Ricevimento studenti - Lunedì 27 giugno 2022

Titolo nota 27/06/2022

$$A = \begin{bmatrix} -4 & 27 & 0 \\ 0 & 5 & 0 \\ 9 & -27 & 5 \end{bmatrix}; \quad p_A(\lambda) = \det \begin{bmatrix} (-4-\lambda) & 27 & 0 \\ 0 & (5-\lambda) & 0 \\ 9 & -27 & (5-\lambda) \end{bmatrix}$$

$$p_A(\lambda) = (5-\lambda) \cdot \det \begin{bmatrix} (-4-\lambda) & 0 \\ 9 & (5-\lambda) \end{bmatrix} = (5-\lambda)^2 (-4-\lambda)^1$$

$\lambda_1 = 5$ $\quad m_a(\lambda_1) = 2$

$\lambda_2 = -4$ $\quad m_a(\lambda_2) = 1$

↳ i 2 autovalori reali distinti

$\lambda_1 = 5$

$$\begin{bmatrix} -9 & 27 & 0 \\ 0 & 0 & 0 \\ 9 & -27 & 0 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad \begin{cases} -9x + 27y = 0 \\ 0 = 0 \\ 9x - 27y = 0 \end{cases}$$

$$\{ x - 3y = 0 \quad \rightarrow \quad x = 3y$$

autovettori $(x, y, z) = (3y, y, z) =$

$$= y(3,1,0) + z(0,0,1) \quad \forall (y,z) \in \mathbb{R}^2 - \{(0,0)\}$$

Base autospazio $E_5 = ((3,1,0),(0,0,1))$

↳ base relativa all'autospazio E_5

$\lambda_2 = -4$

$$\begin{bmatrix} 0 & 27 & 0 \\ 0 & 9 & 0 \\ 9 & -27 & 9 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{cases} 27y = 0 \\ 9y = 0 \\ 9x - 27y + 9z = 0 \end{cases}$$

$$\begin{cases} y = 0 \\ 9x + 9z = 0 \end{cases} \quad \begin{cases} y = 0 \\ x + z = 0 \end{cases} \quad \begin{cases} y = 0 \\ z = -x \end{cases}$$

autovettori $(x,y,z) = (x,0,-x) =$

$= x(1,0,-1) \quad \forall x \in \mathbb{R} - \{0\}$

Base $E_{(-4)} = (1,0,-1)$

oppure $(-1,0,1)$

oppure $(77,0,-77)$

↳ base relativa all'autospazio $E_{(-4)}$

r: $4y + 2z + 1 = x - 1 = 0$

s: $4y + 2z = x + 4y + 2z = 0 \rightarrow O(0,0,0) \in s$

$s: \; 4y+2z = x = 0$

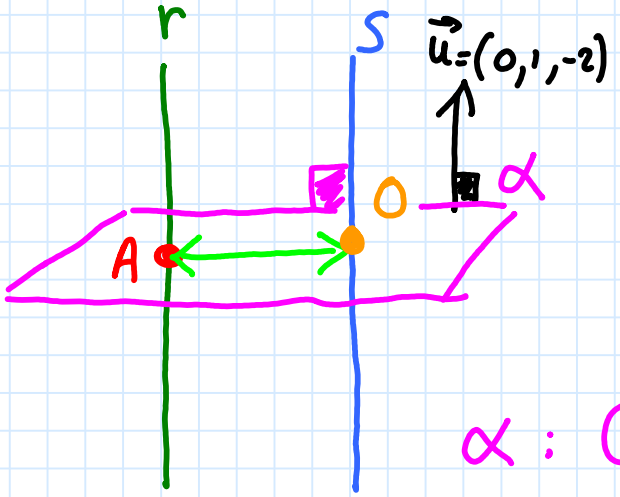
$r: \; 4y+2z+1 = x-1 = 0$

vedo ad occhio che le 2 rette sono parallele

$$\vec{v}_r = \begin{bmatrix} 0 & 4 & 2 \\ 1 & 0 & 0 \end{bmatrix} \begin{matrix} \nearrow l = 0 \\ \rightarrow m = -(-2) = +2 \\ \searrow n = -4 \end{matrix} \qquad \vec{v}_r = (0, 2, -4)$$

$$\vec{v}_s = \begin{bmatrix} 0 & 4 & 2 \\ 1 & 4 & 2 \end{bmatrix} \begin{matrix} \nearrow l' = 0 \\ \rightarrow m' = -(-2) = +2 \\ \searrow n' = -4 \end{matrix} \qquad \vec{v}_s = (0, 2, -4)$$

$\vec{v}_r = \vec{v}_s \Rightarrow \vec{v}_r \parallel \vec{v}_s \Rightarrow r \parallel s$



(1) α : $O \in \alpha$ et $\alpha \perp s$

(2) $\{A\} = \alpha \cap r$

(3) $d(O, A) = d(r, s)$

$\alpha: 0 \cdot x + 1 \cdot y + (-2) \cdot z + 0 = 0$

$\alpha: \; y - 2z = 0$

$$\alpha \cap r: \begin{cases} y - 2z = 0 & \leftarrow \text{piano } \alpha \\ 4y + 2z + 1 = 0 \\ x - 1 = 0 \end{cases} \leftarrow \text{retta } r \qquad \begin{cases} y = 2z \\ 8z + 2z + 1 = 0 \\ x = 1 \end{cases}$$

$10z + 1 = 0 \rightarrow z = -\frac{1}{10} \rightarrow y = -\frac{2}{10}$

$A\left(1, -\frac{2}{10}, -\frac{1}{10}\right)$ $\qquad O(0,0,0)$

$$d(A,O)=\sqrt{(1-0)^2+\left(-\frac{2}{10}-0\right)^2+\left(-\frac{1}{10}-0\right)^2}=$$

$$=\sqrt{1+\frac{4}{100}+\frac{1}{100}}=\sqrt{\frac{105}{100}}=$$

$$=\sqrt{\frac{21}{20}}$$

$$\boxed{d(r,s)=\sqrt{\frac{21}{20}}}$$

$$d(r,s)=\frac{\sqrt{105}}{10}$$ ← è la stessa cosa

$O(0,0,0)$; $\perp$ piano $\alpha: 5x-y+2z=0$;

// retta $r: x=5x-y+2z=0$

$O \in \pi$

π : $\pi \perp \alpha$; $\pi \parallel r$

$\pi: ax+by+cz+d=0$

$O \in \pi \Rightarrow d=0 \Rightarrow \pi: ax+by+cz=0$

$\pi \perp \alpha \Rightarrow 5\cdot a+(-1)\cdot b+2\cdot c=0$

$5a-b+2c=0 \rightarrow b=5a+2c$

$\pi: a\cdot x+(5a+2c)\cdot y+c\cdot z=0$

$\pi \parallel r \iff \det \begin{bmatrix} a & (5a+2c) & c \\ 1 & 0 & 0 \\ 5 & -1 & 2 \end{bmatrix} = 0$

$-1 \cdot \det \begin{bmatrix} (5a+2c) & c \\ -1 & 2 \end{bmatrix} = 0$

$-\left[2 \cdot (5a+2c) + c\right] = 0$

$10a + 5c = 0 \quad ; \quad 2a + c = 0$

$c = -2a$

$\pi : a \cdot x + \left(5a + (-4a)\right) \cdot y + (-2a) z = 0$

$\pi : a \cdot x + a \cdot y - 2a \cdot z = 0 \qquad \forall a \in \mathbb{R} - \{0\}$

scelgo $a = 1$

$\pi : x + y - 2z = 0$

$r : 4x - y + 15 = x + z = 0$

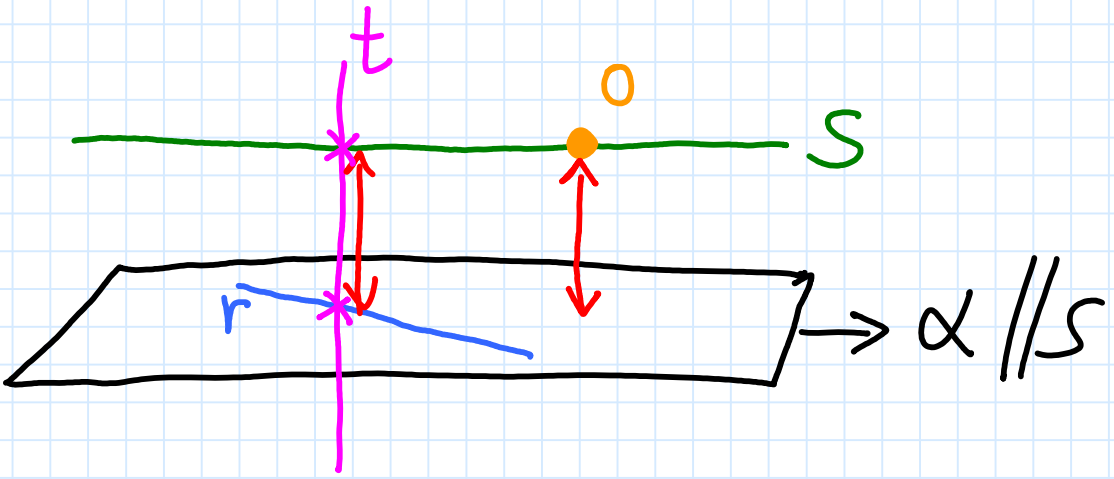
$s : 7y + z = 2x - y - 2z = 0$

sghembe

$\hookrightarrow O(0,0,0)$

t retta di minima distanza tra r e s

VERSORE avente direzione di t



α : $\alpha \in F(r)$ et $\alpha \parallel s$

$F(r)$: $\lambda \cdot (4x - y + 15) + \mu \cdot (x + z) = 0$

$$(4\lambda + \mu) \cdot x - \lambda y + \mu z + 15\lambda = 0$$

$$\alpha \parallel s \iff \det \begin{bmatrix} (4\lambda+\mu) & -\lambda & \mu \\ 0 & 7 & 1 \\ 2 & -1 & -2 \end{bmatrix} = 0$$

$(4\lambda+\mu) \quad -\lambda$

$0 \quad 7$

$2 \quad -1$

$$-14 \cdot (4\lambda + \mu) - 2\lambda - 14\mu + (4\lambda + \mu) = 0$$

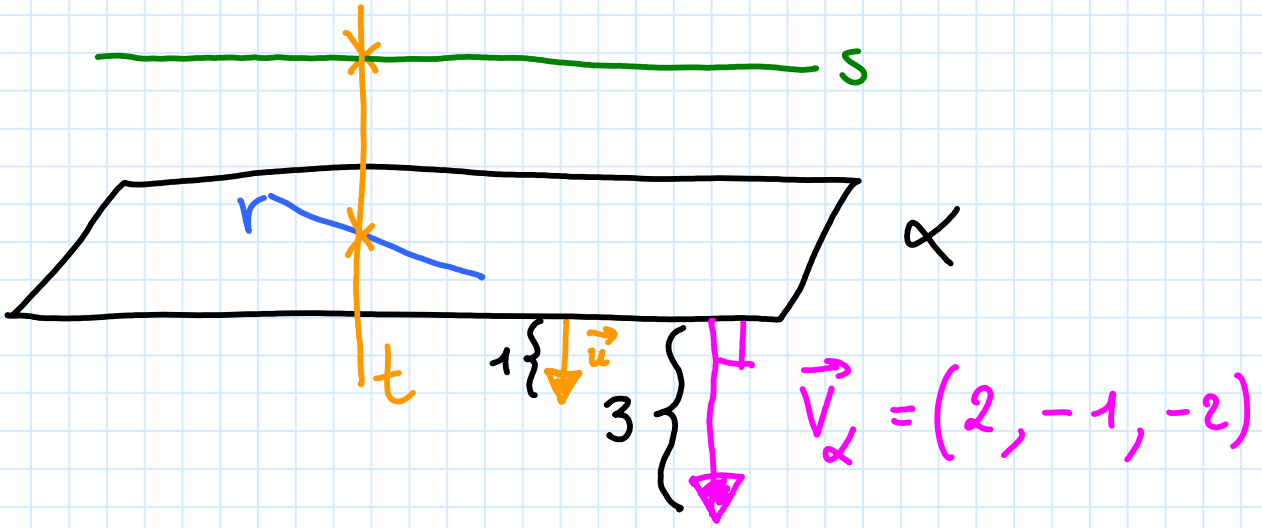
$$-56\lambda - 14\mu - 2\lambda - 14\mu + 4\lambda + \mu = 0$$

$$-54\lambda - 27\mu = 0 \; ; \quad 2\lambda + \mu = 0$$

$\mu = -2\lambda$; scelgo $\lambda = 1$ e ottengo $\mu = -2$

$1 \cdot (4x - y + 15) - 2 \cdot (x + z) = 0$

$$\alpha : 2x - y - 2z + 15 = 0$$



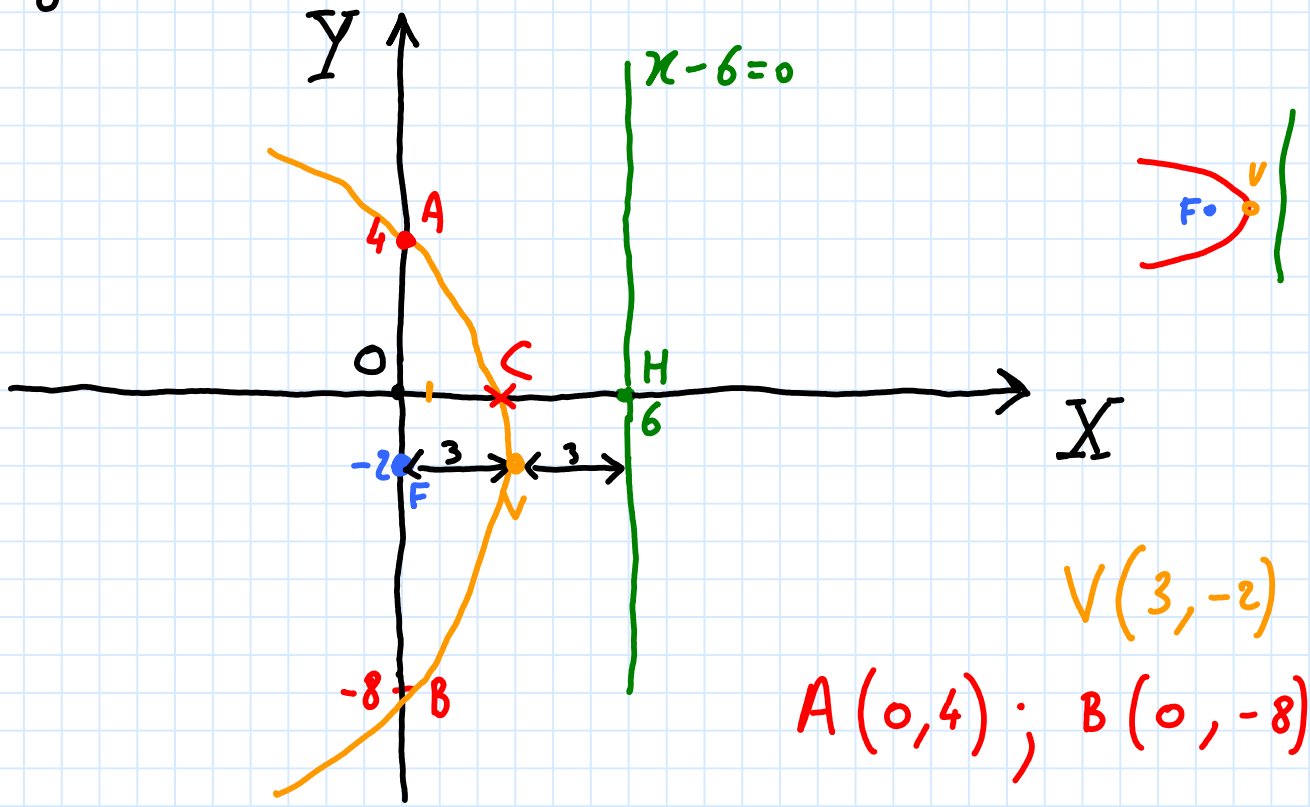
$$\| \vec{V}_\alpha \| = \sqrt{2^2 + (-1)^2 + (-2)^2} = \sqrt{9} = 3$$

VERSORE $\vec{u} = \frac{1}{3} \vec{V}_\alpha = \frac{1}{3}(2, -1, -2)$

VERSORE $\vec{u} = \left(\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3} \right)$

↳ è un versore avente la stessa direzione della retta di minima distanza

Ω parabola avente fuoco $F(0,-2)$ e direttrice la retta di equazione $x-6=0$.
Trovare i punti di intersezione di Ω con gli assi coordinati



$V(3,-2)$

$A(0,4)$; $B(0,-8)$

$C(x,0)$ $F(0,-2)$ $H(6,0)$

$$d(C,F) = d(C,H) \rightarrow [d(C,F)]^2 = [d(C,H)]^2 \rightarrow$$

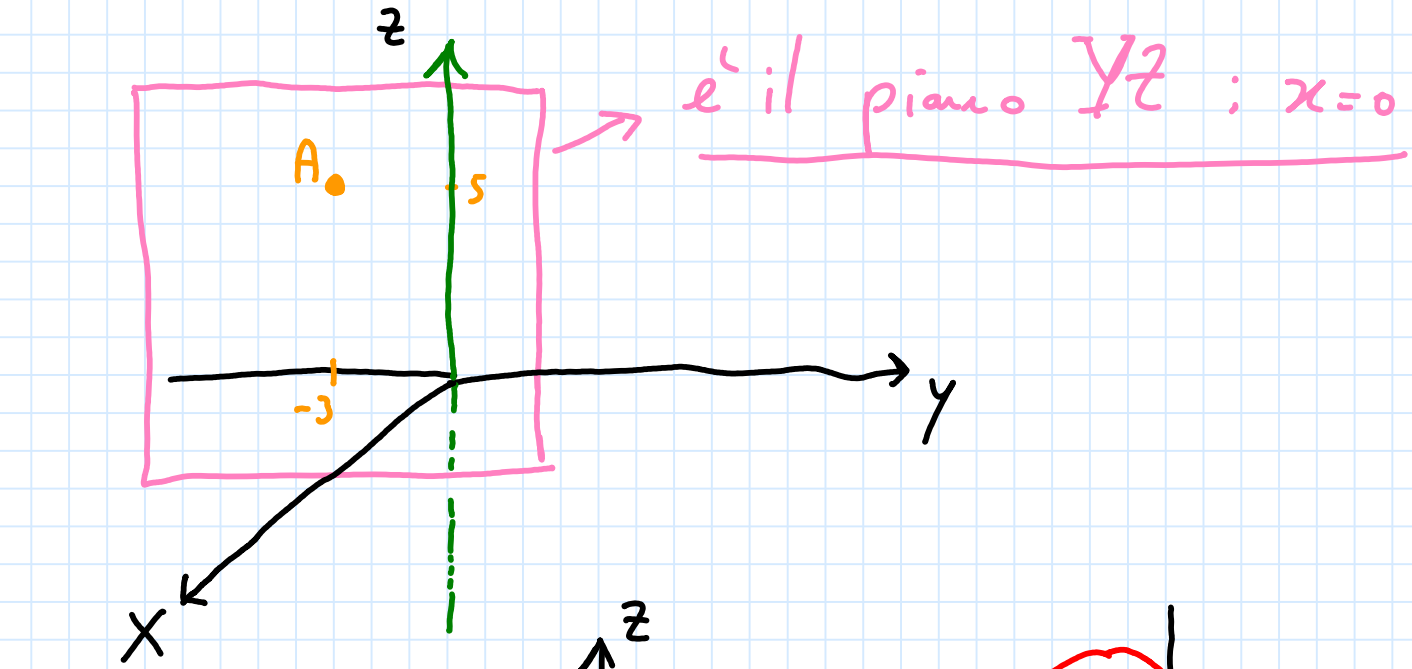
$$\rightarrow (x-0)^2 + (0-(-2))^2 = (x-6)^2 + (0-0)^2 \rightarrow$$

$$x^2 + 4 = x^2 + 36 - 12x$$

$$12x = 32 \; ; \quad 3x = 8 \; ; \quad x = \frac{8}{3}$$

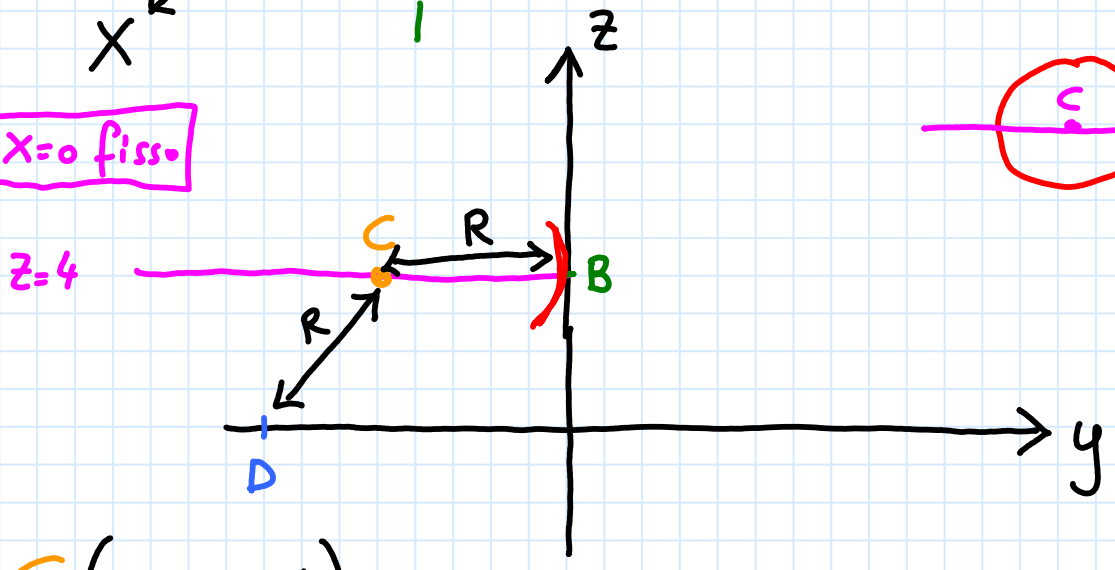
$C\left(\frac{8}{3},0\right)$

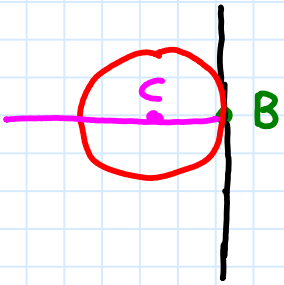
piano Π contenente l'asse z e passante per $A(0,-3,5)$. Sul piano Π sia γ la circonferenza tangente all'asse z nel punto $B(0,0,+4)$ e passante per $D(0,-8,0)$. Trovare (nello spazio) il centro e il raggio della circonferenza



è il piano yz : $x=0$

$x=0$ fisso





$C(0,y,4)$

$B(0,0,4)$

$D(0,-8,0)$

$d(C,B) = d(C,D)$

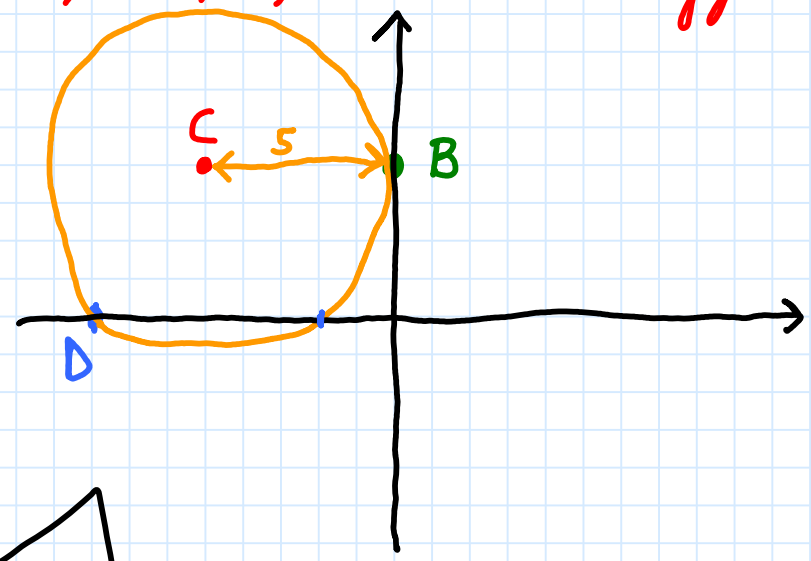
$[d(C,B)]^2 = [d(C,D)]^2$

$$(0-0)^2+(y-0)^2+(4-4)^2=(0-0)^2+(y+8)^2+(4-0)^2$$

$$y^2 = y^2 + 64 + 16y + 16$$

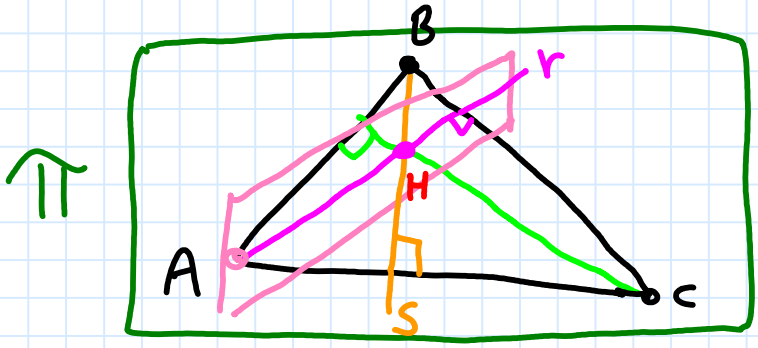
$$16y + 80 = 0 \quad ; \quad y + 5 = 0 \quad ; \quad y = -5$$

$C(0,-5,4) \longrightarrow$ raggio $= 5$



Siano A, B e C i punti di intersezione con il piano di equazione $\pi: x - 2y + z + 18 = 0$ con gli assi coordinati X, Y e Z. Trovare l'ortocentro del triangolo $A\hat{B}C$.

$A(-18,0,0)$; $B(0,9,0)$; $C(0,0,-18)$



r, s } sono 2 rette nello spazio

$r = \alpha \cap \Pi$ $\quad$ $s = \gamma \cap \Pi$

$\alpha = ?$ $\quad$ $\gamma = ?$

α è passante per A e $\perp$ a $[\vec{BC}]$

γ è passante per B e $\perp$ a $[\vec{AC}]$

$A(-18,0,0)$; $B(0,9,0)$; $C(0,0,-18)$;

$[\vec{BC}] = (0,-9,-18)$; $[\vec{CB}] = (0,9,18)$; $\frac{1}{9}[\vec{CB}] = (0,1,2)$

$\alpha : 0\cdot(x-(-18)) + 1\cdot(y-0) + 2\cdot(z-0) = 0$

$\alpha : y + 2z = 0$ $\quad$ $r : \begin{cases} y+2z=0 \\ x-2y+z+18=0 \end{cases}$

$[\vec{AC}] = (18,0,-18)$; $\frac{1}{18}[\vec{AC}] = (1,0,-1)$

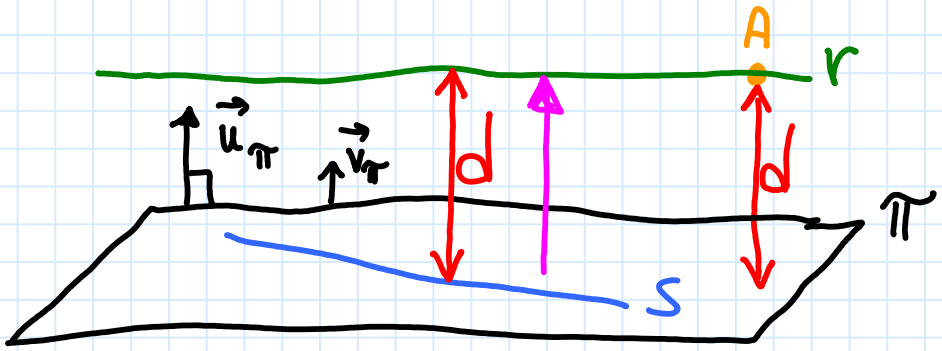
$\gamma : 1\cdot(x-0) + 0\cdot(y-9) - 1\cdot(z-0) = 0$

$\gamma : x - z = 0$ $\quad$ $s : \begin{cases} x-z=0 \\ x-2y+z+18=0 \end{cases}$

$r \cap s : \begin{cases} y+2z=0 \\ x-z=0 \\ x-2y+z+18=0 \end{cases}$ $\quad$ $\begin{cases} y=-2z \\ x=+z \\ z+4z+z+18=0 \end{cases}$

$6z + 18 = 0 \rightarrow z = -3 \rightarrow y = +6$, et $x = -3$

$H(-3,6,-3)$ è l'ortocentro



① $\pi \in F(s)$ et $\pi \parallel r$

② d = distanza di un punto qualsiasi di r da π

$\pi : ax+by+cz+d=0$ $\qquad \vec{u}_\pi = (a,b,c) \perp \pi$

$$\ell = \|\vec{u}_\pi\| = \sqrt{a^2+b^2+c^2}$$

$$\vec{V}_\pi = \frac{1}{\ell} \cdot \vec{u}_\pi \quad \text{VERSORE}$$

Vettore richiesto è $\quad d \cdot \vec{V}_\pi = \frac{d}{\ell} \cdot \vec{u}_\pi$

esempio piano $\pi : x + 2y - 2z + 7 = 0$

$d = 7$

$$\vec{u}_\pi = \sqrt{1^2+2^2+(-2)^2} = \sqrt{9} = 3 \quad \text{VETTORE} \perp \pi$$

$$\frac{1}{3}\vec{u}_\pi = \frac{1}{3}(1,2,-2) = \left(\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right) \text{ VERSORE} \perp \pi$$

$7\left(\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right)$ VETTORE $\perp \pi$ LUNGO 7

$$\begin{cases} 20x - tx + ty - t = 0 \\ tx - 20x + 5y + t = 0 \end{cases} \qquad \begin{cases} (20-t)x + ty = t \\ (20-t)x - 5y = t \end{cases}$$

$$A = \begin{bmatrix} (20-t) & t \\ (20-t) & -5 \end{bmatrix}; \quad \det A = (20-t)(-5-t) = (t-20)\cdot(t+5)$$

$\forall t \in \mathbb{R} - \{-5, 20\}$ $\det A \neq 0 \rightarrow rg\, A = 2 \rightarrow$ sist. CRAMER

quindi <u>per ogni t</u> $\in \mathbb{R} - \{-5, 20\}$ ho un' UNICA soluzione (dipendente da t)

$$x = \frac{\det \begin{bmatrix} t & t \\ t & -5 \end{bmatrix}}{\det A} = \frac{-5t - t^2}{(t-20)(t+5)} = \frac{-t(5+t)}{(t-20)(t+5)} = \frac{t}{20-t}$$

$$y = \frac{\det \begin{bmatrix} (20-t) & t \\ (20-t) & t \end{bmatrix}}{\det A} = \frac{0}{\det A} = 0$$

$\forall t \in \mathbb{R} - \{-5, 20\}$ LA SOLUZIONE E' UNICA

ed è data dalla coppia ordinata $(x, y) = \left(\frac{t}{20-t}, 0\right)$

$\rightarrow$ $\boxed{t = -5}$

$$\begin{cases} 25x - 5y = -5 \\ 25x - 5y = -5 \end{cases} \rightarrow \infty^1 \text{ soluzioni}$$

$5x - y = -1$; $y = 5x+1$

$(x,y) = (x, 5x+1) = x(1,5) + (0,1) \quad \forall x \in \mathbb{R}$

→ $\boxed{t=20}$ $\begin{cases} 20y = 20 \\ -5y = 20 \end{cases}$ $\begin{cases} y = 1 \\ y = -4 \end{cases}$ → $1 = -4$ ASSURDO

il sistema non ha soluzione per $t=20$