Ricevimento studenti - venerdì 26 gennaio 2024

Titolo nota 26/01/2024

Buongiorno e benvenuti al ricevimento studenti.
Io sto lavorando al computer.
Fatevi SENTIRE (ho le cuffie) così accendo la telecamera e il microfono

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A, B, C $\quad \pi : x - 2y + z + 18 = 0$

$\downarrow \ \downarrow \ \downarrow$

$x \quad y \quad z$

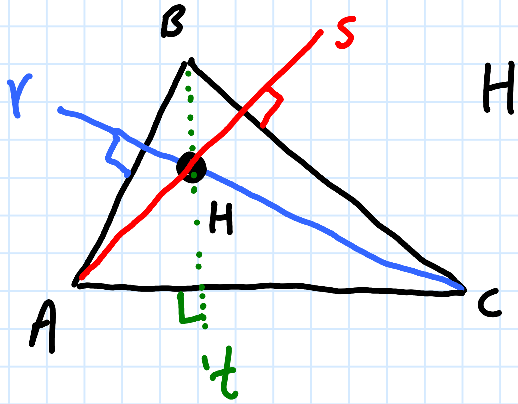
asse X : $y = z = 0 \longrightarrow x + 18 = 0 \rightarrow x = -18$

$A(-18, 0, 0)$

asse Y : $x = z = 0 \longrightarrow -2y + 18 = 0 \rightarrow y = 9 \rightarrow B(0, 9, 0)$

asse z : $x = y = 0 \longrightarrow z + 18 = 0 \longrightarrow z = -18 \rightarrow C(0, 0, -18)$

$A(-18, 0, 0)$; $B(0, 9, 0)$; $C(0, 0, -18)$



H ortocentro

$\{H\} = r \cap s$

$r = \pi \cap \alpha \qquad \alpha = ?$

$s = \pi \cap \beta \qquad \beta = ?$

$\alpha = ?$ α è il piano passante per il punto C e ortogonale al vettore $[\vec{AB}]$

$A(-18,0,0)$; $B(0,9,0)$; $[\overrightarrow{AB}] = (\underset{a}{18}, \underset{b}{9}, \underset{c}{0})$

$C(0,0,-18)$

$\alpha : 18(x-0) + 9(y-0) + 0(z+18) = 0$

$$\boxed{\alpha : 2x + y = 0}$$

$\beta = ?$ β è il piano passante per A e ortogonale a $[\overrightarrow{BC}]$

$B(0,9,0)$; $C(0,0,-18)$; $\longrightarrow$ $[\overrightarrow{BC}] = (\underset{a}{0}, \underset{b}{-9}, \underset{c}{-18})$

$A(-18,0,0)$

$\beta : 0 \cdot (x+18) - 9(y-0) - 18(z-0) = 0$

$$\boxed{\beta : y + 2z = 0}$$

$r = \pi \wedge \alpha$ $\qquad s = \pi \wedge \beta$ $\qquad r \wedge s : \begin{cases} x - 2y + z + 18 = 0 \\ 2x + y = 0 \\ y + 2z = 0 \end{cases}$

$\cdots\cdots\cdots> \quad H(\quad , \quad , \quad)$

$$A = \begin{bmatrix} (t^2-1) & 0 & 1 \\ 1 & (t+1) & 1 \\ 0 & 0 & (t-1) \end{bmatrix}$$

$t \in \mathbb{R}$

un autovalore con

$m_a \geq 2$

(<u>NON</u> ha 3 autovalori <u>distinti</u>)

$$p_A(\lambda) = \det \begin{bmatrix} [(t^2-1)-\lambda] & 0 & 1 \\ 1 & [(t+1)-\lambda] & 1 \\ 0 & 0 & [(t-1)-\lambda] \end{bmatrix} =$$

$$= \Big[\underbrace{(t-1)-\lambda}_{\lambda_1 = t-1}\Big]\cdot\Big[\underbrace{(t^2-1)-\lambda}_{\lambda_2 = t^2-1}\Big]\cdot\Big[\underbrace{(t+1)+\lambda}_{\lambda_3 = t+1}\Big];$$

$\lambda_1 = \lambda_2 \rightarrow t-1 = t^2-1 \rightarrow t\cdot(t-1)=0 \rightarrow t=0$ vel $t=1$

$\lambda_1 = \lambda_3 \rightarrow t-1 = t+1 \rightarrow -1=+1$ impossibile

$\lambda_2 = \lambda_3 \rightarrow t^2-1 = t+1 \rightarrow t^2-t-2=0 \rightarrow (t-2)(t+1)=0 \rightarrow$

$\rightarrow t=2$ vel $t=-1$

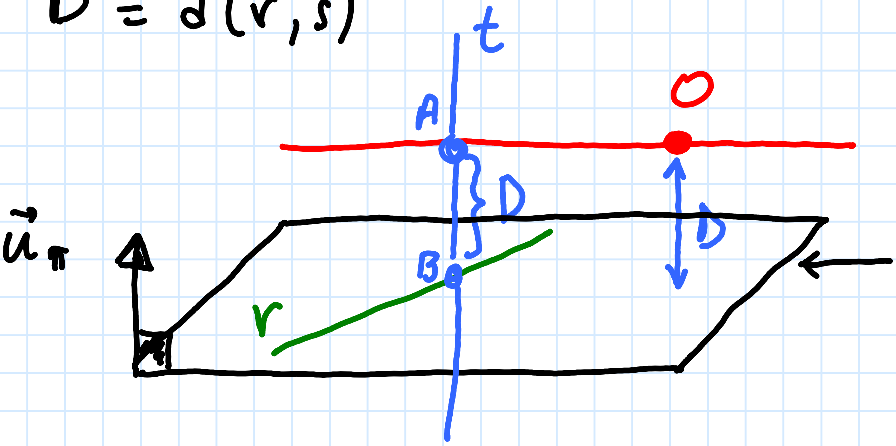
$$t \in \{-1, 0, 1, 2\}$$

$r: x+3y-2z = y-2z = 0$

$s: 3x+5z = x+2y+2z = 0 \rightarrow O(0,0,0) \in s$

t retta minima distanza

$D = d(r,s)$



$\leftarrow \pi \in F(r)$ et $\pi // s$

$\pi \in F(r): \lambda(x+3y-2z) + \mu(y-2z) = 0$

$$\pi: \underbrace{\lambda}_{a}\cdot x + \underbrace{(3\lambda+\mu)}_{b}\cdot y + \underbrace{(-2\mu)}_{c}\cdot z - 2z\lambda = 0$$

$s: 3x+5z = x+2y+2z = 0$ $\begin{bmatrix} 3 & 0 & 5 \\ 1 & 2 & 2 \end{bmatrix}$ $\rightarrow \ell = -10$, $\rightarrow m = -1$, $\rightarrow n = +6$

$\pi // s \iff al + bm + cn = 0 \iff -10\lambda - (3\lambda + \mu) - 12\mu = 0$

$-13\lambda - 13\mu = 0 \qquad \lambda + \mu = 0$

scelgo $\lambda = 1$ e ottengo $\mu = -1$

$$\boxed{\pi : x + 2y + 2z - 27 = 0}$$

$\vec{u}_\pi = (1,2,2) \perp \pi \; ; \quad \vec{u}_\pi // t$

$$D = d(r,s) = d(0,\pi) = \frac{|-27|}{\underbrace{\sqrt{1^2+2^2+2^2}}_{\|\vec{u}_\pi\|}} = \frac{27}{3} = 9$$

$\vec{v}$ $\Big\}\, 9$ $\qquad \vec{u}_\pi$ $\}\, 3$ $\qquad \vec{v} = 3\vec{u}_\pi$ $\qquad \boxed{\vec{v} = (3,6,6)}$

$$A = \begin{bmatrix} (t^3-8) & 3 & 1 \\ 0 & (t^2-2t) & 0 \\ 0 & (t-2) & (t^2-4) \end{bmatrix}$$

$$p_A(\lambda) = \det \begin{bmatrix} [(t^3-8)-\lambda] & 3 & 1 \\ 0 & [(t^2-2t)-\lambda] & 0 \\ 0 & (t-2) & [(t^2-4)-\lambda] \end{bmatrix} =$$

$$= [\underbrace{(t^2-2t)}_{\lambda_1 = t^2-2t} - \lambda] \cdot [\underbrace{(t^3-8)}_{\lambda_2 = t^3-8} - \lambda] \cdot [\underbrace{(t^2-4)}_{\lambda_3 = t^2-4} - \lambda];$$

$$\lambda_1 = \lambda_2 = \lambda_3 = 0 \rightarrow \begin{cases} t^2-2t=0 \\ t^3-8=0 \\ t^2-4=0 \end{cases} \rightarrow \begin{cases} t\cdot(t-2)=0 \\ (t-2)(t^2+4t+4)=0 \\ (t-2)(t+2)=0 \end{cases}$$

la soluzione del sistema è $\boxed{t=2}$

$\alpha: \quad tx + t^2y + tz = 0$

$\beta: \quad x - 6y + t = 0$

$\gamma: \quad z + 6 = 0$

$$C = \begin{bmatrix} t & t^2 & t & 0 \\ 1 & -6 & 0 & t \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

$\text{rg}\, C = 2$

$$B = \begin{bmatrix} -6 & 0 \\ 0 & 1 \end{bmatrix}; \quad \det B = -6 \neq 0$$

$\text{rg}\, B = 2$

$$\det \begin{bmatrix} t & t^2 & t \\ 1 & -6 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 0 \rightarrow \boxed{1\cdot(-6t-t^2)=0}$$

$$\det \begin{bmatrix} t^2 & t & 0 \\ -6 & 0 & t \\ 0 & 1 & 6 \end{bmatrix} = 0 \rightarrow \boxed{-t^3+36t=0}$$

$$\begin{cases} t\cdot(t+6)=0 \\ t\,\underbrace{(36-t^2)}_{(6-t)(6+t)}=0 \end{cases} \qquad \begin{cases} t=0 \quad \text{vel} \quad t=-6 \\ t=0 \quad \text{vel} \quad t=6 \quad \text{vel} \quad t=-6 \end{cases}$$

$$\boxed{t \in \{\cancel{0}, -6\}}$$

si noti che per $t=0$ α non è un piano

$r: 3x-4y-5=0$ direttrice parabole passanti per l'origine $O(0,0)$ e aventi i fuochi sull'asse X. Trovare le coordinate dei fuochi.

$$F(\alpha, 0) \qquad \alpha = ?$$

$O \in$ parabola $\qquad \underbrace{d(O,F)}_{} = d(O, \text{direttrice})$

$$\sqrt{(0-\alpha)^2+(0-0)^2} = \frac{|3\cdot 0-4\cdot 0-5|}{\sqrt{3^2+(-4)^2}}$$

$$\alpha^2 = \frac{25}{25} \; ; \quad \alpha^2 = 1 \; ; \quad \alpha = \pm 1$$

$F_1(1,0) \qquad F_2(-1,0)$