Ricevimento studenti - venerdì 19 gennaio 2024

Titolo nota 19/01/2024

Buongiorno e benvenuti al ricevimento studenti. Io sto lavorando al computer. Fatevi <u>SENTIRE</u> (ho le cuffie) così poi accendo la telecamera e il microfono. Grazie.

$$A = \begin{bmatrix} -1 & 0 & h \\ -3 & 2 & -2 \\ 0 & K & 1 \end{bmatrix} \qquad h, K \in \mathbb{R} \qquad \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix} = X \text{ autovettore}$$

λ suo autovalore

X è un autovettore di A relativo all'autovalore λ $\overset{\text{DEF}}{\Longleftrightarrow}$ $A \cdot X = \lambda \cdot X$

$$\begin{bmatrix} -1 & 0 & h \\ -3 & 2 & -2 \\ 0 & K & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix} = \lambda \cdot \begin{bmatrix} 2 \\ 3 \\ -3 \end{bmatrix}$$

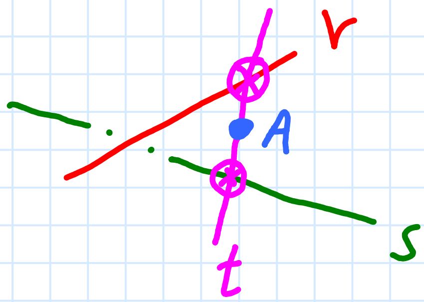
$$\begin{bmatrix} -2-3h \\ 6 \\ 3K-3 \end{bmatrix} = \begin{bmatrix} 2\lambda \\ 3\lambda \\ -3\lambda \end{bmatrix} \Longleftrightarrow \begin{cases} -2-3h = 2\lambda \\ 6 = 3\lambda \rightarrow \lambda = 2 \\ 3K-3 = -3\lambda \end{cases}$$

$$\begin{cases} -2-3h = 4 \\ 3K-3 = -6 \end{cases} \qquad \begin{cases} h = -2 \\ K = -1 \end{cases} \qquad \begin{matrix} h = 2 \\ K = -1 \end{matrix}$$

$A(3,0,-1)$

r: asse X : $y=z=0$ $\qquad A \notin r$

s: $x=z+7=0$ $\qquad A \notin s$



t retta per A che si appoggia sia ad r che ad s

① $\alpha \in F(r)$ et $A \in \alpha$

② $\beta \in F(s)$ et $A \in \beta$

③ $t = \alpha \cap \beta$

① $\alpha \in F(r)$: $\lambda x + \mu y = 0 \quad (\lambda,\mu) \neq (0,0)$

$A \in \alpha \Rightarrow 3 \cdot \lambda + 0 \cdot \mu = 0 \Rightarrow 3\lambda = 0 \Rightarrow \lambda = 0$

$\Rightarrow \mu \neq 0$ scelgo $\mu = 1$

$\boxed{\alpha : y = 0}$

② $\beta \in F(s)$: $\lambda \cdot x + \mu \cdot (z+7) = 0$

$A \in \beta \Rightarrow 3\lambda + 6\mu = 0 \Rightarrow \lambda + 2\mu = 0$

$\Rightarrow \lambda = -2\mu$

scelgo $\mu = -1$ e quindi $\lambda = 2$

$\boxed{\beta : 2x - z - 7 = 0}$

$\boxed{t = \alpha \cap \beta : y = 2x - z - 7 = 0}$ retta richiesta

$V(w,x,y,z) \leq \mathbb{R}^4$

$c_1 = (0,0,2,-1) \qquad c_2 = (0,-1,1,0)$

$v = (w,x,y,z) \in S$

$$A = \begin{array}{c} \begin{array}{cccc} w & x & y & z \end{array} \\ \begin{bmatrix} 0 & \boxed{0} & \boxed{2} & -1 \\ 0 & \boxed{-1} & \boxed{1} & 0 \end{bmatrix} \end{array} \qquad rg\, A = 2$$

$B \longrightarrow$ si vede subito che $rg\, B = 2$

$$\tilde{A} = \begin{bmatrix} w & x & y \\ 0 & 0 & 2 \\ 0 & -1 & +1 \end{bmatrix} \qquad \tilde{\tilde{A}} = \begin{bmatrix} x & y & z \\ 0 & 2 & -1 \\ -1 & 1 & 0 \end{bmatrix} \begin{array}{cc} x & y \\ 0 & 2 \\ -1 & 1 \end{array}$$

$rg\, \tilde{A} = 2$ et $rg\, \tilde{\tilde{A}} = 2$

$\det \tilde{A} = 0$ et $\det \tilde{\tilde{A}} = 0$

$-2 \cdot (-w + 0) = 0$ et $y + 2z + x = 0$

$w = 0$ et $x + y + 2z = 0$

$$\boxed{\begin{cases} w = 0 \\ x + y + 2z = 0 \end{cases}}$$

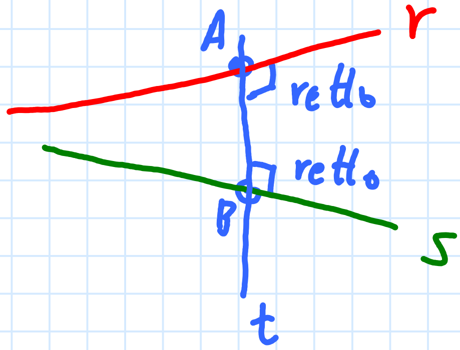
equazioni del sottospazio generato da

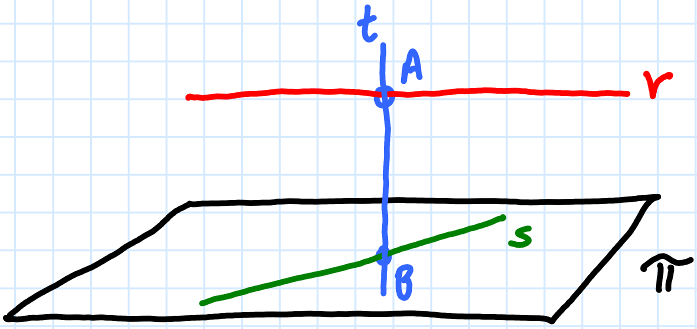
$c_1 = (\underset{w}{0}, \underset{x}{0}, \underset{y}{2}, \underset{z}{-1})$ OK!

$c_2 = (\underset{w}{0}, \underset{x}{-1}, \underset{y}{1}, \underset{z}{0})$ ok!

$r: \; x-3 = 5x-3y = 0$

$s: \; x-5 = x+5z = 0$





in generale

① $\pi : \pi \in F(s)$ et $\pi \,//\, r$

② $\alpha : \alpha \in F(r)$ et $\alpha \perp \pi$

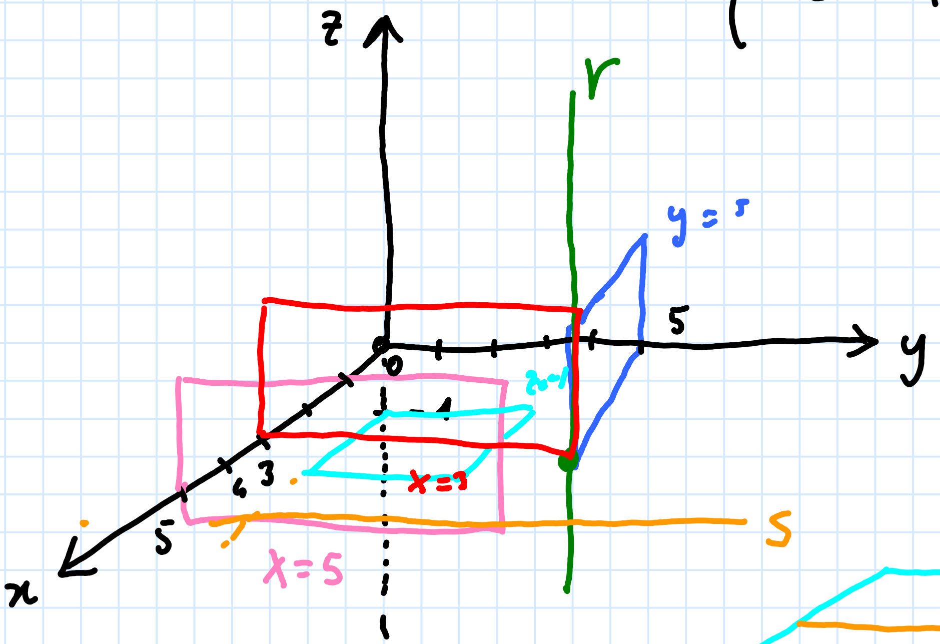
③ $\beta : \beta \in F(s)$ et $\beta \perp \pi$

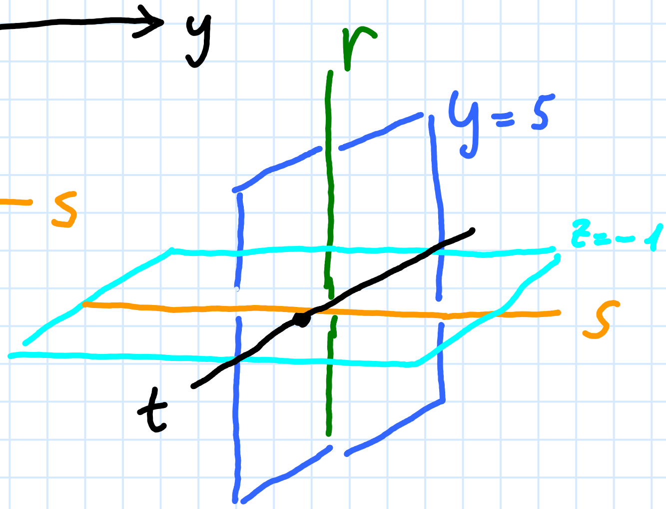
④ $t = \alpha \cap \beta$

nel nostro caso (particolare)

$r: \; x-3 = 5x-3y = 0 \qquad r: \begin{cases} x = 3 \\ y = 5 \end{cases}$

$s: \; x-5 = x+5z = 0 \qquad s: \begin{cases} x = 5 \\ z = -1 \end{cases}$



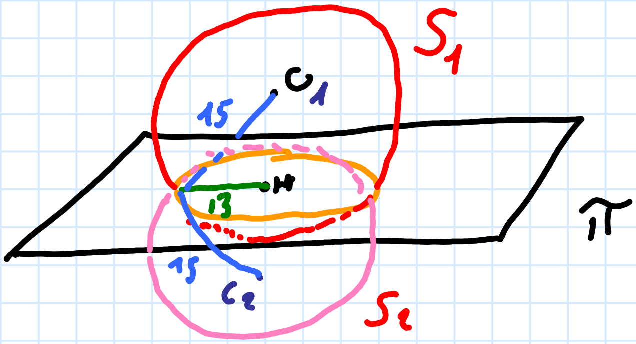


$t: \begin{cases} y = 5 \\ z = -1 \end{cases}$

$R = 15 \quad ; \quad \pi : 2x + 3y - z = 0$

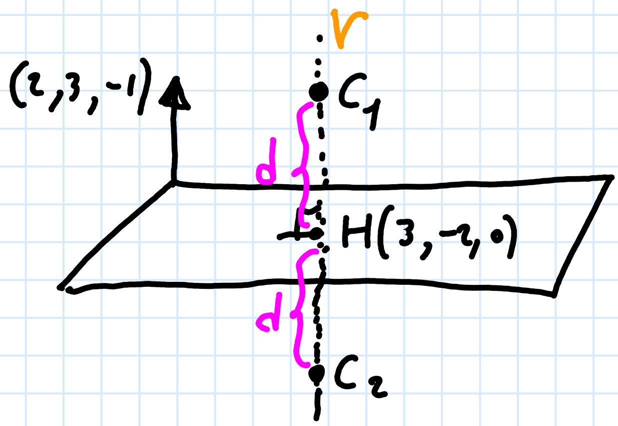
H centro circonferenza $\quad H(3, -2, 0)$

$r = 13$



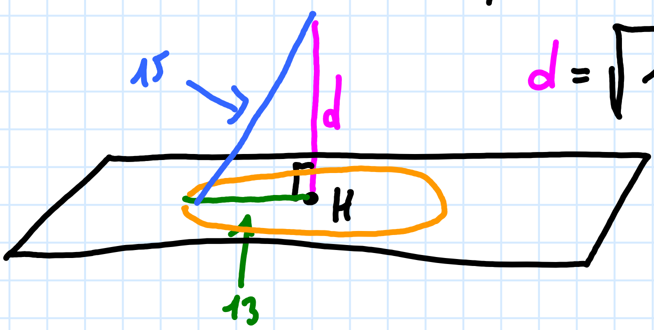
$R = 15$

$C_1 = ? \qquad C_2 = ?$

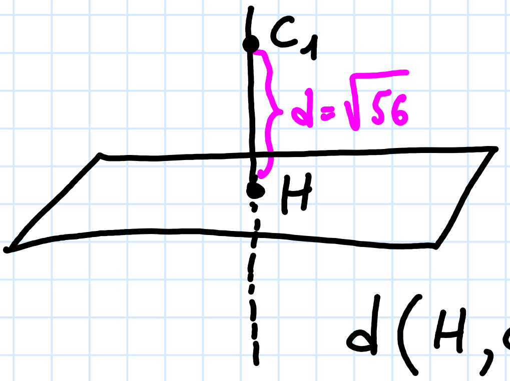


$$r : \begin{cases} x = 2 \cdot t + 3 \\ y = 3 \cdot t + (-2) \\ z = -1 \cdot t + 0 \end{cases}$$

$$C(2t+3, 3t-2, -t)$$



$$d = \sqrt{15^2 - 13^2} = \sqrt{56}$$



$$H(3, -2, 0)$$

$$C(2t+3, 3t-2, -t)$$

$$d(H, C) = \sqrt{56}$$

$$\sqrt{(2t)^2 + (3t)^2 + (-t)^2} = \sqrt{56}$$

$$4t^2 + 9t^2 + t^2 = 56 \quad ; \quad 14t^2 = 56 \quad ; \quad t^2 = 4$$

$$t = \pm 2$$

$t_1 = +2 \longrightarrow C_1(7, 4, -2)$

$t_2 = -2 \longrightarrow C_2(-1, -8, +2)$

$R = 15$

$S_1: (x-7)^2 + (y-4)^2 + (z+2)^2 = 15^2$

$S_2: (x+1)^2 + (y+8)^2 + (z-2)^2 = 15^2$

NEL PIANO $r: 3x - 4y - 5 = 0$ direttrice parabole passanti per l'origine e aventi i fuochi sull'asse X delle ascisse.
Trovare coordinate dei fuochi.

$O(0,0) \in$ parabola

$F(\alpha, 0) \in$ asse X

$$d(O, \underset{\substack{\uparrow \\ \text{fuoco}}}{F}) = d(O, \underset{\substack{\uparrow \\ \text{direttrice}}}{r})$$

$$\sqrt{(0-\alpha)^2 + (0-0)^2} = \frac{|3\cdot 0 - 4\cdot 0 - 5|}{\sqrt{3^2 + (-4)^2}}$$

$$\alpha^2 = \frac{25}{25} \; ; \quad \alpha^2 = 1 \; ; \quad \alpha = \pm 1$$

$F_1(1, 0) \qquad F_2(-1, 0)$

$$A=\begin{bmatrix}1&0&1\\0&1&0\\1&0&1\end{bmatrix}; \quad p_A(\lambda)=\det\begin{bmatrix}(1-\lambda)&0&1\\0&(1-\lambda)&0\\1&0&(1-\lambda)\end{bmatrix}=$$

$$=(1-\lambda)\cdot\det\begin{bmatrix}(1-\lambda)&1\\1&(1-\lambda)\end{bmatrix}=(1-\lambda)\cdot\left[(1-\lambda)^2-1\right]=$$

$$=(1-\lambda)\cdot(\lambda^2-2\lambda)=\lambda\cdot(1-\lambda)\cdot(\lambda-2)$$

$\lambda_1=0$ $\quad$ $\lambda_2=1$ $\quad$ $\lambda_3=2$

3 autovalori a due a due distinti $\Rightarrow$ $\exists$ base di autovettori $\rightarrow$ è diagonalizzabile

$\boxed{\lambda_1=0}$

$$\begin{bmatrix}1&0&1\\0&1&0\\1&0&1\end{bmatrix}\cdot\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \qquad \begin{cases}x+z=0 & \longrightarrow z=-x\\ y=0 & \longrightarrow y=0\\ x+z=0\end{cases}$$

$$(x,y,z)=(x,0,-x)=x\cdot\boxed{(1,0,-1)} \qquad v_1=(1,0,-1)$$

$\boxed{\lambda_2=1}$

$$\begin{bmatrix}0&0&1\\0&0&0\\1&0&0\end{bmatrix}\cdot\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \qquad \begin{cases}z=0\\0=0\\x=0\end{cases} \qquad y \text{ LIBERA}$$

$$(x,y,z)=(0,y,0)=y\boxed{(0,1,0)} \qquad v_2=(0,1,0)$$

$\boxed{\lambda_3=2}$

$$\begin{bmatrix}-1&0&1\\0&-1&0\\1&0&-1\end{bmatrix}\cdot\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix} \qquad \begin{cases}-x+z=0 & \rightarrow z=x\\ -y=0 & \rightarrow y=0\\ x-z=0\end{cases}$$

$$(x,y,z)=(x,0,x)=x\boxed{(1,0,1)} \qquad v_3=(1,0,1)$$

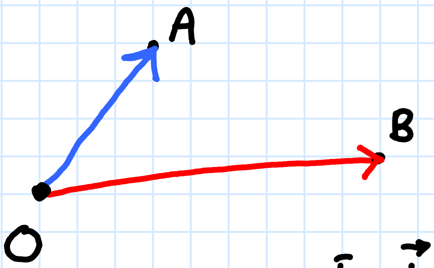
$$\Lambda = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix} ; \quad C = \left[\begin{array}{c:c:c} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{array}\right]$$

si può verificare che $A \cdot C = C \cdot \Lambda$

ovvero $A = C \cdot \Lambda \cdot C^{-1}$

$O(0,0,0)$; $A(t,-t,t)$; $B(0,-12,12)$;

area $A\hat{O}B = 156$ $\qquad t \in \mathbb{R}$



$[\overrightarrow{OA}] = (t, -t, t)$

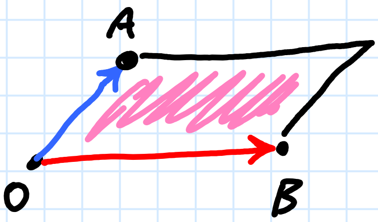
$[\overrightarrow{OB}] = (0, -12, 12)$

$$\vec{w} = [\overrightarrow{OA}] \wedge [\overrightarrow{OB}] = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ t & -t & t \\ 0 & -12 & 12 \end{bmatrix} = 0\,\vec{i} + (-12t)\,\vec{j} + (-12t)\,\vec{k}$$

$$\vec{w} = (-12t)\,\vec{j} + (-12t)\,\vec{k}$$

$$\underbrace{\|\vec{w}\|}_{\geq 0} = \sqrt{(-12t)^2 + (-12t)^2} = \sqrt{2 \cdot 144 \cdot t^2} = 12\sqrt{2} \cdot |t|$$

$\|\vec{w}\|$ = area del parallelogramma



$$\frac{12\sqrt{2}\,|t|}{2} = 156$$

$$|t| = \frac{156}{12} \cdot \frac{2}{\sqrt{2}} = 13 \cdot \sqrt{2}$$

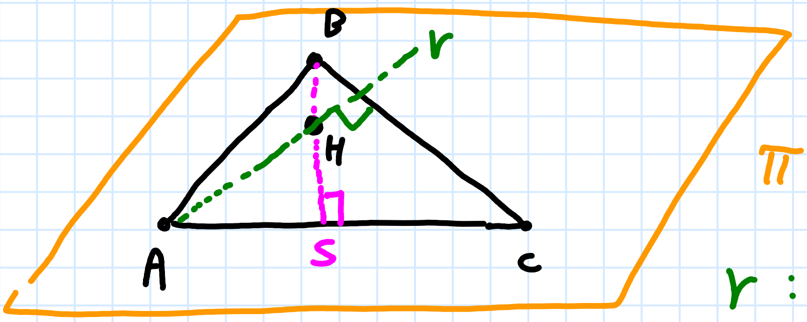
$|t| = 13\sqrt{2} \longrightarrow t = \pm 13\sqrt{2}$

$\pi : x - 2y + z + 18 = 0$

A, B, C intersezioni con gli assi coordinati

$A(-18, 0, 0)$; $B(0, 9, 0)$; $C(0, 0, -18)$

triangolo $A\hat{B}C$. Trovare suo ORTOCENTRO.



$\pi : x - 2y + z + 18 = 0$

$r : \begin{cases} \pi \\ \alpha \to ? \end{cases}$

$\alpha :$ $A \in \alpha$ et $\alpha \perp [\overrightarrow{BC}]$

$\beta :$ $B \in \beta$ et $\beta \perp [\overrightarrow{AC}]$

$s : \begin{cases} \pi \\ \beta \to ? \end{cases}$

$A(-18, 0, 0)$; $B(0, 9, 0)$; $C(0, 0, -18)$

$[\overrightarrow{BC}] = (0, -9, -18)$ $\qquad \vec{u} = (0, 1, 2) \parallel [\overrightarrow{BC}]$

$[\overrightarrow{AC}] = (18, 0, -18)$ $\qquad \vec{v} = (1, 0, -1) \parallel [\overrightarrow{AC}]$

$\alpha : 0 \cdot (x + 18) + 1 \cdot (y - 0) + 2(z - 0) = 0$

$$\boxed{\alpha : y + 2z = 0}$$

$\beta : 1 \cdot (x - 0) + 0 \cdot (y - 9) + (-1) \cdot (z - 0) = 0$

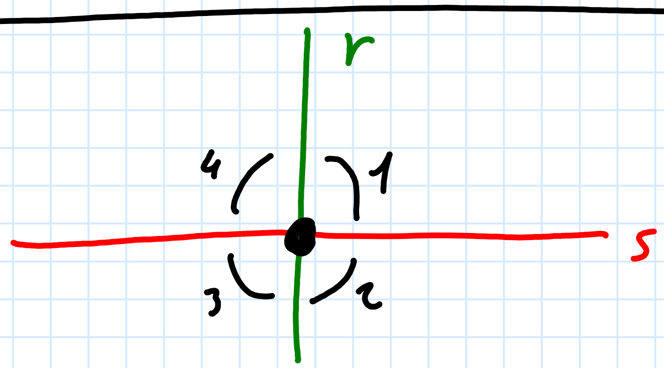
$$\boxed{\beta : x - z = 0}$$

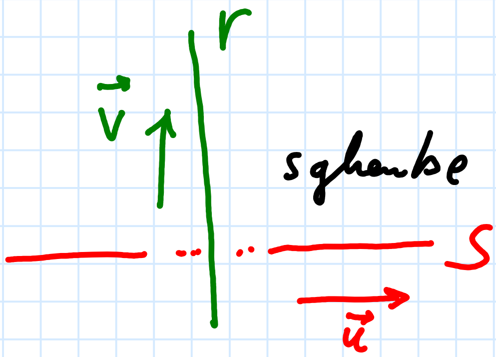
$r : \begin{cases} x - 2y + z + 18 = 0 \\ y + 2z = 0 \end{cases}$ $\qquad s : \begin{cases} x - 2y + z + 18 = 0 \\ x - z = 0 \end{cases}$

$$r \cap s : \begin{cases} x-2y+z+18=0 \\ y+2z=0 \\ x-z=0 \end{cases} \qquad \begin{cases} x=z \\ y=-2z \\ z+4z+z+18=0 \end{cases}$$

$$\begin{cases} z=-3 \\ x=-3 \\ y=+6 \end{cases} \qquad H(-3,\ 6,\ -3)$$

ortocentro





$\vec{v} \perp \vec{u}$

$\vec{v}=(\ell, m, n)$

$\vec{u}=(\ell', m', n')$ $\qquad r \perp s \iff \ell\ell' + m m' + n \cdot n' = 0$

$$A = \begin{bmatrix} (t^2-1) & 0 & 1 \\ 1 & (t+1) & 1 \\ 0 & 0 & (t-1) \end{bmatrix}$$

$t \in \mathbb{R}$

A deve avere <u>ALMENO</u> due autovalori uguali

$$p_A(\lambda) = \det \begin{bmatrix} [(t^2-1)-\lambda] & 0 & 1 \\ 1 & [(t+1)-\lambda] & 1 \\ 0 & 0 & [(t-1)-\lambda] \end{bmatrix} =$$

$$= [(t-1)-\lambda] \cdot [(t^2-1)-\lambda] \cdot [(t+1)-\lambda]$$

$\lambda_1 = t-1 \qquad \lambda_2 = t^2-1 \qquad \lambda_3 = t+1$

$\lambda_1 = \lambda_2 \Rightarrow t-1 = t^2-1 \Rightarrow t^2-t=0 \Rightarrow t\cdot(t-1)=0$

$t_1 = 0 \qquad t_2 = 1$

$\lambda_1 = \lambda_3 \Rightarrow t-1 = t+1 \Rightarrow -1 = +1$ ↯ impossibile

$\lambda_2 = \lambda_3 \Rightarrow t^2-1 = t+1 \Rightarrow t^2-t-2=0 \Rightarrow$

$\Rightarrow (t-2)(t+1)=0$

$t_3 = +2 \qquad t_4 = -1$

$$\boxed{t \in \{-1, 0, +1, +2\}}$$