Ricevimento studenti - venerdì 2 febbraio 2024

Titolo nota 02/02/2024

Buongiorno e benvenuti al ricevimento studenti.

$r \parallel$ asse X $\quad A(1, 9, -3\sqrt{3}) \in r$

$\alpha \in F(r)$: $\quad \widehat{\alpha, \text{piano } XY} = \frac{\pi}{6}$ radianti

asse $X \longrightarrow (\ell, m, n) = (1, 0, 0)$

$r \parallel$ asse $X \longrightarrow (\ell, m, n) = (1, 0, 0)$

$$r: \begin{cases} x = 1 \cdot t + 1 \\ y = 0 \cdot t + 9 \\ z = 0 \cdot t - 3\sqrt{3} \end{cases}$$

$$r: \begin{cases} x = t + 1 \\ y - 9 = 0 \\ z + 3\sqrt{3} = 0 \end{cases} \qquad r: \begin{cases} y - 9 = 0 \\ z + 3\sqrt{3} = 0 \end{cases}$$

$\alpha \in F(r)$: $\lambda \cdot (y - 9) + \mu \cdot (z + 3\sqrt{3}) = 0$

$$\alpha : \underbrace{0}_{a} \cdot x + \underbrace{\lambda}_{b} \cdot y + \underbrace{\mu}_{c} \cdot z + \underbrace{(-9\lambda + 3\sqrt{3}\mu)}_{d} = 0$$

piano XY : $z = 0 \qquad \underbrace{0}_{a'} \cdot x + \underbrace{0}_{b'} \cdot y + \underbrace{1}_{c'} \cdot z + \underbrace{0}_{d'} = 0$

$\theta = \frac{\pi}{6}$ radianti $= 30^\circ \longrightarrow \cos\theta = \frac{\sqrt{3}}{2}$

$$\cos\theta = \pm \frac{a \cdot a' + b \cdot b' + c \cdot c'}{\sqrt{a^2 + b^2 + c^2} \cdot \sqrt{(a')^2 + (b')^2 + (c')^2}}$$

$$\frac{\sqrt{3}}{2} = \pm \frac{\mu}{\sqrt{0^2+\lambda^2+\mu^2}\cdot\sqrt{0^2+0^2+1^2}}$$

$$\sqrt{3}\cdot\sqrt{\lambda^2+\mu^2} = \pm 2\mu \longrightarrow 3\cdot(\lambda^2+\mu^2) = 4\mu^2$$

$\mu^2 = 3\lambda^2$; scelgo $\lambda = 1 \longrightarrow \mu = \pm\sqrt{3}$

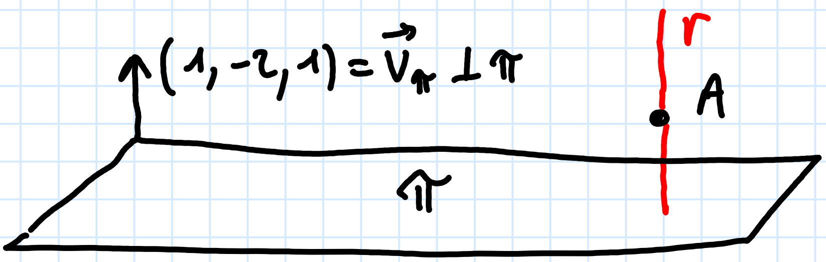
2 soluzioni $(\lambda,\mu) = (1,\sqrt{3})$ e $(\lambda,\mu) = (1,-\sqrt{3})$

$$\lambda\cdot(y-9) + \mu\cdot(z+3\sqrt{3}) = 0$$

1. $y - 9 + \sqrt{3}z + 9 = 0 \longrightarrow y + \sqrt{3}z = 0$
2. $y - 9 - \sqrt{3}z - 9 = 0 \longrightarrow y - \sqrt{3}z - 18 = 0$

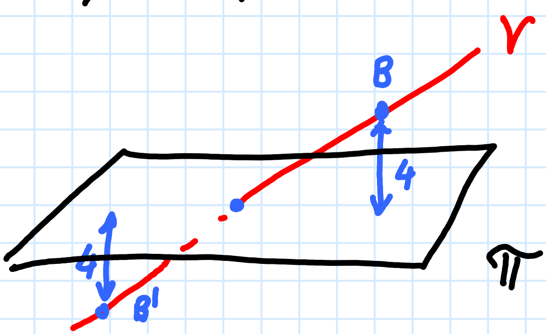
$A(3,0,-2)$; $\pi: x - 2y + z = 0$; $A \in r$ et $r \perp \pi$

punti di r a distanza 4 da $\pi': x - 4 = 0$.



$$r: \begin{cases} x = 1\cdot t + 3 \\ y = -2\cdot t + 0 \\ z = 1\cdot t - 2 \end{cases}$$

$B(t+3, -2t, t-2) \in r \quad \forall t \in \mathbb{R}$



$d(B,\pi') = 4$

$\pi': x - 4 = 0$

$$\frac{|(t+3)-4|}{\sqrt{1^2+0^2+0^2}} = 4 \rightarrow |t-1| = 4$$

$$t-1 = \pm 4 \rightarrow t = 1 \pm 4 \begin{cases} t_1 = 5 \\ t_2 = -3 \end{cases}$$

$B(8,-10,3)$ $B'(0,+6,-5)$

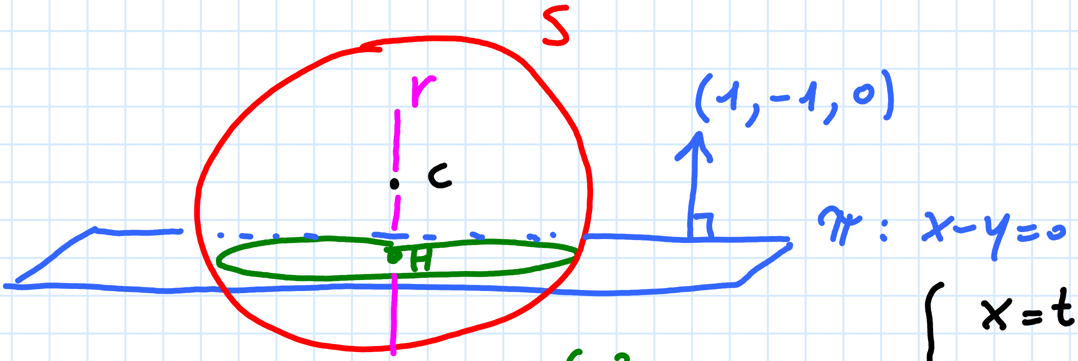
$x^2+y^2+z^2+4y+2\sqrt{2}z = x-y=0$ circonferenza

Trovare il suo centro e il suo raggio

Sfera $x^2+y^2+z^2+4y+2\sqrt{2}z=0$

centro sfera $C(0,-2,-\sqrt{2})$

raggio sfera $R = \frac{1}{2}\sqrt{0^2+4^2+8-0} = \sqrt{6}$



$$r: \begin{cases} x = 1\cdot t + 0 \\ y = -1\cdot t - 2 \\ z = 0\cdot t - \sqrt{2} \end{cases}$$

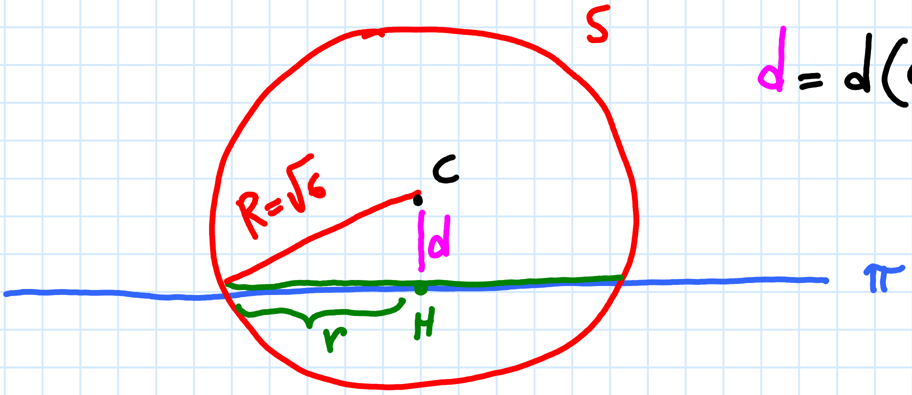
$$r: \begin{cases} x = t \\ y = -t-2 \\ z = -\sqrt{2} \end{cases}$$

$$\{H\} = r \cap \pi : \begin{cases} x = t \\ y = -t-2 \\ z = -\sqrt{2} \\ x-y=0 \end{cases} \rightarrow$$

$\rightarrow t-(-t-2)=0 \rightarrow 2t+2=0 \Rightarrow$

$\rightarrow t+1=0 \rightarrow t=-1$

$H(-1,-1,-\sqrt{2})$



$d = d(C,H) = \dots$

$r = \sqrt{R^2 - d^2}$

$C(0,-2,-\sqrt{2})$; $H(-1,-1,-\sqrt{2})$

$d(C,H) = \sqrt{1+1} = \sqrt{2}$

$r = \sqrt{6-2} = \sqrt{4} = 2$

$r = 2$

$r \parallel$ asse y ; $A(-\sqrt{3},2,3) \in r$;

$\alpha \in F(r)$ $\qquad \widehat{\alpha, \text{piano } yz} = \frac{\pi}{6}$ radianti

asse $y \longrightarrow (l,m,n) = (0,1,0)$

$r \parallel$ asse $y \longrightarrow (l,m,n) = (0,1,0)$

$$r: \begin{cases} x = 0 \cdot t - \sqrt{3} \\ y = 1 \cdot t + 2 \\ z = 0 \cdot t + 3 \end{cases}$$

$$r: \begin{cases} x + \sqrt{3} = 0 \\ y = t + 2 \\ z - 3 = 0 \end{cases} \qquad r: \begin{cases} x + \sqrt{3} = 0 \\ z - 3 = 0 \end{cases}$$

$$F(r): \ \lambda \cdot (x + \sqrt{3}) + \mu \cdot (z-3) = 0$$

$$\underbrace{\lambda}_{a} \cdot x + \underbrace{0}_{b} \cdot y + \underbrace{\mu}_{c} \cdot z + (\sqrt{3}\lambda - 3\mu) = 0$$

piano yz : $x = 0$ $\qquad \underbrace{1}_{a'} \cdot x + \underbrace{0}_{b'} \cdot y + \underbrace{0}_{c'} \cdot z = 0$

$$\theta = \frac{\pi}{6} \text{ radianti} = 30° \longrightarrow \cos\theta = \frac{\sqrt{3}}{2}$$

$$\frac{\sqrt{3}}{2} = \pm \frac{\lambda}{\sqrt{\lambda^2+\mu^2} \cdot \sqrt{1}}$$

$$\sqrt{3} \cdot \sqrt{\lambda^2+\mu^2} = \pm 2\lambda \longrightarrow 3 \cdot (\lambda^2+\mu^2) = 4\lambda^2 \longrightarrow$$

$\boxed{\lambda^2 = 3 \cdot \mu^2}$ scelgo $\mu = 1 \longrightarrow \lambda^2 = 3 \longrightarrow \lambda = \pm\sqrt{3}$

$(\mu, \lambda) = (1, \sqrt{3})$ 1 e $(\mu, \lambda) = (1, -\sqrt{3})$ 2

1 $\sqrt{3} \cdot (x + \sqrt{3}) + 1 \cdot (z - 3) = 0 \longrightarrow \sqrt{3}x \cancel{+3} + z \cancel{-3} = 0$

$$\sqrt{3}x + z = 0$$

2 $-\sqrt{3} \cdot (x + \sqrt{3}) + 1(z - 3) = 0 \longrightarrow -\sqrt{3}x - 3 + z - 3 = 0$

$$-\sqrt{3} \cdot x + z - 6 = 0$$

$$\sqrt{3}x - z + 6 = 0$$

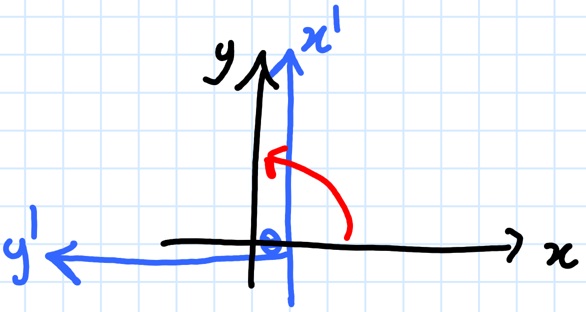
$$-x^2 + 15y^2 - 15 = 0$$

$$x^2 - 15y^2 + 15 = 0$$

$$x^2 - 15y^2 = -15$$

$$\frac{x^2}{15} - y^2 = -1 \quad ; \quad y^2 - \frac{x^2}{15} = +1$$

rotazione di 90° gradi degli assi (orario)



$$\begin{cases} y = x' \\ x = -y' \end{cases}$$

$$(x')^2 - \frac{(-y')^2}{15} = +1$$

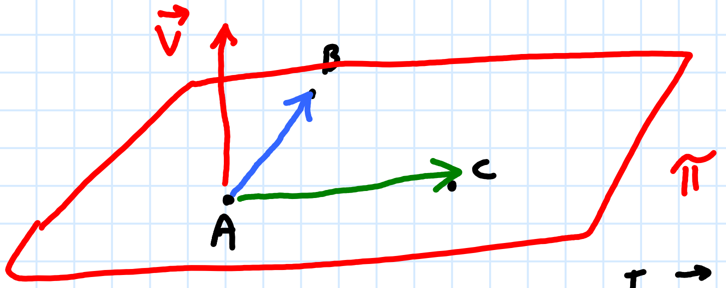
$$(x')^2 - \frac{(y')^2}{15} = +1$$

IPERBOLE

$A(-1,0,0)$; $B(0,t,0)$; $C(0,0,\sqrt{11})$;

π piano passante per A, B e C

$\pi' =$ piano XZ $\qquad \widehat{\pi,\pi'} = \frac{2}{3}\pi$ radianti



$[\vec{AB}] = (1, t, 0)$

$[\vec{AC}] = (1, 0, \sqrt{11})$

$$\vec{V} = [\vec{AB}] \wedge [\vec{AC}] = \det \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & t & 0 \\ 1 & 0 & \sqrt{11} \end{bmatrix} = (t\sqrt{11})\,\vec{i} + (-\sqrt{11})\,\vec{j} + (-t)\,\vec{k}$$

$$\vec{V} = \underbrace{(t\sqrt{11})}_{a}\,\vec{i} + \underbrace{(-\sqrt{11})}_{b}\,\vec{j} + \underbrace{(-t)}_{c}\,\vec{k} \leftarrow \text{piano } \pi$$

$\pi' =$ piano XZ : $y = 0$ $\qquad \underbrace{0}_{a'} \cdot x + \underbrace{1}_{b'} \cdot y + \underbrace{0}_{c'} \cdot z = 0$

$$\theta = \frac{2}{3}\pi \text{ radianti} \longrightarrow \theta = 120° \longrightarrow \cos\theta = -\frac{1}{2}$$

$$-\frac{1}{2} = \pm \frac{-\sqrt{11}}{\sqrt{11t^2+11+t^2} \cdot \sqrt{0^2+1^2+0^2}}$$

$$-\sqrt{12t^2+11} = \mp 2\sqrt{11} \longrightarrow 12t^2+11 = 44 \rightarrow 12t^2 = 33$$

$$4t^2 = 11 \rightarrow t^2 = \frac{11}{4} \longrightarrow t = \pm\frac{\sqrt{11}}{2}$$

$$t_1 = \frac{\sqrt{11}}{2}$$

$$t_2 = -\frac{\sqrt{11}}{2}$$