Ricevimento studenti - lunedì 9 gennaio 2023

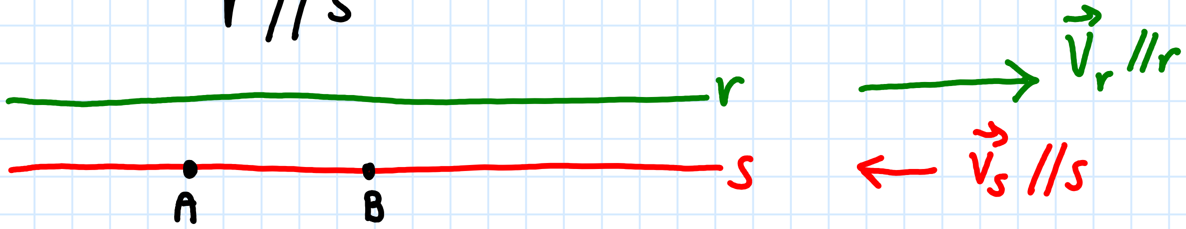
Titolo nota — 09/01/2023

$$\begin{cases} x+y+2z=0 \\ 3x+3y+6z=0 \\ -x-y-2z=0 \end{cases} \qquad \infty^2$$

$t \in \mathbb{R}$ $\qquad r: (t-1)\cdot x + 5y + (t^2-12)z = y+13=0$

$s: \quad A(2-4t, 5, -9) \in s \; ; \; B(2-3t, 5, t-8) \in s$

$r // s$



$$\vec{v}_s = [\vec{AB}] = (t, 0, t+1)$$

$$\begin{bmatrix} (t-1) & 5 & (t^2-12) \\ 0 & 1 & 0 \end{bmatrix} \quad \begin{aligned} \ell &= -(t^2-12) \\ m &= 0 \\ n &= t-1 \end{aligned}$$

$$\vec{v}_r = (12-t^2, 0, t-1)$$

$\vec{v}_r // \vec{v}_s \iff \vec{v}_r, \vec{v}_s$ lin. DIP.

$$rg \begin{bmatrix} t & 0 & (t+1) \\ (12-t^2) & 0 & (t-1) \end{bmatrix} = 1 \iff rg \begin{bmatrix} t & (t+1) \\ (12-t^2) & (t-1) \end{bmatrix} = 1 \iff$$

$$\det \begin{bmatrix} t & (t+1) \\ (12-t^2) & (t-1) \end{bmatrix} = 0 \iff t\cdot(t-1) - (12-t^2)(t+1) = 0$$

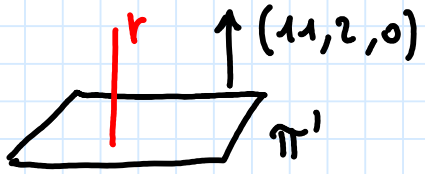
$$t^2 - t - 12t - 12 + t^3 + t^2 = 0$$

$$t^3 + 2t^2 - 13t - 12 = 0$$ trovare le sue radici

$\pi : \; 3x + 3y + 7z - 14 = 0$

$\{A\} = \pi \wedge$ asse z : $x = y = 0$ $\qquad A(0,0,2)$

Sia $r \perp \pi' : 11x + 2y + 3 = 0$ $\qquad O(0,0,0) \in r$



$$r : \begin{cases} x = 11 \cdot t + 0 \\ y = 2 \cdot t + 0 \\ z = 0 \cdot t + 0 \end{cases}$$

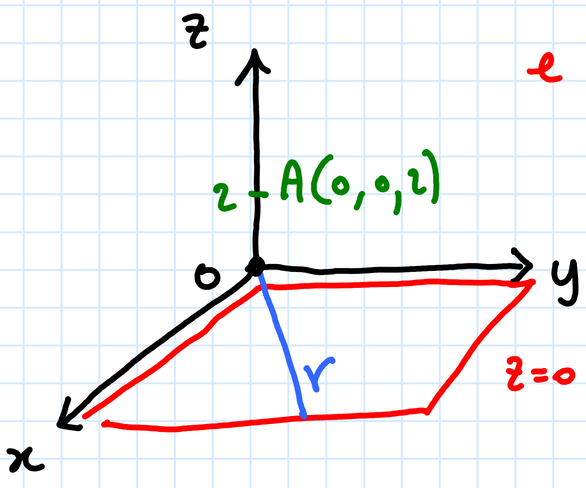
Sia h la distanza di A dalla retta r.

Siano B e C i punti di r aventi distanza $4h$ da A. Calcolare l'area del triangolo $\overset{\triangle}{ABC}$

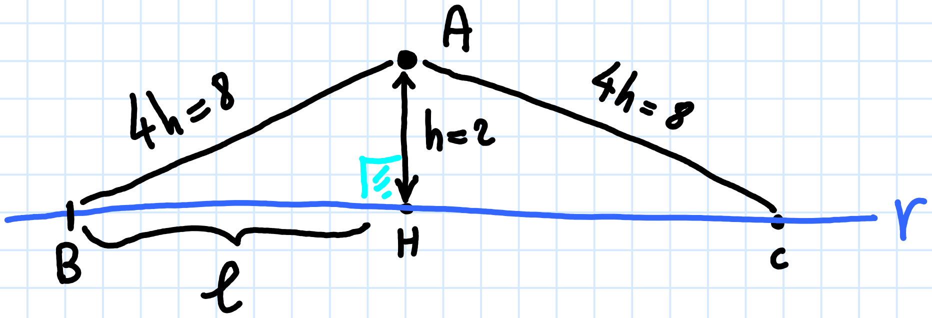
$$r : \begin{cases} x = 11t \\ y = 2t \\ z = 0 \end{cases} \rightarrow t = \frac{1}{2}y \rightarrow x = \frac{11}{2}y \rightarrow 2x - 11y = 0$$

$$\boxed{r : \; 2x - 11y = z = 0}$$

la retta r si trova sul piano $z=0$ ovvero il piano xy e inoltre passa per l'origine



$$h = d(A, r) = 2$$



$$\ell = \sqrt{8^2 - 2^2} = \sqrt{64-4} = \sqrt{60} = 2\sqrt{15}$$

$$\boxed{\text{area } \overset{\triangle}{ABC}} = 2 \cdot \text{area } \overset{\triangle}{ABH} = 2 \cdot \left(2 \cdot \sqrt{15} \cdot 2 \cdot \frac{1}{2}\right) = \boxed{4\sqrt{15}}$$

$$\begin{cases} 3x - y + 14z - 10 = 0 \\ 1x - y + 12z - 4 = 0 \\ 2x + y - 9z - 5 = 0 \end{cases}$$

Trovare la soluzione generale

$$\begin{bmatrix} 1 & -1 & 12 & -4 \\ 3 & -1 & 14 & -10 \\ 2 & 1 & -9 & -5 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & -1 & 12 & -4 \\ 0 & 2 & -22 & 2 \\ 0 & 3 & -33 & 3 \end{bmatrix} \rightsquigarrow$$

$$\begin{bmatrix} 1 & -1 & 12 & -4 \\ 0 & 1 & -11 & 1 \\ 0 & 1 & -11 & 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & -1 & 12 & -4 \\ 0 & 1 & -11 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{cases} x - y + 12z - 4 = 0 \\ y - 11z + 1 = 0 \end{cases} \uparrow \qquad \begin{cases} x - (11z - 1) + 12z - 4 = 0 \\ y = 11z - 1 \end{cases}$$

$$\begin{cases} x + z - 3 = 0 \\ y = 11z - 1 \end{cases} \qquad \begin{cases} x = 3 - z \\ y = 11z - 1 \end{cases}$$

$(x, y, z) = (3 - z, 11z - 1, z)$ soluzione generale

$$(x, y, z) = (3, -1, 0) + z(-1, 11, 1)$$

$(3,-1,0)$ soluzione particolare

$z(-1,11,1)$ soluzione generale dell'omogeneo associato

$(-1,11,1)$ base dello spazio delle soluzioni dell'omogeneo associato

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$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Se possibile, diagonalizzare A

$$P_A(\lambda) = \det \begin{bmatrix} (1-\lambda) & 0 & 1 \\ 0 & (1-\lambda) & 0 \\ 1 & 0 & (1-\lambda) \end{bmatrix} = (1-\lambda)\cdot\left[(1-\lambda)^2 - 1\right] =$$

$$= (1-\lambda)\cdot(\lambda^2 - 2\lambda) = \lambda\cdot(1-\lambda)\cdot(\lambda-2)$$

$\lambda_1 = 0$

$\lambda_2 = 1$

$\lambda_3 = 2$

$$\Lambda = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$ è la matrice diagonale

$\boxed{\lambda_1 = 0}$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{cases} x + z = 0 \rightarrow x = -z \\ y = 0 \\ \cancel{x + z = 0} \end{cases}$$

$$(x,y,z) = (-z, 0, z) = z(-1,0,1) \qquad \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$\boxed{\lambda_2 = 1}$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \begin{cases} z = 0 \\ 0 = 0 \\ x = 0 \end{cases}$$

$$(x,y,z)=(0,y,0)=y\cdot(0,1,0) \qquad \begin{bmatrix}0\\1\\0\end{bmatrix}$$

$\hookrightarrow$ libera

$\boxed{\lambda_3=2}$

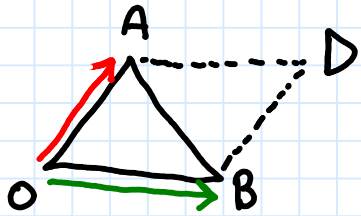
$$\begin{bmatrix}-1&0&1\\0&-1&0\\1&0&-1\end{bmatrix}\cdot\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \qquad \begin{cases}-x+z=0 \to z=x\\ -y=0 \to y=0\\ x\cancel{-}z=0\end{cases}$$

$$(x,y,z)=(x,0,x)=x\cdot(1,0,1) \qquad \begin{bmatrix}1\\0\\1\end{bmatrix}$$

$$C=\begin{bmatrix}-1&0&1\\0&1&0\\1&0&1\end{bmatrix}; \quad \Lambda=\begin{bmatrix}0&0&0\\0&1&0\\0&0&2\end{bmatrix}$$

sono le due matrici che diagonalizzano A

$A(t,-t,t)$; $B(0,-12,12)$; $O(0,0,0)$



area $O\hat{A}B=156$

$[\overrightarrow{OA}]=(t,-t,t)$ $\qquad$ $[\overrightarrow{OB}]=(0,-12,12)$

$$\vec{w}=[\overrightarrow{OA}]\wedge[\overrightarrow{OB}]=\det\begin{bmatrix}\vec{i}&\vec{j}&\vec{k}\\t&-t&t\\0&-12&12\end{bmatrix}=0\cdot\vec{i}-(12t)\vec{j}-(12t)\vec{k}$$

$$\|\vec{w}\|=\text{area } OADB=\sqrt{0^2+(-12t)^2+(-12t)^2}=$$

$$=\sqrt{288t^2}=12|t|\sqrt{2}$$

$$\text{area } O\hat{A}B=\frac{1}{2}\text{ area } OADB= 6|t|\cdot\sqrt{2}$$

$$6|t|\cdot\sqrt{2} = 156$$

$$|t|\cdot\sqrt{2} = 26 \quad ; \quad |t| = \frac{26}{\sqrt{2}} = 13\sqrt{2}$$

$$\boxed{t = \pm 13\sqrt{2}}$$

$$A = \begin{bmatrix} -1 & 0 & h \\ 1 & -2 & 4 \\ 0 & K & 1 \end{bmatrix} \qquad v = \begin{bmatrix} -4 \\ -1 \\ 1 \end{bmatrix} \text{ autovettore di } A$$

$$\exists\, \lambda \in \mathbb{R} : \quad A \cdot v = \lambda \cdot v$$

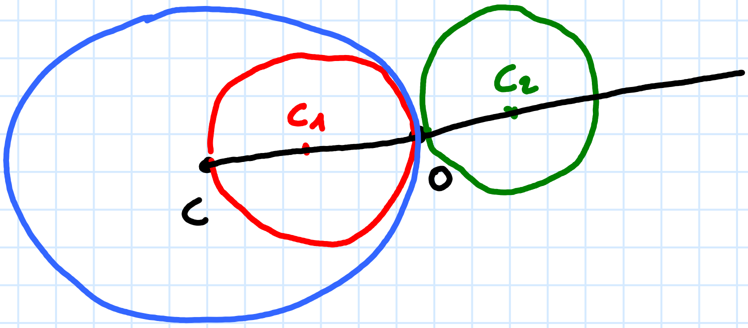
$$\begin{bmatrix} -1 & 0 & h \\ 1 & -2 & 4 \\ 0 & K & 1 \end{bmatrix} \cdot \begin{bmatrix} -4 \\ -1 \\ 1 \end{bmatrix} = \lambda \cdot \begin{bmatrix} -4 \\ -1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 4+h \\ -4+2+4 \\ -K+1 \end{bmatrix} = \begin{bmatrix} -4\lambda \\ -\lambda \\ \lambda \end{bmatrix} ; \quad \begin{cases} 4+h = -4\lambda \\ 2 = -\lambda \\ -K+1 = \lambda \end{cases} ; \quad \begin{cases} \boxed{\lambda = -2} \\ \boxed{K = 3} \\ \boxed{h = 4} \end{cases}$$

$$S : x^2+y^2+z^2+x+2y+3z=0 \qquad O(0,0,0) \in S$$

$$C\left(-\frac{1}{2}, -1, -\frac{3}{2}\right)$$

$$R = \frac{1}{2}\sqrt{1^2+2^2+3^2-4\cdot 0} = \frac{1}{2}\sqrt{14}$$



$C \xleftarrow{\quad\quad} O$ $\quad [\vec{OC}] = \left(-\frac{1}{2}, -1, -\frac{3}{2}\right)$

$C_1 \xleftarrow{\quad} O$ $\quad [\vec{OC_1}] = \frac{1}{2}[\vec{OC}] = \left(-\frac{1}{4}, -\frac{1}{2}, -\frac{3}{4}\right)$

$C_1\left(-\frac{1}{4}, -\frac{1}{2}, -\frac{3}{4}\right) \quad R_1 = \frac{1}{4}\sqrt{14}$

$S_1 : \left(x+\frac{1}{4}\right)^2 + \left(y+\frac{1}{2}\right)^2 + \left(z+\frac{3}{4}\right)^2 = \left(\frac{1}{4}\sqrt{14}\right)^2$

↳ interna

$O \xrightarrow{\quad\quad} C_2$ $\quad [\vec{OC_2}] = -\frac{1}{2}[\vec{OC}] = \left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\right)$

$S_2 : \left(x-\frac{1}{4}\right)^2 + \left(y-\frac{1}{2}\right)^2 + \left(z-\frac{3}{4}\right)^2 = \left(\frac{1}{4}\sqrt{14}\right)^2$

↳ esterna
