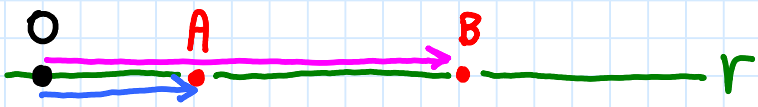
Ricevimento studenti - lunedì 16 gennaio 2023

Titolo nota 16/01/2023

$A(4, t-2, -t)$; $B(t-4, 2-t, t-1)$

r passante per A e B

$t \in \mathbb{R}$ per cui r passa per $O(0,0,0)$



$$O \in r \iff [\vec{OA}] \,//\, [\vec{OB}] \iff [\vec{OA}], [\vec{OB}] \text{ lin. DIP.}$$

$$rg\begin{bmatrix} \boxed{4} & (t-2) & -t \\ (t-4) & (2-t) & (t-1) \end{bmatrix} = 1 \iff$$

$$\iff \begin{cases} \det\begin{bmatrix} 4 & (t-2) \\ (t-4) & (2-t) \end{bmatrix} = 0 \\ \det\begin{bmatrix} 4 & -t \\ (t-4) & (t-1) \end{bmatrix} = 0 \end{cases} \iff \begin{cases} 4(2-t) - (t-4)(t-2) = 0 \\ 4(t-1) - (t-4)(-t) = 0 \end{cases}$$

$$\iff \begin{cases} 4(2-t) + (t-4)(2-t) = 0 \\ 4(t-1) + (t-4)t = 0 \end{cases} \iff \begin{cases} (2-t)\cdot t = 0 \\ \cancel{4t} - 4 + t^2 - \cancel{4t} = 0 \end{cases} \quad (t-2)(t+2) = 0$$

$$\iff \begin{cases} t = 0 \text{ vel } t = 2 \\ t = -2 \text{ vel } t = 2 \end{cases} \iff t = 2$$

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altro modo: $\exists \alpha \in \mathbb{R} : [\vec{OB}] = \alpha \cdot [\vec{OA}]$

↳ essendo $[\vec{OA}] \neq \vec{0}$

$(t-4,\ 2-t,\ t-1) = \alpha \cdot (4,\ t-2,\ -t)$

$(t-4,\ 2-t,\ t-1) = (4\alpha,\ \alpha\cdot(t-2),\ -\alpha\cdot t)$

$$\begin{cases} t-4 = 4\alpha \\ 2-t = \alpha\cdot(t-2) \\ t-1 = -\alpha t \end{cases} \qquad \begin{cases} t-4 = 4\alpha \\ t-2 = -\alpha\cdot(t-2) \\ t-1 = -\alpha\cdot t \end{cases}$$

$\boxed{t \neq 2}$ da $t-2 = -\alpha\cdot(t-2)$ si ha che $\alpha = -1$

sostituendo nelle altre due equazioni si ha

$$\begin{cases} t-4 = 4\cdot(-1) \\ t-1 = -(-1)\cdot t \end{cases} \qquad \begin{cases} t-4 = -4 \\ t-1 = t \end{cases} \qquad \begin{cases} t = 0 \\ -1 = 0 \end{cases} ↯$$

quindi l'unico valore possibile resta $t=2$

$\boxed{t=2}$ $[\overrightarrow{OA}] = (4,\ 0,\ -2)$

$[\overrightarrow{OB}] = (-2,\ 0,\ 1)$

e si vede subito che $[\overrightarrow{OB}] = \underbrace{(-\tfrac{1}{2})}_{\alpha = -\frac{1}{2}} \cdot [\overrightarrow{OA}]$

$A(0,0,3)$; $\pi : 2x - 3y + z = 0$;

$s : x+3 = 3y + z - 6 = 0$;

Trovare la retta r tale che: $A \in r$, $r // \pi$, $r \perp s$

$$r : \begin{cases} x = \ell \cdot t + 0 \\ y = m \cdot t + 0 \\ z = n \cdot t + 3 \end{cases} \qquad \vec{u}_r = (\ell, m, n)$$

$\vec{v}_\pi = (2, -3, 1) \perp \pi$

$$r // \pi \iff \vec{u}_r \perp \vec{v}_\pi \iff \vec{u}_r \bullet \vec{v}_r = 0 \iff$$

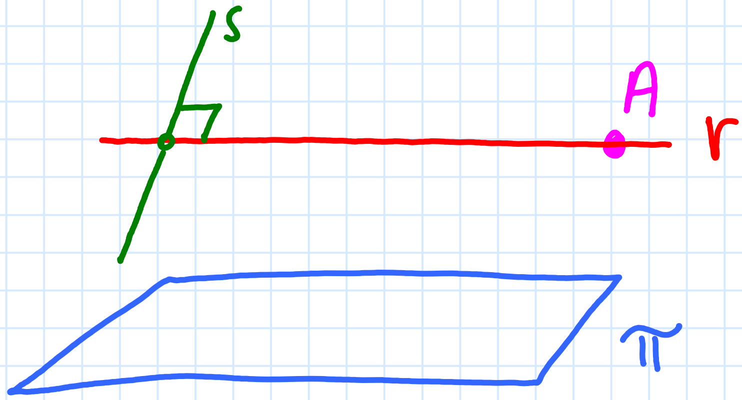
$$\iff 2\cdot l - 3\cdot m + 1\cdot n = 0 \iff$$

$$\iff n = -2\cdot l + 3\cdot m$$

$$\vec{u}_r = (l, m, -2l+3m)$$

$$s: \quad x+3 = 3y+z-6 = 0$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 1 \end{bmatrix} \to \begin{matrix} 0 \\ -1 \\ 3 \end{matrix} \qquad \vec{u}_s = (0, -1, 3)$$



$$r \perp s \iff \vec{u}_r \perp \vec{u}_s \iff \vec{u}_r \bullet \vec{u}_s = 0 \iff$$

$$\iff l\cdot 0 + m\cdot(-1) + (-2l+3m)\cdot 3 = 0 \iff$$

$$\iff -m - 6l + 9m = 0 \iff 8m - 6l = 0 \iff$$

$$\iff 4m - 3l = 0$$

scelgo $m = 3$ et $l = 4$

$$n = -2\cdot l + 3\cdot m = -8 + 9 = 1$$

$(\ell, m, n) = (4, 3, 1) \qquad A(0,0,3)$

$$r: \begin{cases} x = 4\cdot t \\ y = 3\cdot t \\ z = t+3 \end{cases}$$

$\rightsquigarrow$ se vuole, può passare alle cartesiane

$t = z-3$

$$r: \begin{cases} x = 4\cdot(z-3) \\ y = 3\cdot(z-3) \end{cases} \qquad r: \begin{cases} x - 4z + 12 = 0 \\ y - 3z + 9 = 0 \end{cases}$$