y = 2\end{cases}$ è la stessa

$$5x + y = 1 \Rightarrow y = 1 - 5x$$

$$(x, y) = (x, 1-5x) = (0,1) + x\cdot(1,-5) \quad \forall x \in \mathbb{R}$$

quindi, per $t=2$ ho ∞^1 soluzioni

infine per $t=12$ ho $\begin{cases} 0\cdot x + 12y = 12 \\ 0\cdot x + 2y = 12 \end{cases}$

$$\left.\begin{cases} y=1 \\ y=6 \end{cases}\right] \Rightarrow 1=6 \qquad \underline{\text{ASSURDO}}$$

per $t=12$ il sistema <u>NON</u> ha soluzioni

$$7x^2 - 2\sqrt{21}\,xy + 3y^2 + 2\sqrt{21}\,x - 6y + 23 = 0$$

$$A = \begin{bmatrix} 7 & -\sqrt{21} \\ -\sqrt{21} & 3 \end{bmatrix}; \quad p_A(\lambda) = \dots = \lambda^2 - (7+3)\lambda + 0 =$$

$$= \lambda^2 - 10\lambda = \lambda\cdot(\lambda - 10)$$

$\lambda_1 = 0$ $\qquad$ $\lambda_2 = 10$

$\boxed{\lambda_1 = 0}$ troviamo un auto VERSORE

$$\begin{bmatrix} 7 & -\sqrt{21} \\ -\sqrt{21} & 3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \left.\begin{cases} 7x - \sqrt{21}\,y = 0 \\ -\sqrt{21}\,x + 3y = 0 \end{cases}\right] \text{L.DIP}$$

$$-\sqrt{21}\,x + 3y = 0 \Rightarrow 3y = \sqrt{21}\cdot x \Rightarrow y = \frac{\sqrt{21}}{3}\cdot x$$

$$(x,y) = \left(x, \frac{\sqrt{21}}{3}\cdot x\right) = \underbrace{x\cdot\left(1, \frac{\sqrt{21}}{3}\right) \quad \forall x \neq 0}_{\text{auto vettore}}$$

$$\left\| \left(1, \frac{\sqrt{21}}{3}\right) \right\| = \sqrt{1^2 + \left(\frac{\sqrt{21}}{3}\right)^2} = \sqrt{1 + \frac{21}{9}} = \sqrt{\frac{30}{9}}$$

$$= \sqrt{\frac{10}{3}}$$

scegliendo $x = \dfrac{1}{\sqrt{\frac{10}{3}}} = \sqrt{\dfrac{3}{10}}$ ottengo

un autoversore $\sqrt{\frac{3}{10}} \cdot \left(1, \frac{\sqrt{21}}{3}\right) =$

$$= \left(\sqrt{\frac{3}{10}}, \sqrt{\frac{63}{90}}\right) = \left(\sqrt{\frac{3}{10}}, \sqrt{\frac{7}{10}}\right)$$

auto VERSORE relativo a $\lambda_1 = 0$

$$\left(\sqrt{\frac{3}{10}}, \sqrt{\frac{7}{10}}\right)$$

$$\Lambda = \begin{bmatrix} 0 & 0 \\ 0 & 10 \end{bmatrix}; \quad C = \begin{bmatrix} \sqrt{\frac{3}{10}} & -\sqrt{\frac{7}{10}} \\ \sqrt{\frac{7}{10}} & \sqrt{\frac{3}{10}} \end{bmatrix}$$

$\downarrow$

matrice associata alla rotazione

Dopo la rotazione l'equazione della conica diventa

$$0\cdot(x')^2 + 10\cdot(y')^2 + \alpha\cdot x' + \beta\cdot y' + 23 = 0$$

dove $[\alpha \quad \beta] = [2\sqrt{21} \quad -6]\cdot\begin{bmatrix}\sqrt{\frac{3}{10}} & -\sqrt{\frac{7}{10}} \\ \sqrt{\frac{7}{10}} & \sqrt{\frac{3}{10}}\end{bmatrix}$

$$\alpha = 2\cdot\sqrt{21}\cdot\sqrt{\frac{3}{10}} - 6\sqrt{\frac{7}{10}} = \sqrt{\frac{4\cdot 63}{10}} - \sqrt{\frac{36\cdot 7}{10}}$$

$$\beta = 2\cdot\sqrt{21}\cdot\left(-\sqrt{\frac{7}{10}}\right) - 6\cdot\sqrt{\frac{3}{10}} = -\sqrt{\frac{4\cdot 147}{10}} - \sqrt{\frac{108}{10}}$$

$$\alpha = \sqrt{\frac{126}{5}} - \sqrt{\frac{126}{5}} = 0 \; ; \quad \beta = -\sqrt{\frac{294}{5}} - \sqrt{\frac{54}{5}}$$

$$\beta = -7\cdot\sqrt{\frac{6}{5}} - 3\cdot\sqrt{\frac{6}{5}} = -10\sqrt{\frac{6}{5}} = -2\cdot\sqrt{30}$$

$$10\cdot(y')^2 - 2\sqrt{30}\,y' + 23 = 0 \; ; \quad 10\cdot\left(y' - \frac{\sqrt{30}}{10}\right)^2 - 3 + 23 = 0$$

traslazione $y'' = y' - \frac{\sqrt{30}}{10}$; $10\cdot(y'')^2 + 20 = 0$

$(y'')^2 + 2 = 0$ conica (degenere) senza punti reali

$$A = \begin{bmatrix} -1 & 0 & h \\ 1 & -2 & 4 \\ 0 & K & 1 \end{bmatrix}$$

$(h, K) \in \mathbb{R}^2$ per i quali il vettore $(-4, -1, 1)$ è un autovettore di A.

Scrivere anche il suo autovalore

H è autovettore di A relativo all'autovalore λ se (per definizione) $A \cdot H = \lambda \cdot H$

$$\underbrace{\begin{bmatrix} -1 & 0 & h \\ 1 & -2 & 4 \\ 0 & K & 1 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ -1 \\ 1 \end{bmatrix}} = \underbrace{\lambda \cdot \begin{bmatrix} -4 \\ -1 \\ 1 \end{bmatrix}}$$

$$\begin{bmatrix} (-1)\cdot(-4) + 0\cdot(-1) + h\cdot 1 \\ 1\cdot(-4) - 2\cdot(-1) + 4\cdot 1 \\ 0\cdot(-4) + K\cdot(-1) + 1(1) \end{bmatrix} = \begin{bmatrix} -4\cdot\lambda \\ -1\cdot\lambda \\ 1\cdot\lambda \end{bmatrix}$$

$$\begin{bmatrix} h+4 \\ 2 \\ -K+1 \end{bmatrix} = \begin{bmatrix} -4\lambda \\ -\lambda \\ \lambda \end{bmatrix} \Rightarrow \begin{cases} h+4 = -4\lambda \\ 2 = -\lambda \\ -K+1 = \lambda \end{cases} \Rightarrow \boxed{\lambda = -2}$$

$$\begin{cases} h+4 = +8 \Rightarrow h = 4 \\ -K+1 = -2 \Rightarrow K = 3 \end{cases}$$

$r: \; 2x+7 = 2x+5z+7 = 0$

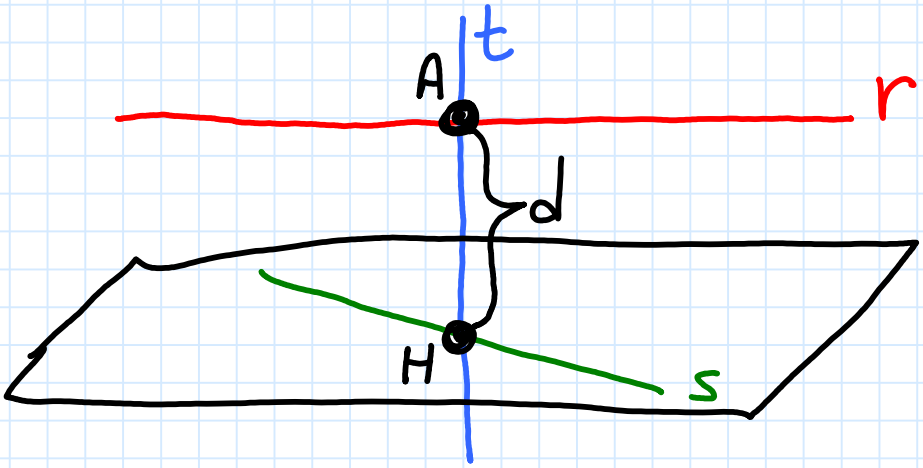
$s: \; 3y+20 = 3y+5z = 0$

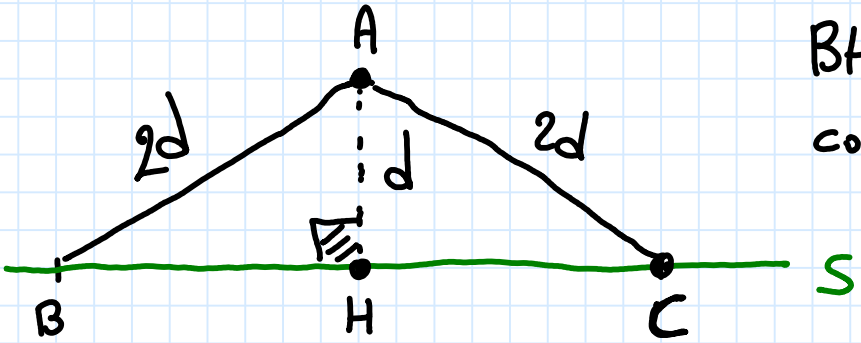
sghembe

sia t la retta di minima distanza tra r e s

sia d la distanza tra i punti A e H di intersezione di t con r e s rispettivamente.

Siano B e C i punti di s aventi distanza 2d da A. Calcolare l'area di $\overset{\triangle}{ABC}$.





$BH = ? = \sqrt{(2d)^2 - d^2} = \sqrt{3} \cdot d$

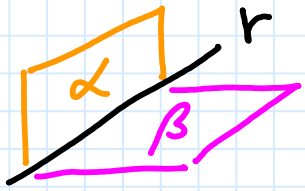
con Pitagora

$BC = 2 \cdot BH = 2 \cdot d \cdot \sqrt{3}$

area $\overset{\triangle}{ABC} = \frac{1}{2} \cdot (2 \cdot d \cdot \sqrt{3}) \cdot d = d^2 \cdot \sqrt{3}$

Quindi, una volta trovato d ho subito l'area.

Ora calcoliamo d



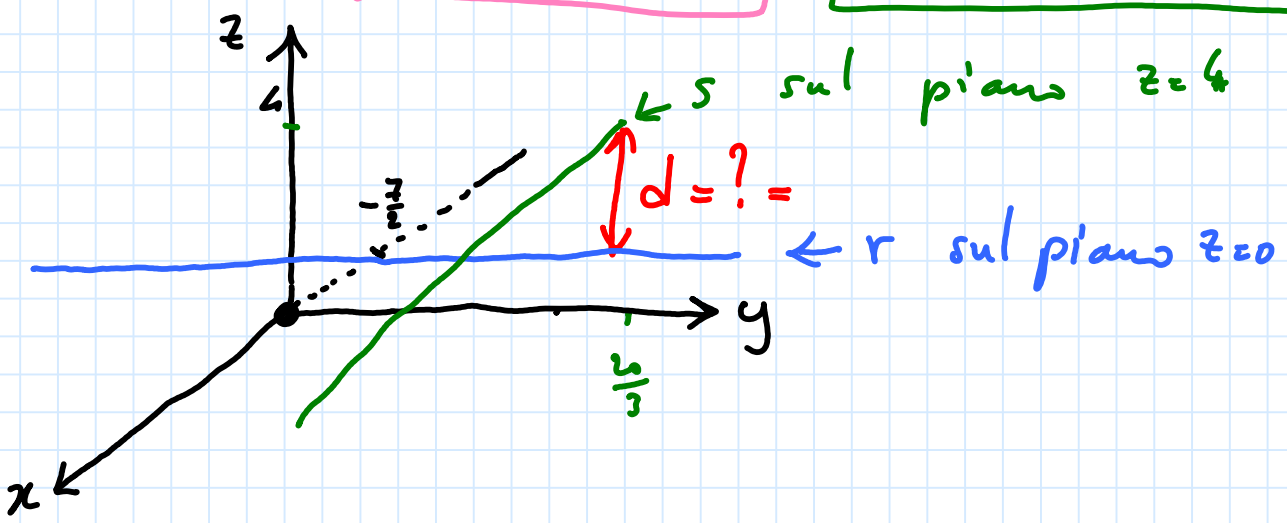
$r: \underbrace{2x+7}_{\alpha} = \underbrace{2x+5z+7}_{\beta} = 0$

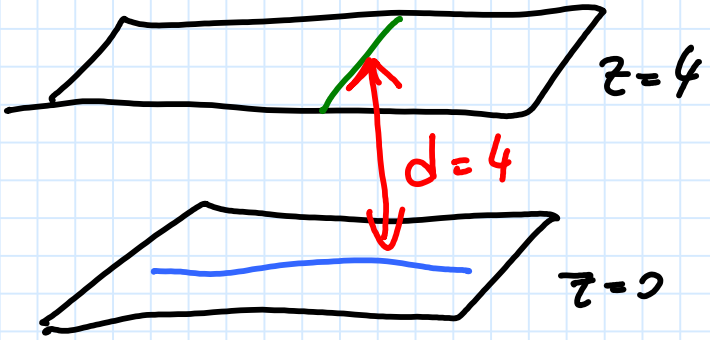
$s: 3y+20 = 3y+5z = 0$

$r: 2x+7 = 5z = 0$

$\boxed{r: 2x+7 = z = 0}$

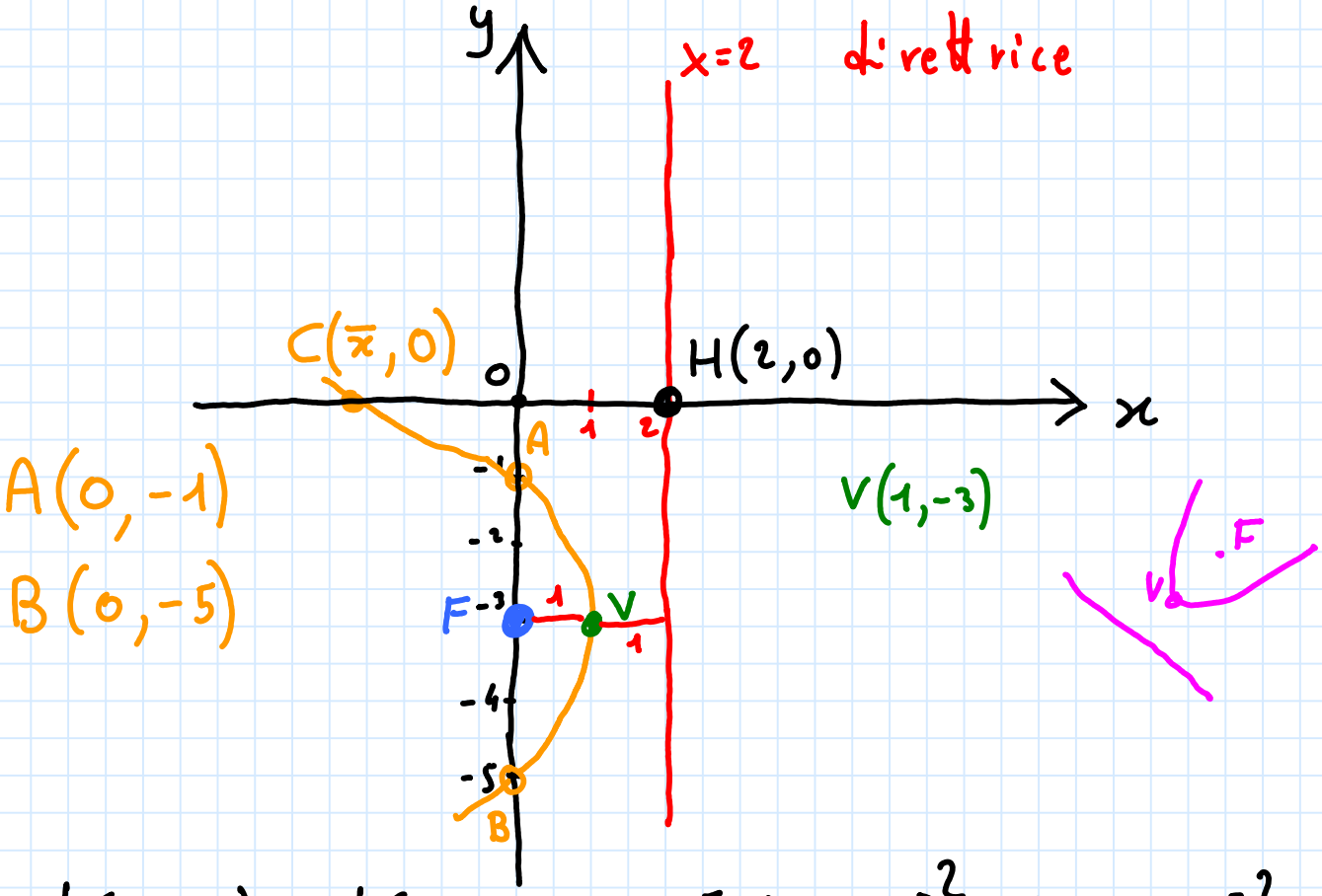
$s: \; 3y - 20 = \boxed{-20 + 5z = 0}$ $\qquad$ $\boxed{s: \; 3y - 20 = z - 4 = 0}$





$d = 4$; area $\overset{\triangle}{ABC} = 16 \cdot \sqrt{3}$

parabola avente come direttrice la retta $x = 2$ e come fuoco il punto $F(0, -3)$.
Trovare i punti d'intersezione con gli assi coordinati



$d(C,F) = d(C,H)$; $[d(C,F)]^2 = [d(C,H)]^2$

$C(\bar{x},0)$

$H(2,0)$

$F(0,-3)$

$$(\bar{x}-0)^2+(0-(-3))^2 = (\bar{x}-2)^2+(0-0)^2$$

$$\cancel{\bar{x}^2}+9 = \cancel{\bar{x}^2}+4-4\bar{x}$$

$$4\bar{x} = -5 \qquad \bar{x} = -\frac{5}{4}$$

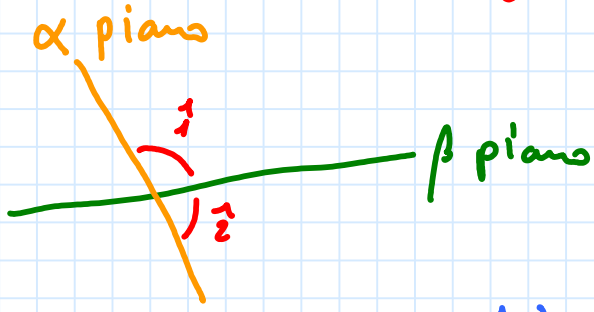
$$C\left(-\frac{5}{4},0\right)$$

$A(t,0,0)$; $B(0,-2,0)$; $C(0,0,\sqrt{23})$

trovare i valori reali di t per i quali il piano α passante per A, B e C forma

col piano YZ un angolo di $\frac{2}{3}\pi$ rad.

problema di angolo tra 2 piani

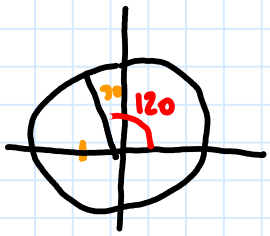


$\alpha: ax+by+cz+d=0$

$\beta: a'x+b'y+c'z+d'=0$

$$\cos(\widehat{\alpha,\beta}) = \pm \frac{a\cdot a'+b\cdot b'+c\cdot c'}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{(a')^2+(b')^2+(c')^2}}$$

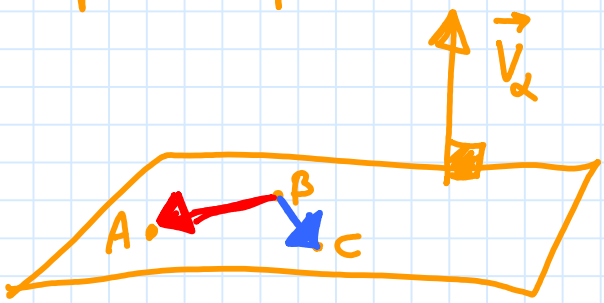
$\frac{2}{3}\pi$ rad $= 120°$



$\cos(120°) = ? = -\frac{1}{2}$

β sia il piano YZ : $x=0$ → $a'=1$, $b'=0$, $c'=0$

α il piano per i tre punti A, B, C



$\alpha: ax+by+cz+d=0$

$\vec{V}_\alpha = (a,b,c)$

mi serve un (qualsiasi) vettore $\perp$ al piano α

$A(t,0,0)$; $B(0,-2,0)$; $C(0,0,\sqrt{23})$

$[\vec{BA}] = (t, 2, 0)$ $\qquad$ $[\vec{BC}] = (0, 2, \sqrt{23})$

$$[\vec{BA}] \wedge [\vec{BC}] = \det\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ t & 2 & 0 \\ 0 & 2 & \sqrt{23} \end{bmatrix} =$$

$$= 2\cdot\sqrt{23}\cdot\vec{i} - t\cdot\sqrt{23}\cdot\vec{j} + 2\cdot t\cdot\vec{k} \perp \alpha$$

$a = 2\cdot\sqrt{23}$ $\quad b = -t\cdot\sqrt{23}$ $\quad c = 2\cdot t$

$a' = 1$ $\quad b' = 0$ $\quad c' = 0$

$\cos(120°) = -\frac{1}{2}$

$$-\frac{1}{2} = \pm\frac{2\cdot\sqrt{23} + 0 + 0}{\sqrt{92 + 23t^2 + 4t^2}\cdot\underbrace{\sqrt{1^2+0^2+0^2}}_{1}}$$

$$-\frac{1}{2} = \pm\frac{2\sqrt{23}}{\sqrt{92 + 27t^2}}$$

$$-\sqrt{92 + 27t^2} = \pm 2\cdot 2\cdot\sqrt{23} = \pm 4\sqrt{23}$$

$$92 + 27t^2 = 16\cdot 23$$

$$27t^2 = 16\cdot 23 - 92 = 368 - 92 = 276$$

$$27t^2 = 276$$

$$9t^2 = 92$$

$$t^2 = \frac{92}{9} = \frac{4\cdot 23}{9}$$

$$\boxed{t = \pm\frac{2}{3}\cdot\sqrt{23}}$$