Ricevimento studenti – lunedì 17 luglio 2023

Titolo nota 17/07/2023

Buongiorno e benvenuti al ricevimento studenti.
Io sto lavorando al computer. Fatevi <u>SENTIRE</u> che accendo il microfono e la telecamera.

$$A = \begin{bmatrix} 1 & -t & 2 \\ 0 & -5 & (t-4) \\ 0 & 0 & 1 \end{bmatrix} \qquad t \in \mathbb{R}$$

diagonalizzabile ?

$$p_A(\lambda) = \det \begin{bmatrix} (1-\lambda) & -t & 2 \\ 0 & (-5-\lambda) & (t-4) \\ 0 & 0 & (1-\lambda) \end{bmatrix}$$

$$p_A(\lambda) = (1-\lambda)(-5-\lambda)(1-\lambda) = -(5+\lambda)^1 \cdot (1-\lambda)^2$$

autovalori $\lambda_1 = -5$; $m_a(-5) = 1$

$\lambda_2 = +1$; $m_a(1) = 2$

$m_g(-5) = 1$; $m_g(1) = ?$

$$m_g(1) = \overset{\text{numero incognite}}{\overset{\downarrow}{n}} - rg(A - 1 \cdot I_3) =$$

$$= 3 - rg(A - 1 \cdot I_3)$$

Siccome noi vogliamo che

$m_g(1) = m_a(1) = 2$ allora deve

essere $rg(A - 1 \cdot I_3) = 1$

$$A - 1 \cdot I_3 = \begin{bmatrix} 0 & -t & 2 \\ 0 & -6 & (t-4) \\ 0 & 0 & 0 \end{bmatrix}$$

$$rg(A - 1 \cdot I_3) = 1 \iff rg\begin{bmatrix} -t & 2 \\ -6 & (t-4) \end{bmatrix} = 1 \iff$$

$$\iff \det\begin{bmatrix} -t & 2 \\ -6 & (t-4) \end{bmatrix} = 0 \iff -t \cdot (t-4) + 12 = 0 \iff$$

$$\iff -t^2 + 4t + 12 = 0 \iff t^2 - 4t - 12 = 0 \iff$$

$$\iff (t-6) \cdot (t+2) = 0 \iff t_1 = +6 \text{ or } t_2 = -2$$

$$\begin{cases} 3x - y + 10z - 22 = 0 \\ 1x - y + 8z - 8 = 0 \\ 2x + y - 5z - 13 = 0 \end{cases} \qquad C = \begin{bmatrix} 1 & -1 & 8 & -8 \\ 3 & -1 & 10 & -22 \\ 2 & 1 & -5 & -13 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 8 & -3 \\ 0 & 2 & -14 & +2 \\ 0 & 3 & -21 & +3 \end{bmatrix} ; \begin{bmatrix} 1 & -1 & 8 & -3 \\ 0 & 1 & -7 & 1 \\ \cancel{0} & \cancel{1} & \cancel{-7} & \cancel{1} \end{bmatrix}$$

$$\begin{cases} x - y + 8z - 3 = 0 \\ y - 7z + 1 = 0 \end{cases} \uparrow \begin{cases} x - (7z - 1) + 8z - 3 = 0 \\ y = 7z - 1 \end{cases}$$

$$\begin{cases} x + z - 2 = 0 \\ y = 7z - 1 \end{cases} \qquad \begin{cases} x = -z + 2 \\ y = 7z - 1 \end{cases}$$

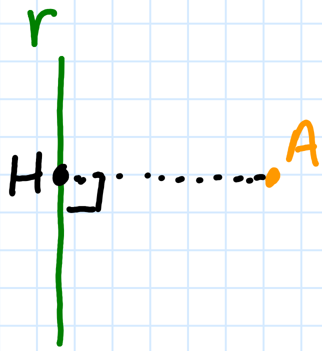
$$(x, y, z) = (-z + 2,\ 7z - 1,\ z) =$$

$$= \underbrace{(2, -1, 0)}_{X_p \leftarrow \text{particolare}} + \underbrace{z \cdot (-1, 7, 1)}_{X_0 \leftarrow \text{omogeneo associato}} \leftarrow \text{SOL. GENERALE}$$

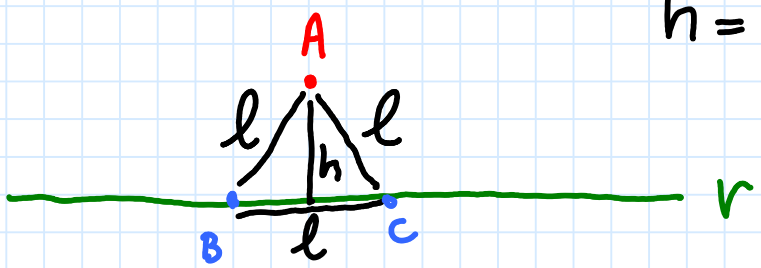
$$\forall z \in \mathbb{R} \rightarrow \infty^1 \text{ soluzione}$$

$A(10, \sqrt{44}, 3)$; $r:\ y = x - 8 = 0$

$B, C \in r$ tali che $\overset{\triangle}{ABC}$ sia equilatero



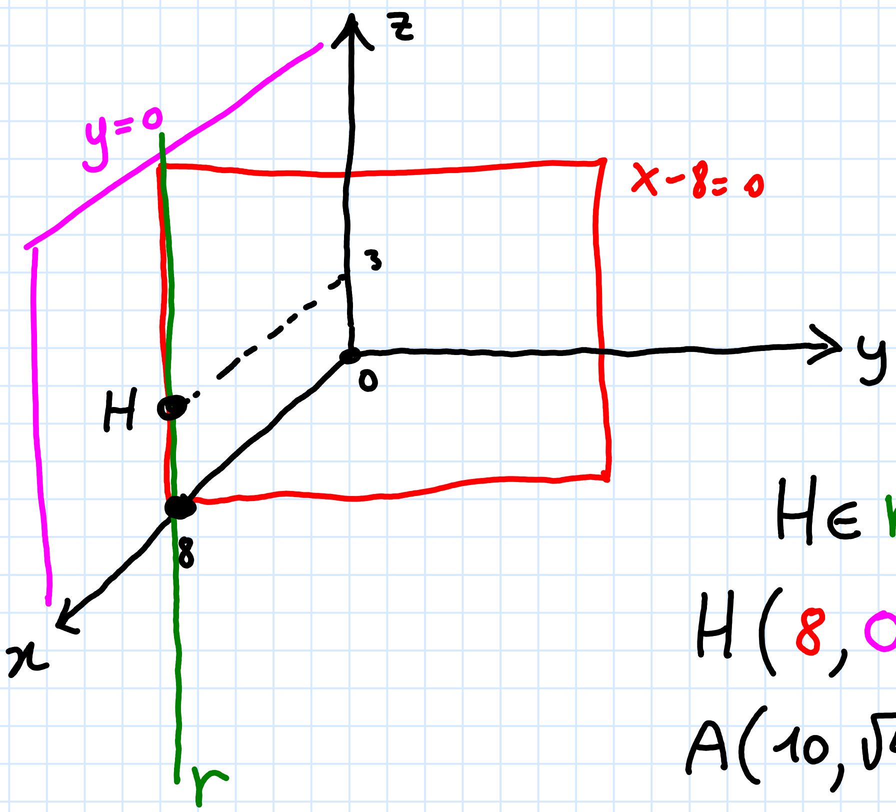
trovare la distanza di A dalla retta r



$$h = \frac{\sqrt{3}}{2} \cdot \ell$$

$$\ell = \frac{2}{\sqrt{3}} \cdot h$$

$$\frac{\ell}{2} = \frac{1}{\sqrt{3}} \cdot h$$



$H \in r$

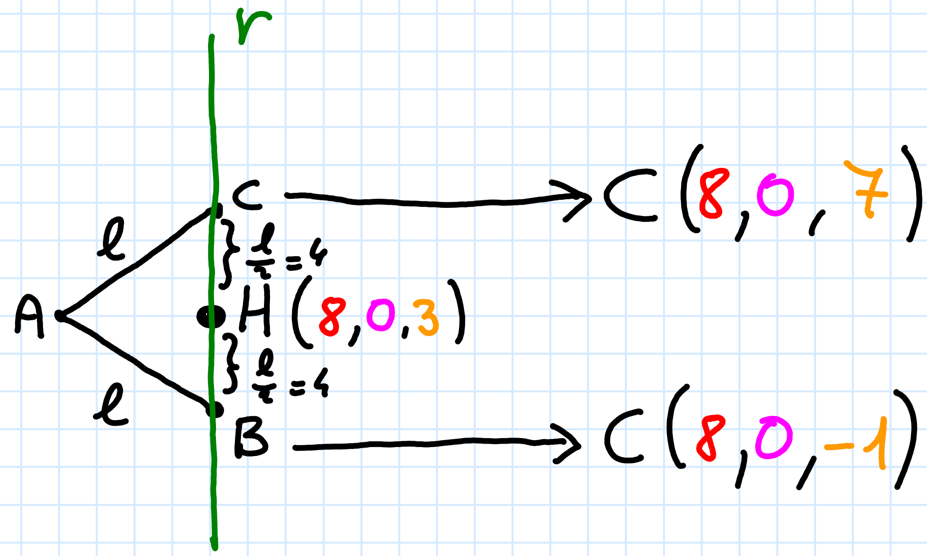
$H(8, 0, 3)$

$A(10, \sqrt{44}, 3)$

$$h = d(A,H) = \sqrt{(10-8)^2 + (\sqrt{44}-0)^2 + (3-3)^2} = \sqrt{48}$$

$$h = 4\sqrt{3} \Rightarrow \frac{\ell}{2} = \frac{1}{\sqrt{3}} \cdot h = \frac{1}{\cancel{\sqrt{3}}} \cdot 4 \cdot \cancel{\sqrt{3}} = 4$$

$$\frac{\ell}{2} = 4$$



$A(t,0,0)$; $B(0,-2,0)$, $C(0,0,\sqrt{23})$

α piano passante per A, B e C

$\beta =$ piano Yz : $x=0$

$\theta = \frac{2}{3}\pi$ radianti

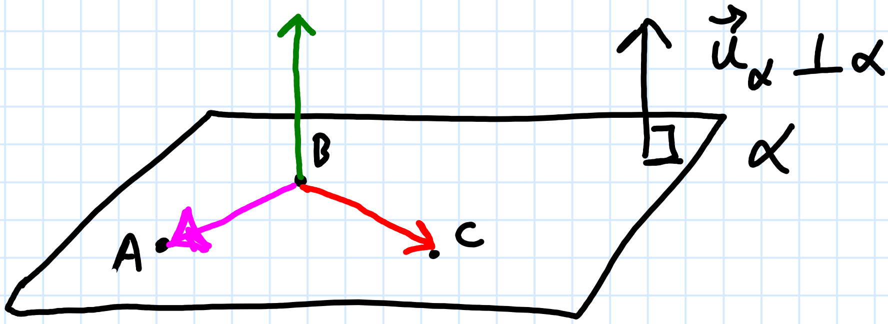
angolo tra 2 piani

$$\cos\theta = \pm \frac{aa' + bb' + cc'}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{(a')^2+(b')^2+(c')^2}}$$

$\alpha \rightarrow \vec{u}_\alpha \perp \alpha \rightarrow \vec{u}_\alpha = (a, b, c)$

$\beta \rightarrow \vec{u}_\beta \perp \beta \rightarrow \vec{u}_\beta = (a', b', c')$

$\beta =$ piano Yz : $x=0 \Rightarrow \vec{u}_\beta = (1,0,0)$



$$A(t,0,0)\,;\ B(0,-2,0) \rightarrow \vec{BA} = (t,\ 2,\ 0)$$

$$C(0,0,\sqrt{23})\,;\ B(0,-2,0) \rightarrow \vec{BC} = (0,\ 2,\ \sqrt{23})$$

$$\vec{u}_\alpha = \vec{BA} \wedge \vec{BC} = \det\begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ t & 2 & 0 \\ 0 & 2 & \sqrt{23} \end{bmatrix} =$$

$$\vec{u}_\alpha = \underbrace{2\sqrt{23}}_{a}\,\vec{i}\ \underbrace{-\,t\sqrt{23}}_{b}\,\vec{j} + \underbrace{2t}_{c}\,\vec{k}$$

$$\vec{u}_\beta = \underbrace{1}_{a'}\cdot\vec{i} + \underbrace{0}_{b'}\,\vec{j} + \underbrace{0}_{c'}\,\vec{k}$$

$$\theta = \frac{2}{3}\pi \text{ rad} = 120^\circ$$

$$\cos\theta = -\frac{1}{2}$$

$$-\frac{1}{2} = \pm\frac{2\sqrt{23}}{\sqrt{92+23t^2+4t^2}\cdot\sqrt{1^2}}$$

$$-\sqrt{92+27t^2} = \pm 4\sqrt{23}$$

$$92+27t^2 = 16\cdot 23 = 368$$

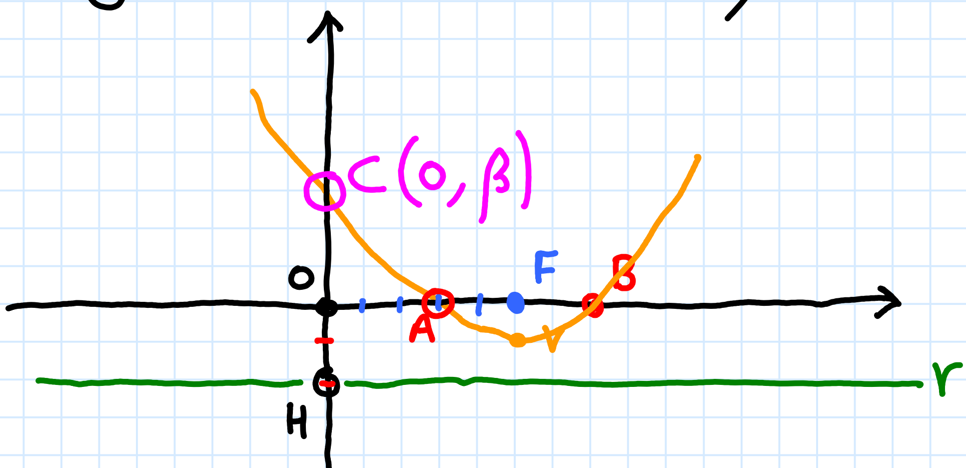
$$27t^2 = 368-92 = 276$$

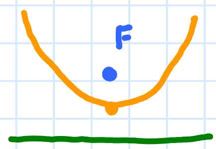
$$27\cdot t^2 = 276$$

$$9\cdot t^2 = 92$$

$$t^2 = \frac{92}{9} \longrightarrow t = \pm\frac{\sqrt{92}}{3}$$

$r: y=-2$ direttrice ; $F(5,0)$





$A(3,0)$

$B(7,0)$

$$d(C,r) = d(C,F)$$

$$d(C,H) = d(C,F)$$

$C(0,\beta)$; $H(0,-2)$; $F(5,0)$

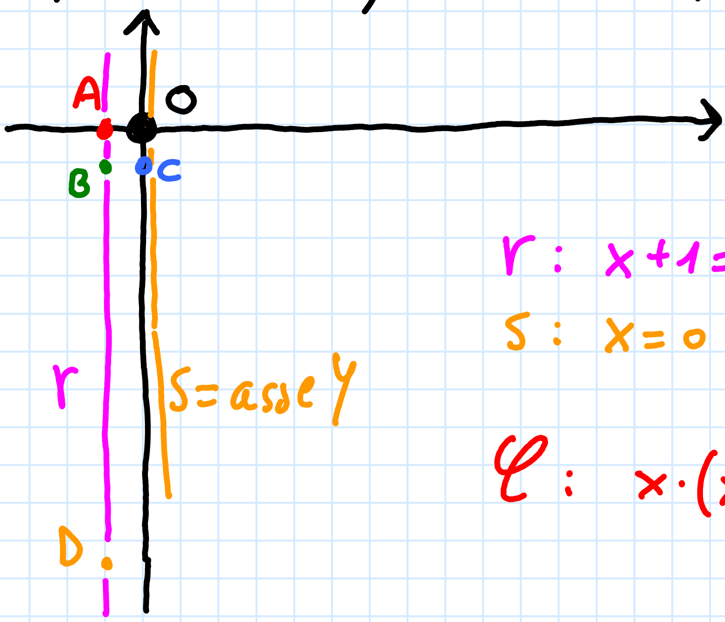
$$\sqrt{(0-0)^2+(\beta-(-2))^2} = \sqrt{(0-5)^2+(\beta-0)^2}$$

$$\cancel{\beta^2}+4\beta+4 = 25+\cancel{\beta^2}$$

$4\beta = 21 \qquad \beta = \frac{21}{4}$

$C\left(0, \frac{21}{4}\right)$

$O(0,0)$; $A(-1,0)$; $B(-1,-1)$; $C(0,-1)$; $D(-1,-12)$



$r: \ x+1=0$

$s: \ x=0$

$\mathscr{C}: \ x\cdot(x+1)=0$