Ricevimento studenti – lunedì 3 luglio 2023

Titolo nota 03/07/2023

Buongiorno e benvenuti al ricevimento studenti.
Io sto lavorando al computer. Fatevi <u>SENTIRE</u>
che accendo la telecamera e il microfono.

Sia

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$p_A(\lambda) \overset{DEF}{=} \det(A - \lambda I_3) = \begin{bmatrix} (1-\lambda) & 0 & 1 \\ \boxed{0 \quad (1-\lambda) \quad 0} \\ 1 & 0 & (1-\lambda) \end{bmatrix} =$$

$$= (1-\lambda)\cdot\left[(1-\lambda)\cdot(1-\lambda) - 1\cdot 1\right] = (1-\lambda)\cdot(\lambda^2 - 2\lambda) =$$

$$= \lambda\cdot(1-\lambda)\cdot(\lambda-2)$$

$$\lambda_1 = 0$$
$$\lambda_2 = 1$$
$$\lambda_3 = 2$$

$$(A - \lambda_1 \cdot I_3)\cdot X = 0 \qquad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} x + z = 0 \\ y = 0 \\ x + z = 0 \end{cases} \qquad \begin{cases} y = 0 \\ z = -x \end{cases}$$

$$(x,y,z)=(x,0,-x)=x\cdot(1,0,-1) \quad \forall x\in\mathbb{R}-\{0\}$$

$\lambda_2=1$ $\qquad (A-\lambda_2 I_3)\cdot X=0$

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \left\{\begin{array}{l} z=0 \\ 0=0 \\ x=0 \end{array}\right.$$

$$(x,y,z)=(0,y,0)=y\cdot(0,1,0) \quad \forall y\in\mathbb{R}-\{0\}$$

$\lambda_3=2$

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}\cdot\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left\{\begin{array}{l} -x+z=0 \\ -y=0 \\ x-z=0 \end{array}\right. \qquad \left\{\begin{array}{l} y=0 \\ z=x \end{array}\right. \qquad (x,y,z)=(x,0,x)=x(1,0,1) \quad \forall x\in\mathbb{R}-\{0\}$$

$\lambda_1=0 \longrightarrow (1,0,-1)$

$\lambda_2=1 \longrightarrow (0,1,0)$

$\lambda_3=2 \longrightarrow (1,0,1)$

$$\Lambda=\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}; \quad C=\left[\begin{array}{c:c:c} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{array}\right]$$

$O(0,0,0)$; $\alpha: 5x - y + 2z = 0$;

$r: \; x = 5x - y + 2z = 0$

$\pi: \; O \in \pi, \; \pi \perp \alpha, \; \pi \parallel r$

$\pi: ax + by + cz + d = 0$

$O(0,0,0) \in \pi \Rightarrow d = 0 \Rightarrow \pi: ax + by + cz = 0$

$\pi \perp \alpha: 5x - y + 2z = 0 \Rightarrow (a,b,c) \perp (5,-1,2) \Rightarrow$

$\Rightarrow (a,b,c) \bullet (5,-1,2) = 0 \Rightarrow 5a - b + 2c = 0 \Rightarrow$

$\Rightarrow b = 5a + 2c \Rightarrow \pi: ax + (5a+2c)y + cz = 0$

$\pi \parallel r : x = 5x - y + 2z = 0 \Rightarrow$

$$\Rightarrow \det \begin{bmatrix} a & (5a+2c) & c \\ 1 & 0 & 0 \\ 5 & -1 & 2 \end{bmatrix} = 0 \Rightarrow$$

$$\Rightarrow -1 \cdot \det \begin{bmatrix} (5a+2c) & c \\ -1 & 2 \end{bmatrix} = 0 \Rightarrow$$

$$\Rightarrow -\left[2 \cdot (5a+2c) + c \right] = 0 \Rightarrow$$

$\Rightarrow 10a + 5c = 0 \Rightarrow 2a + c = 0 \Rightarrow c = -2a$

$\pi: ax + (5a+2c)y + cz = 0 \quad$ con $c = -2a$

$\pi: ax + ay - 2az = 0$

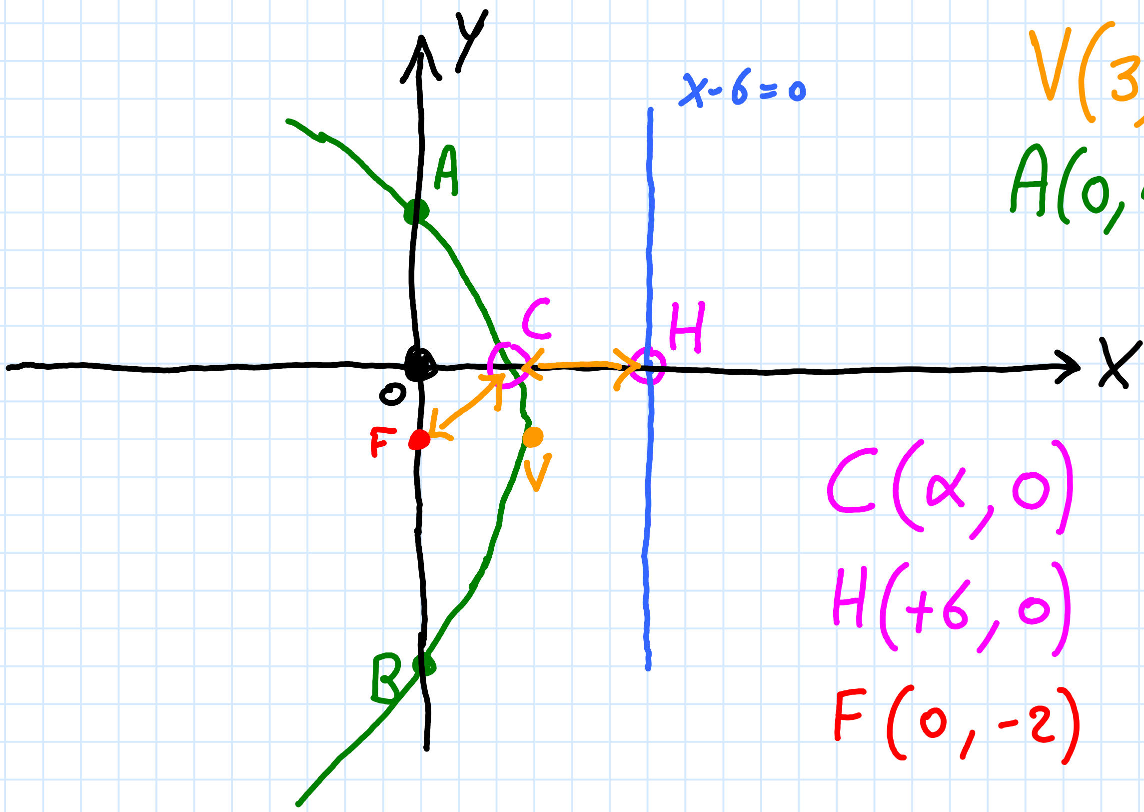
Scelgo $a \neq 0$ a piacere, $a = 1$

$\pi: x + y - 2z = 0$

Ω parabola $F(0,-2)$ fuoco

direttrice $x - 6 = 0$.

Punti intersezione con assi coordinati



$V(3,-2)$

$A(0,4)$

$B(0,-8)$

$C(\alpha, 0)$

$H(+6, 0)$

$F(0,-2)$

$d(F,C) = d(H,C) \; ; \; [d(F,C)]^2 = [d(H,C)]^2$

$(0-\alpha)^2 + (-2-0)^2 = (+6-\alpha)^2 + (0-0)^2$

$$\cancel{\alpha^2}+4=36+\cancel{\alpha^2}-12\alpha$$

$$12\alpha=32 \qquad 3\alpha=8 \qquad \alpha=\frac{8}{3}$$

$$C\left(\frac{8}{3},0\right) \qquad \frac{8}{3}<\frac{9}{3}=3$$

$r \parallel$ asse y ; $A(\sqrt{3},-6,1)\in r$;

piani contenenti r che formano angolo di $\frac{\pi}{3}$ radianti con asse z

$$\text{asse } y \parallel \vec{j}=\begin{array}{ccc} l & m & n \\ (0, & 1, & 0) \end{array} \qquad r:\begin{cases} x=0\cdot t+\sqrt{3} \\ y=1\cdot t+(-6) \\ z=0\cdot t+1 \end{cases}$$

$$r:\begin{cases} x=\sqrt{3} \\ y=t-6 \\ z=1 \end{cases} \qquad r:\begin{cases} x-\sqrt{3}=0 \\ z-1=0 \end{cases}$$

$$\mathcal{F}(r):\ \lambda\cdot(x-\sqrt{3})+\mu\cdot(z-1)=0$$

$$\underbrace{\lambda}_{a}\cdot x+\underbrace{0}_{b}\cdot y+\underbrace{\mu}_{c}\cdot z\underbrace{-(+\sqrt{3}\lambda+\mu)}_{d}=0$$

$$\text{asse } z \parallel \vec{k}=(\underset{l}{0},\underset{m}{0},\underset{n}{1})$$

ANGOLO RETTA - PIANO

$$\theta = \frac{\pi}{3}\,\text{rad}$$

$$\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$$

$$\sin\theta = \frac{|al + bm + cn|}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{l^2+m^2+n^2}}$$

$$\frac{\sqrt{3}}{2} = \frac{|0 + 0 + \mu|}{\sqrt{\lambda^2+\mu^2}\cdot\sqrt{1}}$$

$$\frac{\sqrt{3}}{2} = \frac{|\mu|}{\sqrt{\lambda^2+\mu^2}} \quad ; \quad \sqrt{3}\cdot\sqrt{\lambda^2+\mu^2} = 2|\mu|$$

$$3\cdot(\lambda^2+\mu^2) = 4\mu^2$$

$3\lambda^2 = \mu^2$ scelgo $\lambda = 1$

$$\mu^2 = 3 \qquad \mu = \pm 1$$

$$1\cdot x + 0\cdot y \pm 1\cdot z - \left(\sqrt{3}\cdot 1 + (\pm 1)\right) = 0$$

$$\pi_1 : \; x + z - \sqrt{3} - 1 = 0$$

$$\pi_2 : \; x - z - \sqrt{3} + 1 = 0$$

$$9x^2 - 10xy + 9y^2 + 44x - 12y + 32 = 0$$

$$A = \begin{bmatrix} 9 & -5 \\ -5 & 9 \end{bmatrix} ; \quad p_A(\lambda) = \lambda^2 - 18\lambda + 56 = 0$$

$$(\lambda-4)\cdot(\lambda-14)=0$$

$$\lambda_1=4 \qquad \lambda_2=14$$

$\boxed{\lambda_1=4}$ $\to$ suoi autovettori

$$\begin{bmatrix}5 & -5\\ -5 & 5\end{bmatrix}\cdot\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix} \quad ; \quad \begin{cases}5x-5y=0\\ -5x+5y=0\end{cases}$$

$$x-y=0 \quad \to \quad x=y$$

$$(x,y)=(y,y)=y(1,1) \quad \forall y\neq 0$$

$\lambda_1=4 \to$ auto VERSORE $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\lambda_2=14 \to$ auto VERSORE $\left(-\frac{\sqrt{2}}{2}, +\frac{\sqrt{2}}{2}\right)$

$$C=\begin{bmatrix}\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{bmatrix}$$ matrice della rotazione

$$4\cdot(x')^2+14(y')^2+\alpha\cdot x'+\beta\cdot y'+32=0$$

$$\begin{bmatrix}44 & -12\end{bmatrix}\begin{bmatrix}\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{bmatrix}=\begin{bmatrix}\alpha & \beta\end{bmatrix}$$

$$\alpha=44\cdot\frac{\sqrt{2}}{2}-12\cdot\frac{\sqrt{2}}{2}=16\sqrt{2}$$

$$\beta=44\cdot\left(-\frac{\sqrt{2}}{2}\right)-12\cdot\frac{\sqrt{2}}{2}=-28\sqrt{2}$$

$$4(x')^2 + 14(y')^2 + 16\sqrt{2}\,x' - 28\sqrt{2}\,y' + 32 = 0$$

$$\boxed{2\cdot(x')^2} + \boxed{7\cdot(y')^2} + \boxed{8\sqrt{2}\,x'} \boxed{-14\sqrt{2}\,y'} + 16 = 0$$

$$\boxed{2\cdot\left(x' + 2\sqrt{2}\right)^2 - 16} + \boxed{7\cdot\left(y' - \sqrt{2}\right)^2 - 14} + 16 = 0$$

traslazione : $\begin{cases} x'' = x' + 2\sqrt{2} \\ y'' = y' - \sqrt{2} \end{cases}$

$$2\cdot(x'')^2 + 7\cdot(y'')^2 - 14 = 0$$

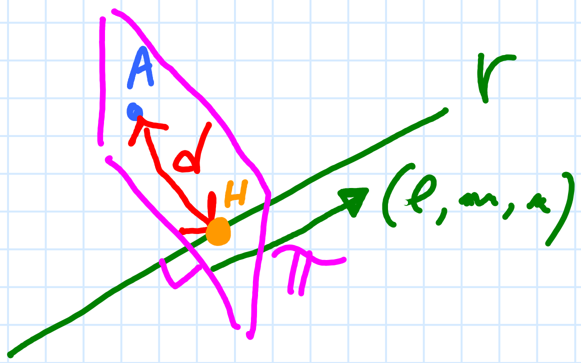
$$2\cdot(x'')^2 + 7\cdot(y'')^2 = +14$$

$$\boxed{\frac{(x'')^2}{7} + \frac{(y'')^2}{2} = +1}$$

eq. canonica ELLISSE

$A(\sqrt{3}, 1, -7)$; $r:\ y = z + 11 = 0$;

distanza di A da r



π : $A \in \pi$ et $\pi \perp r$

$r: \; y = z + 11 = 0$ $\quad \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\to l = 1, \; m = 0, \; n = 0$

r è parallela all'asse X (si vedeva)

per π prendo $a=1$, $b=0$, $c=0$

$$1 \cdot (x - \sqrt{3}) + 0 \cdot (y-1) + 0 \cdot (z+7) = 0$$

$$\pi: \; x - \sqrt{3} = 0$$

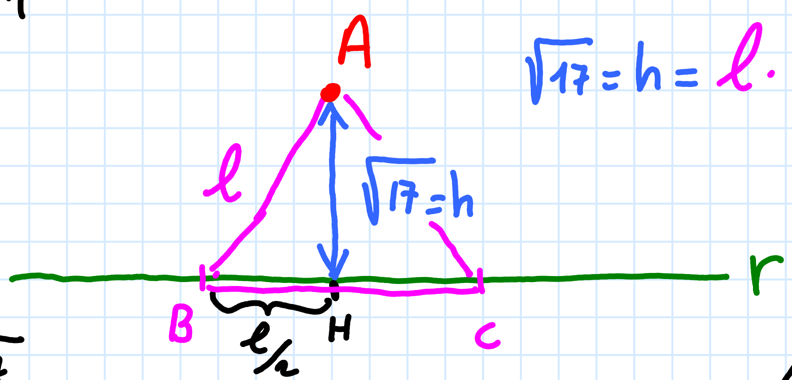
$\{H\} = \pi \cap r: \begin{cases} x - \sqrt{3} = 0 \\ y = 0 \\ z + 11 = 0 \end{cases}$ $\quad H(\sqrt{3}, 0, -11)$

$A(\sqrt{3}, 1, -7)$

$$d(A,H) = \sqrt{(\sqrt{3}-\sqrt{3})^2 + (0-1)^2 + [-11-(-7)]^2}$$

$$d(A,H) = \sqrt{17}$$

$\sqrt{17} = h = l \cdot \frac{\sqrt{3}}{2}$



$\frac{l}{2} = \frac{\sqrt{17}}{\sqrt{3}} = \sqrt{\frac{17}{3}} \longrightarrow$ valore "strano" (forse dati iniziali non esatti)