Ricevimento studenti - lunedì 23 gennaio 2023

Titolo nota 23/01/2023

$$r: \begin{cases} ax+by+cz+d=0 \\ a'x+b'y+c'z+d'=0 \end{cases}$$

- $F(r): \ \lambda(ax+by+cz+d') + \mu(a'x+b'y+c'z+d')=0$

$$\pi: \underbrace{(\lambda a+\mu a')}_{a''}x + \underbrace{(\lambda b+\mu b')}_{b''}y + \underbrace{(\lambda c+\mu c')}_{c''}z + (\lambda d+\mu d')=0$$

$s \to (l, m, n)$ param. direttori di s

$$\pi // s \iff a''\cdot l + b''\cdot m + c''\cdot n = 0$$

$\downarrow$

eq. lineare omogenea in (λ, μ)
di cui deve trovare un'AUTOSOLUZIONE

$$11x^2+16xy-y^2-50x+50y+35=0$$

$$A=\begin{bmatrix} 11 & 8 \\ 8 & -1 \end{bmatrix}; \quad P_A(\lambda)=\lambda^2-10\lambda-75 =$$
$$=(\lambda-15)\cdot(\lambda+5)$$

$$\lambda_1=15 \ ; \quad \lambda_2=-5$$

$$\begin{bmatrix} -4 & 8 \\ 8 & -16 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad \begin{cases} -4x+8y=0 \\ 8x-16y=0 \end{cases} \quad \begin{cases} x-2y=0 \\ x-2y=0 \end{cases}$$

$$x=2y$$

$$(x,y) = (x, 2x) = x(1,2) \qquad \|(1,2)\| = \sqrt{5}$$

scelgo $x = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$ → autoversore $\left(\dfrac{\sqrt{5}}{5}, \dfrac{2\sqrt{5}}{5}\right)$

$$C = \begin{bmatrix} \frac{\sqrt{5}}{5} & -\frac{2\sqrt{5}}{5} \\ \frac{2\sqrt{5}}{5} & \frac{\sqrt{5}}{5} \end{bmatrix}$$ (matrice della rotazione)

$$\begin{bmatrix} -50 & 50 \end{bmatrix} \cdot \begin{bmatrix} \frac{\sqrt{5}}{5} & -\frac{2\sqrt{5}}{5} \\ \frac{2\sqrt{5}}{5} & \frac{\sqrt{5}}{5} \end{bmatrix} = \begin{bmatrix} 10\sqrt{5} & 30\sqrt{5} \end{bmatrix}$$

dopo la rotazione

$$15(x')^2 - 5(y')^2 + 10\sqrt{5}\,x' + 30\sqrt{5}\,y' + 35 = 0$$

$$\boxed{15(x')^2 + 10\sqrt{5}\,x'} \quad \boxed{-5(y')^2 + 30\sqrt{5}\,y'} + 35 = 0$$

$$\boxed{15\cdot\left(x' + \frac{\sqrt{5}}{3}\right)^2 - \frac{25}{3}} \quad \boxed{-5\left(y' - 3\sqrt{5}\right)^2 + 225} + 35 = 0$$

traslazione $\begin{cases} x'' = x' + \frac{\sqrt{5}}{3} \\ y'' = y' - 3\sqrt{5} \end{cases}$

$$15(x'')^2 - 5(y'')^2 + \frac{755}{3} = 0$$

$$5(y'')^2 - 15(x'')^2 = \frac{755}{3} \qquad |5|$$

$$(y'')^2 - 3(x'')^2 = \frac{151}{3}$$

$$\frac{3}{151}\cdot(y'')^2 - \frac{9}{151}\cdot(x'')^2 = 1$$

$$\frac{(y'')^2}{\left(\frac{151}{3}\right)} - \frac{(x'')}{\left(\frac{151}{9}\right)} = 1$$ iperbole

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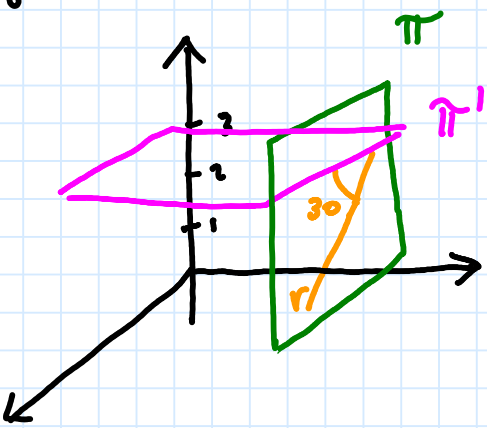
$\pi : x - 4y + 12 = 0$ rette su π

e formano angolo $\frac{\pi}{6}$ radianti

col piano $\pi' : z + 3 = 0$

$$\swarrow$$

$$(a', b', c') = (0, 0, 1)$$



$r \subseteq \pi \Rightarrow r // \pi$

(l, m, n) parametri direttori di r

$$r // \pi \iff 1 \cdot l + (-4)\cdot m + 0\cdot(n) = 0 \iff$$

$$\iff l = 4m$$

$$(4m, m, n)$$

$\widehat{r, \pi'} = \frac{\pi}{6}$ rad $= 30°$ angolo retta-piano

$$\sin\left(\frac{\pi}{6}\,\text{rad}\right) = \frac{|4m\cdot 0 + m\cdot 0 + n\cdot 1|}{\sqrt{(4m)^2+m^2+n^2}\cdot\sqrt{0^2+0^2+1^2}}$$

$$\frac{1}{2} = \frac{|n|}{\sqrt{17m^2+n^2}}$$

$$\sqrt{17m^2+n^2} = 2|n|$$

$$17m^2+n^2 = 4n^2$$

$$17m^2 = 3n^2 \qquad n^2 = \frac{17}{3}m^2$$

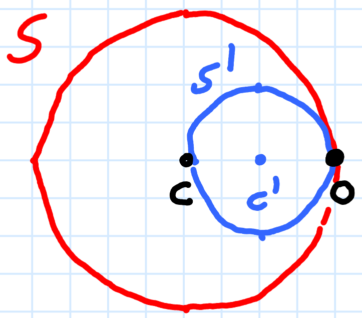
scelgo $m = 3 \rightarrow n^2 = 51 \rightarrow n = \pm\sqrt{51}$

$r_1(12, 3, \sqrt{51})$; $r_2(12, 3, -\sqrt{51})$

$S: x^2+y^2+z^2+x+2y+3z=0$

tangente in $O(0,0,0)$ alla sfera S'

dove $r' = \frac{1}{2}r$



$O(0,0,0)$

$C\left(-\frac{1}{2}, -1, -\frac{3}{2}\right)$

$C\left(-\frac{1}{2}, -1, -\frac{3}{2}\right)$

$C'\left(-\frac{1}{4}, -\frac{1}{2}, -\frac{3}{4}\right)$

$S': x^2+y^2+z^2+\frac{1}{2}x+y+\frac{3}{2}z=0$

$$V(w,x,y,z) \leqslant \mathbb{R}^4$$

$$w - x + z = 4x - y + 2z = 0$$

$$\begin{bmatrix} 1 & -1 & 0 & 1 \\ 0 & 4 & -1 & 2 \end{bmatrix} \cdot \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$w = x - z$$
$$y = 4x + 2z$$

$$(w,x,y,z) = (x-z,\, x,\, 4x+2z,\, z) =$$

$$= x(1,1,4,0) + z(-1,0,2,1)$$

Base di $V = ((1,1,4,0),(1,0,2,1))$

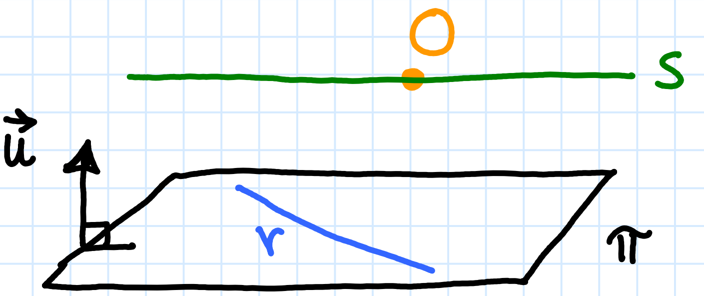
$r:\ x - 5y - 90 = 3y + 2z = 0$

$s:\ 5x + z = x - 2y + 2z = 0 \longrightarrow O \in s$

t retta di minima distanza

D distanza tra r ed s

$\vec{v} \parallel t$ e $\|\vec{v}\| = D$



$\pi \in F(r)$ et $\pi \parallel s$

$$\pi \in F(r):\ \lambda \cdot (x - 5y - 90) + \mu(3y + 2z) = 0$$

$$\pi : \underbrace{\lambda}_{a} \cdot x + \underbrace{(3\mu - 5\lambda)}_{b} \cdot y + \underbrace{(2\mu)}_{c} \cdot z - 90\lambda = 0$$

$$s : 5x + z = x - 2y + 2z = 0$$

$$\begin{bmatrix} 5 & 0 & 1 \\ 1 & -2 & 2 \end{bmatrix} = \begin{cases} l = 2 \\ m = -9 \\ n = -10 \end{cases}$$

$$s \parallel \pi \iff al + bm + cn = 0 \iff 2 \cdot \lambda - 9 \cdot (3\mu - 5\lambda) - 10(2\mu) = 0$$

$$2\lambda - 27\mu + 45\lambda - 20\mu = 0$$

$$47\lambda - 47\mu = 0 \qquad \lambda - \mu = 0$$

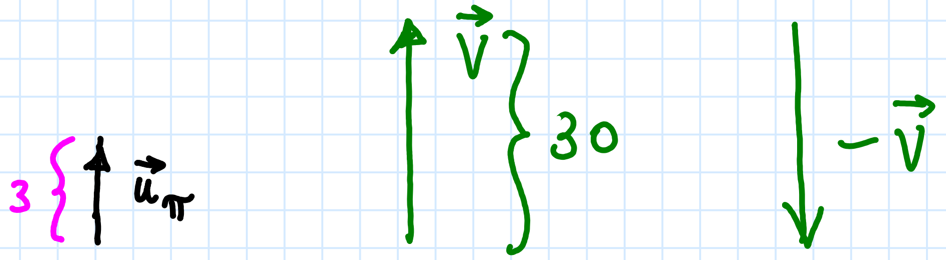
scelgo la coppia $(\lambda, \mu) = (1, 1)$

$$\pi : x - 2y + 2z - 90 = 0$$

$$D = d(O, \pi) = \frac{|0 - 2\cdot 0 + 2 \cdot 0 - 90|}{\sqrt{1^2 + (-2)^2 + 2^2}} = \frac{|-90|}{\sqrt{9}} = \frac{90}{3} = 30$$

$$D = 30$$

$$\vec{u}_\pi = (1, -2, 2) \qquad \|\vec{u}\| = \sqrt{9} = 3$$



$$\vec{v} = 10\,\vec{u}_\pi = 10(1, -2, 2) = (10, -20, 20)$$

$-\vec{v} = (-10, 20, -20)$

$$A = \begin{bmatrix} 3t & 2t^2 \\ 6t & -t \end{bmatrix} \qquad p_A(\lambda) = \det \begin{bmatrix} (3t-\lambda) & 2t^2 \\ 6t & (-t-\lambda) \end{bmatrix}$$

$$p_A(\lambda) = \lambda^2 - (2t)\lambda + (-3t^2 - 12t^3)$$

$$\Delta = [-(2t)]^2 - 4 \cdot 1 \cdot (-3t^2 - 12t^3)$$

$$\Delta = 4t^2 + 12t^2 + 48t^3 = 16t^2 + 48t^3 =$$
$$= 16t^2 \cdot (1 + 3t)$$

$$\Delta > 0 \qquad 16t^2 \cdot (1+3t) > 0$$
$$t^2 \cdot (1+3t) > 0$$

per $t \neq 0$ $\quad t^2 > 0 \qquad 1 + 3t > 0$

$$\boxed{t > -\frac{1}{3}}$$

cosa accade per $t=0$? $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ è già diagonale

$$\boxed{t > -\frac{1}{3}}$$

$A(-2, 0, 0) \in r$

r incidente S : $z + 3 = y - 12x = 0$

r incidente t : $y-4 = z+16x = 0$

$F(s)$: $\lambda(z+3)+\mu(y-12x)=0$

$A \in \pi$: $\lambda(0+3)+\mu(0-12\cdot(-2))=0$

$$3\lambda+24\mu=0 \qquad \lambda+8\mu=0$$

scelgo $(\lambda,\mu)=(8,-1)$

$$8(z+3)-1\cdot(y-12x)=0$$

$$\pi : 12x-y+8z+24=0$$

$F(t)$: $\lambda\cdot(y-4)+\mu\cdot(z+16x)=0$

$A \in \pi'$: $\lambda\cdot(0-4)+\mu\cdot(0+16(-2))=0$

$$-4\lambda-32\mu=0 \qquad \lambda+8\mu=0$$

scelgo $(\lambda,\mu)=(8,-1)$

$$8\cdot(y-4)-1(z+16x)=0$$

$$-16x+8y-z-32=0$$

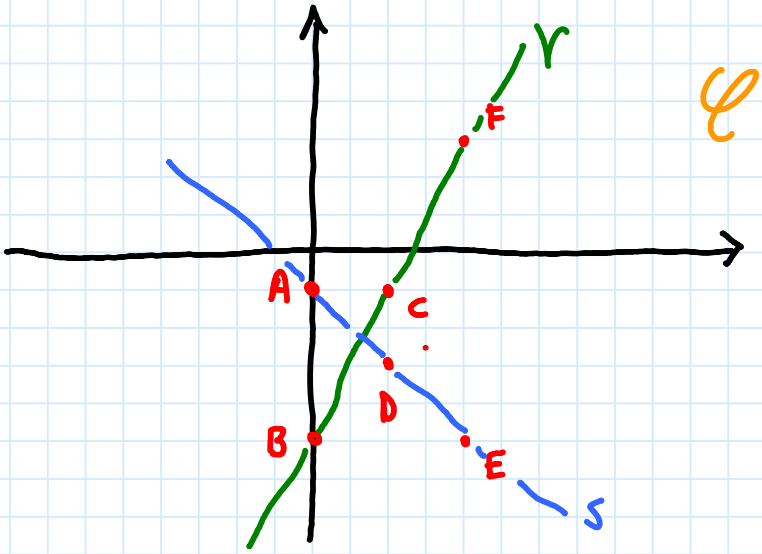
$$\pi' : 16x-8y+z+32=0$$

$$r : \begin{cases} 12x+8y-z+24=0 \\ 16x-8y+z+32=0 \end{cases}$$

$$\begin{vmatrix} 12 & 8 & -1 \\ 16 & -8 & 1 \end{vmatrix} \begin{matrix} \nearrow l = \\ \rightarrow m = \\ \searrow n = \end{matrix}$$

$A(0,-1)$; $B(0,-5)$; $C(2,-1)$; $D(2,-3)$; $E(4,-5)$

$F(4,3)$



$\mathcal{C} = r \cup s$

$r: \quad y = 2x - 5 \qquad \boxed{2x - y - 5 = 0}$

$s: \quad y = -x - 1 \qquad \boxed{x + y + 1 = 0}$

$$\boxed{\mathcal{C}: \quad (2x - y - 5) \cdot (x + y + 1) = 0}$$

$r \parallel$ asse X e $A(1, 9, 3\sqrt{3}) \in r$

piani contenenti r e formano angolo $\frac{\pi}{6}$ radianti col piano XY

$r \parallel$ asse $X \parallel \overset{l\ \ m\ \ n}{(1, 0, 0)}$

$$r: \begin{cases} x = 1t + 1 & = t + 1 \\ y = 0t + 9 & = 9 \\ z = 0t + 3\sqrt{3} & = 3\sqrt{3} \end{cases}$$

$r: \quad y - 9 = z - 3\sqrt{3} = 0$

$\pi \in F(r): \quad \lambda(y-9)+\mu(z-3\sqrt{3})=0$

$$\underbrace{0}_{a}\cdot x+\underbrace{\lambda}_{b}\cdot y+\underbrace{\mu}_{c}\cdot z-(9\lambda+3\sqrt{3}\mu)=0$$

$\pi' =$ piano XY: $\quad \underbrace{0}_{a'}\cdot x+\underbrace{0}_{b'}\cdot y+\underbrace{1}_{c'}\cdot z+0=0$

angolo tra i 2 piani π e π' di $\frac{\pi}{6}$ radianti

$$\cos\left(\frac{\pi}{6}\text{rad}\right)=\pm\frac{0\cdot 0+\lambda\cdot 0+\mu\cdot 1}{\sqrt{0^2+\lambda^2+\mu^2}\cdot\sqrt{0^2+0^2+1^2}}=\frac{\sqrt{3}}{2}$$

$$\pm\frac{\mu}{\sqrt{\lambda^2+\mu^2}}=\frac{\sqrt{3}}{2}$$

$$\pm 2\mu=\sqrt{3}\cdot\sqrt{\lambda^2+\mu^2}$$

$$4\mu^2=3\lambda^2+3\mu^2$$

$\mu^2=3\lambda^2 \qquad$ scelgo $\lambda=1$

$\mu^2=3 \rightarrow \mu=\pm\sqrt{3}$

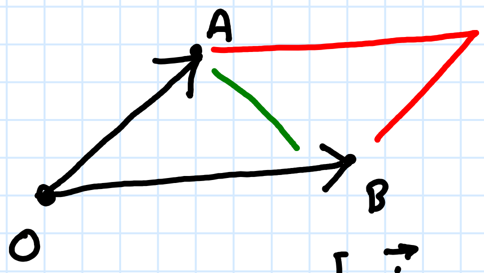
$$1\cdot(y-9)\pm\sqrt{3}\cdot(z-3\sqrt{3})=0$$

$y-9+\sqrt{3}z-9=0 \qquad \pi_2: \; y+\sqrt{3}z-18=0$

$y-9-\sqrt{3}z+9=0 \qquad \pi_1: \; y-\sqrt{3}z=0$

$O(0,0,0)$; $A(t,-t,t)$; $B(0,-12,12)$

area $\overset{\triangle}{OAB} = 156$



$[\vec{OA}] = (t, -t, t)$

$[\vec{OB}] = (0, -12, 12)$

$$[\vec{OA}] \wedge [\vec{OB}] = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ t & -t & t \\ 0 & -12 & 12 \end{bmatrix} = 0 \cdot \vec{i} - 12t\,\vec{j} - 12t\,\vec{k}$$

$$\left\| [\vec{OA}] \wedge [\vec{OB}] \right\| = \sqrt{0^2 + (-12t)^2 + (-12t)^2} = \sqrt{2 \cdot 144 t^2} =$$

$$= 12|t| \cdot \sqrt{2} \overset{\text{teoria}}{=} \text{area parallelogramma}$$

quindi $\dfrac{12|t| \cdot \sqrt{2}}{2} = 156$

$$6|t| \cdot \sqrt{2} = 156$$

$$|t| \cdot \sqrt{2} = 26 = 2 \cdot 13$$

$$|t| = 13\sqrt{2}$$

$$\boxed{t = \pm 13 \cdot \sqrt{2}}$$