Ricevimento studenti giovedì 9 gennaio 2025

Titolo nota 09/01/2025

$$A = \begin{bmatrix} -6 & 0 & 0 \\ t+6 & 1 & 0 \\ -1 & -t & -6 \end{bmatrix}; \quad t \in \mathbb{R}, \quad A \text{ diagonalizzabile}$$

$$p_A(\lambda) = \det \begin{bmatrix} (-6-\lambda) & 0 & 0 \\ t+6 & (1-\lambda) & 0 \\ -1 & -t & (-6-\lambda) \end{bmatrix} = (6+\lambda)^2 \cdot (1-\lambda)$$

$\lambda_1 = -6$ $\qquad$ $\lambda_2 = 1$

$$m_a(\lambda_2) = 1 \implies m_g(\lambda_2) = 1$$

$$m_a(\lambda_1) = 2 \implies 1 \leq m_g(\lambda_1) \leq 2$$

$$A \text{ è diagonalizzabile} \iff m_g(-6) = 2$$

impongo

$$m_g(-6) \overset{\text{DEF}}{=\!=} \dim E(-6) = \color{red}{3} - rg(A - (-6) I_3) \overset{!}{=} 2$$

$$rg(A - (-6) \cdot I_3) = 1$$

$$rg \begin{bmatrix} 0 & 0 & 0 \\ (t+6) & 7 & 0 \\ -1 & -t & 0 \end{bmatrix} = 1 \iff rg \begin{bmatrix} (t+6) & 7 \\ -1 & -t \end{bmatrix} = 1 \iff$$

$$\iff \det \begin{bmatrix} (t+6) & 7 \\ -1 & -t \end{bmatrix} = 0 \iff (t+6) \cdot (-t) + 7 = 0 \iff$$

$$\iff -t^2 - 6t + 7 = 0 \iff t^2 + 6t - 7 = 0 \iff (t+7)(t-1) = 0$$

$$\boxed{t=-7;\ t=+1;}$$ RISULTATO

$$4x^2-2\sqrt{3}xy+2y^2-8x+2\sqrt{3}y-1=0$$

$$A=\begin{bmatrix}4 & -\sqrt{3}\\ -\sqrt{3} & 2\end{bmatrix};\quad p_A(\lambda)=\ldots=\lambda^2-6\lambda+5=(\lambda-1)(\lambda-5)$$

$\lambda_1=1$ $\quad$ $\lambda_2=5$

$\boxed{\lambda_1=1}$ $\Rightarrow$ autovettori

$$\begin{bmatrix}3 & -\sqrt{3}\\ -\sqrt{3} & 1\end{bmatrix}\cdot\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}\qquad \begin{cases}\text{———}\\ -\sqrt{3}x+y=0\end{cases}\qquad y=\sqrt{3}x$$

$$(x,\sqrt{3}x)=x(1,\sqrt{3})\quad \forall x\neq 0\qquad \infty \text{ autovettori}$$

scelgo $x=\frac{1}{2}$ $\qquad$ autoVERSORE $\left(\frac{1}{2},\frac{\sqrt{3}}{2}\right)$

$$C=\begin{bmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}$$

autoVERSORE $\left(-\frac{\sqrt{3}}{2},\frac{1}{2}\right)$

$\det C=+1$ (rotazione)

$$\begin{bmatrix}-8 & 2\sqrt{3}\end{bmatrix}\cdot\begin{bmatrix}\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & \frac{1}{2}\end{bmatrix}=\begin{bmatrix}-1 & 5\sqrt{3}\end{bmatrix}$$

dopo la rotazione l'equazione è

$$1\cdot(x')^2+5\cdot(y')^2-1x'+5\sqrt{3}y'-1=0$$

$$(x')^2 - x' + 5(y')^2 + 5\sqrt{3}\,y' - 1 = 0$$

$$\left(x'-\frac{1}{2}\right)^2 - \frac{1}{4} + 5\cdot\left(y'+\frac{\sqrt{3}}{2}\right)^2 - \frac{15}{4} - 1 = 0$$

$$\underbrace{\left(x'-\frac{1}{2}\right)}_{x''}{}^2 + 5\cdot\underbrace{\left(y'+\frac{\sqrt{3}}{2}\right)}_{y''}{}^2 = 5$$

$$(x'')^2 + 5(y'')^2 = 5$$

$$\boxed{\frac{(x'')^2}{5} + \frac{(y'')^2}{1} = +1}$$

eq. canonica

ELLISSE

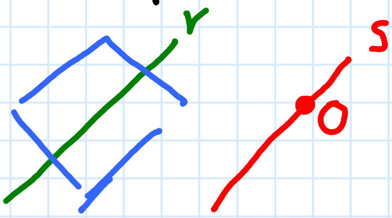
$$r: \begin{cases} 3x+4y+2z+4=0 \\ x+2=0 \end{cases} \qquad s: \begin{cases} 2y+z=0 \\ x+2y+z=0 \end{cases} \qquad O(0,0,0)\in s$$

$$\begin{bmatrix} 3 & 4 & 2 \\ 1 & 0 & 0 \end{bmatrix} \to l=0,\ m=2,\ n=-4 \qquad \begin{bmatrix} 0 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix} \to l=0,\ m=1,\ n=-2$$

$$\vec{u}_r = (0,2,-4) \qquad \vec{u}_s = (0,1,-2)$$

$$\vec{u}_r = 2\cdot\vec{u}_s \Rightarrow \vec{u}_r \parallel \vec{u}_s \Rightarrow r \parallel s \Rightarrow \text{complanari}$$

Trovare il piano che le contiene



$O\in s,\ O\notin r$

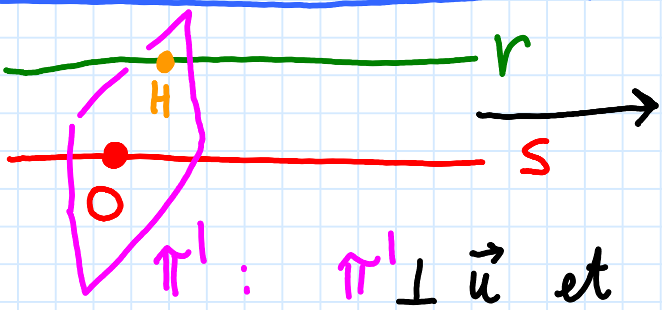
$\pi \in F(r)$ et $O \in \pi$

$$\pi : \lambda(3x+4y+2z+4) + \mu(x+2) = 0$$

$$(3\lambda+\mu)\cdot x + (4\lambda)\cdot y + (2\lambda)\cdot z + (4\lambda+2\mu) = 0$$

$$O \in \pi \Rightarrow 4\lambda + 2\mu = 0 \Rightarrow 2\lambda + \mu = 0 \rightarrow \text{scelgo} \begin{cases} \lambda = 1 \\ \mu = -2 \end{cases}$$

$\boxed{\pi : x + 4y + 2z = 0}$ piano che le contiene



$\vec{u} = (0, 1, -2)$

$\pi' :$ $\pi' \perp \vec{u}$ et $O \in \pi'$

$$\{H\} = \pi' \cap r$$

$$d(r, s) = d(H, O)$$

$$\pi' : y - 2z = 0$$

$$\pi' \cap r : \begin{cases} y - 2z = 0 \\ 3x + 4y + 2z + 4 = 0 \\ x + 2 = 0 \end{cases} \qquad \begin{cases} y = 2z \\ x = -2 \\ -6 + 8z + 2z + 4 = 0 \end{cases}$$

$$\begin{cases} y = 2z \\ x = -2 \\ z = \frac{1}{5} \end{cases} \qquad \begin{cases} x = -2 \\ y = \frac{2}{5} \\ z = \frac{1}{5} \end{cases}$$

$H\left(-2, \frac{2}{5}, \frac{1}{5}\right)$

$O(0, 0, 0)$

$$d(H, O) = \sqrt{(-2)^2 + \left(\frac{2}{5}\right)^2 + \left(\frac{1}{5}\right)^2} = \ldots\ldots$$

$r: 2x-3z+2=y=0$ $\qquad \theta=\frac{\pi}{6}$ rad

$\alpha: \ x-z=0$

$\beta \in F(r): \ \lambda(2x-3z+2)+\mu\cdot y=0$

$$\underbrace{(2\cdot\lambda)}_{a}\cdot x+\underbrace{\mu}_{b}\cdot y+\underbrace{(-3\lambda)}_{c}\cdot z+2\lambda=0$$

$$\alpha: \ \underbrace{1}_{a'}\cdot x+\underbrace{0}_{b'}\cdot y+\underbrace{(-1)}_{c'}z=0$$

$$\cos\theta=\pm\frac{a\cdot a'+b\cdot b'+c\cdot c'}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{(a')^2+(b')^2+(c')^2}}$$

$\theta=\frac{\pi}{6}$ rad $=30°$

$$\frac{\sqrt{3}}{2}=\pm\frac{5\lambda}{\sqrt{13\lambda^2+\mu^2}\cdot\sqrt{2}}$$

$$6\cdot(13\lambda^2+\mu^2)=100\lambda^2$$

$78\lambda^2+6\mu^2=100\lambda^2$

$22\lambda^2=6\mu^2 \qquad 11\lambda^2=3\mu^2 \qquad \mu^2=\frac{11}{3}\lambda^2$

scelgo $\lambda=1 \qquad \mu=\pm\sqrt{\frac{11}{3}}$

$(2\lambda)\cdot x+\mu\cdot y+(-3\lambda)\cdot z+2\lambda=0$

$2\cdot x\pm\sqrt{\frac{11}{3}}\cdot y-3z+2=0$ $\quad$ i 2 piani richiesti

$$A = \begin{bmatrix} 3t & 2t^2 \\ 6t & -t \end{bmatrix} ; \quad p_A(\lambda) = \dots = \lambda^2 - 2t\lambda + (-3t^2 - 12t^3)$$

$$p_A(\lambda) = \lambda^2 + (-2t)\cdot\lambda + (-3t^2 - 12t^3)$$

$$\Delta = (-t)^2 - (-3t^2 - 12t^3) = 12t^3 + 2t^2 = 2t^2 \cdot (6t+1)$$

$$\Delta > 0 \implies t \neq 0 \text{ et } (6t+1) > 0 \Rightarrow t \neq 0 \text{ et } t > -\frac{1}{6}$$

$$t = 0 \Rightarrow A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ matrice diagonale}$$

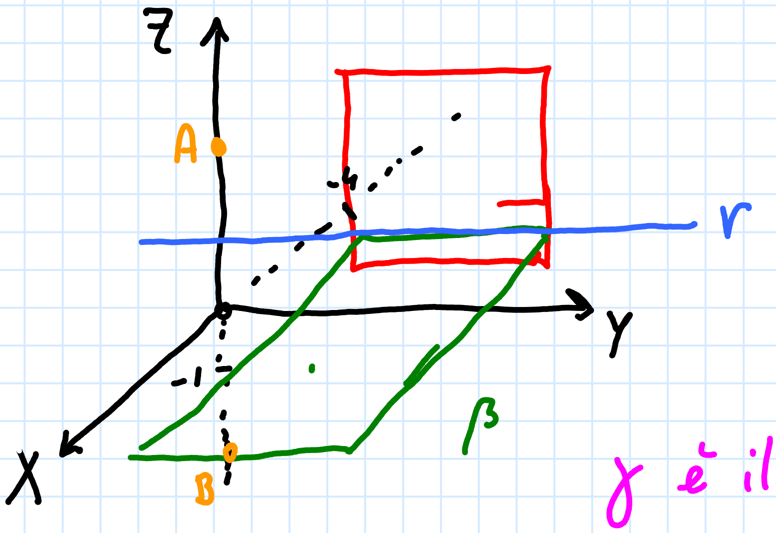
RISULTATO FINALE $\boxed{t > -\frac{1}{6}}$

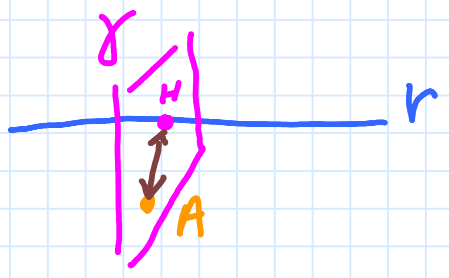
$r: \; x+4 = z+1 = 0$ $\qquad$ $A, B \in$ asse z

$d(A, r) = d(B, r) = 5$

$\alpha \quad x+4=0 \qquad x=-4$

$\beta \quad z+1=0 \qquad z=-1$





γ è il piano XZ : $y=0$

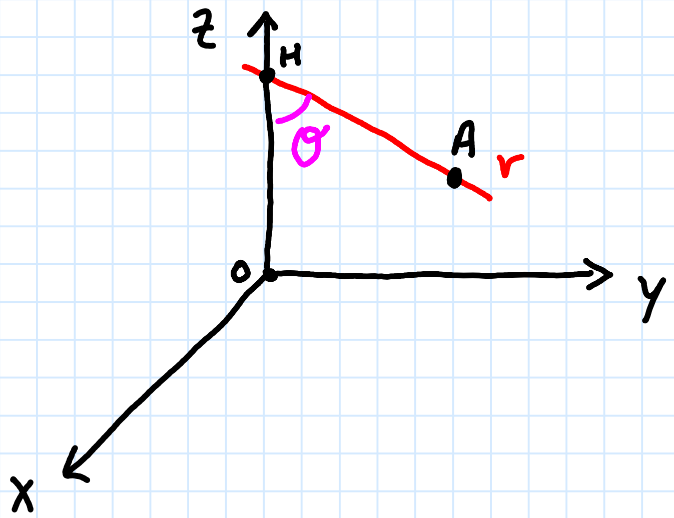
$H(-4,0,-1)$ $\quad$ $A(0,0,w)$

$d(A,H)=5$ $\qquad \sqrt{(-4)^2+0^2+(-1-w)^2}=5$

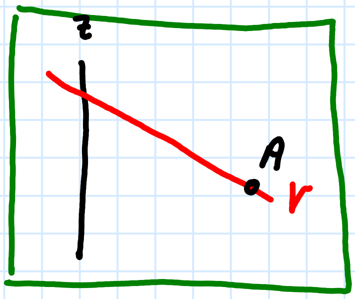
$16+1+w^2+2w=25$

$w^2+2w-8=0 \quad \begin{array}{l} \nearrow w_1= \qquad \longrightarrow A \\ \searrow w_2= \qquad \longrightarrow B \end{array}$

$A(1,2,1)$; asse z ; $\theta=\frac{\pi}{4}$ rad



$r=\alpha\cap\beta$



α

$\alpha\in F(\text{asse } z)$ et $A\in\alpha$

$\downarrow$

$x=y=0$

$\alpha:\ \lambda\cdot x+\mu\cdot y=0$

$A(1,2,1)\in\alpha \Rightarrow \lambda+2\mu=0 \Rightarrow \lambda=2$ et $\mu=-1$

$\boxed{\alpha:\ 2x-y=0} \longrightarrow (a,b,c)=(2,-1,0)$

$\vec{u}_r // r \qquad \vec{u}_r=(l,m,n)$

$r\subseteq\alpha \Rightarrow r//\alpha \Rightarrow al+bm+cn=0$

$$2l - m = 0 \Rightarrow m = 2l$$

$\vec{u}_r // r \qquad \vec{u}_r = (l, 2l, m)$

$\vec{k} //$ asse $z \qquad \vec{k} = (0, 0, 1)$

$$\widehat{\vec{u}_r, \vec{k}} = \theta = \frac{\pi}{4} \text{rad} = 45°$$

$$\cos\theta = \pm \frac{m}{\sqrt{5l^2 + m^2} \cdot 1} = \frac{\sqrt{2}}{2}$$

$$4m^2 = 2 \cdot (5l^2 + m^2) \ ; \quad 2m^2 = 10 l^2 ; \quad m^2 = 5l^2$$

scelgo $l = 1 \Rightarrow m = \pm\sqrt{5}$

$\vec{u}_r = (1, 2, \sqrt{5}) \qquad \vec{u}_s = (1, 2, -\sqrt{5})$

$A(1, 2, 1)$

$$r: \begin{cases} x = 1t + 1 \\ y = 2t + 2 \\ z = \sqrt{5}t + 1 \end{cases} \qquad s: \begin{cases} x = 1t + 1 \\ y = 2t + 2 \\ z = -\sqrt{5}t + 1 \end{cases}$$

$$\begin{cases} 3x - y + 14z - 10 = 0 \\ 1x - y + 12z - 4 = 0 \\ 2x + y - 9z - 5 = 0 \end{cases} \qquad C = \begin{bmatrix} 1 & -1 & 12 & -4 \\ 3 & -1 & 14 & -10 \\ 2 & 1 & -9 & -5 \end{bmatrix} \rightarrow$$

$$\begin{bmatrix} 1 & -1 & 12 & -4 \\ 0 & 2 & -22 & 2 \\ 0 & 3 & -33 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & -1 & 12 & -4 \\ 0 & 1 & -11 & 1 \\ 0 & \cancel{1} & \cancel{-11} & 1 \end{bmatrix}$$

$$\begin{cases} x - y + 12z - 4 = 0 \\ y - 11z + 1 = 0 \end{cases} \qquad \begin{cases} x - (11z - 1) + 12z - 4 = 0 \\ y = 11z - 1 \end{cases}$$

$$\begin{cases} x + z - 3 = 0 \\ y = 11z - 1 \end{cases} \qquad \begin{cases} x = 3 - z \\ y = 11z - 1 \end{cases}$$

$$\boxed{(x, y, z) = (3 - z,\ 11z - 1,\ z) \quad \forall z \in \mathbb{R}}$$

soluzione generale

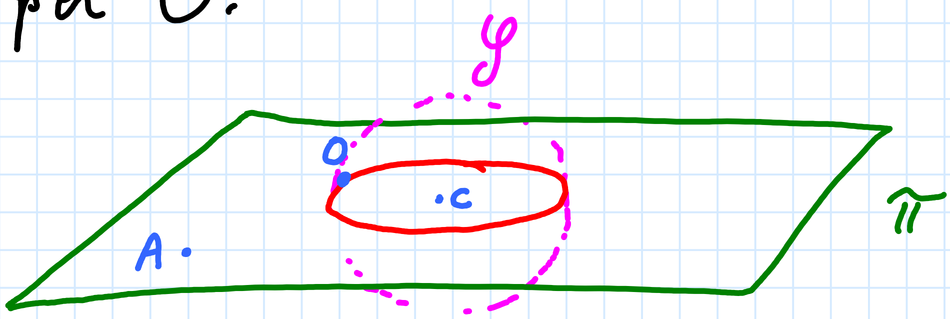
$$(x, y, z) = \underbrace{(3, -1, 0)}_{\text{soluzione PARTICOLARE}} + z(-1, 11, 1)$$

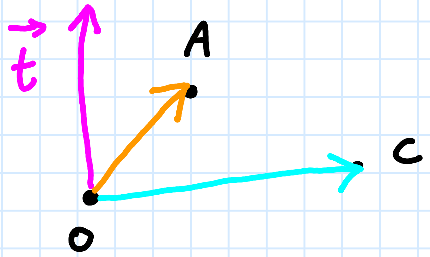
$B = (-1, 11, 1)$ base dello spazio delle soluzioni del sistema lineare omogeneo ASSOCIATO

$O(0,0,0)$, $A(0,2,-5)$, $C(3,0,0)$

π piano passante per O, A e C

circonferenza sul piano π, centro in C e passa per O.





$[\vec{OA}] = (0, 2, -5)$

$[\vec{OC}] = (3, 0, 0)$

$\vec{t} = [\vec{OA}] \wedge [\vec{OC}]$

$$\begin{bmatrix} 0 & 2 & -5 \\ 3 & 0 & 0 \end{bmatrix} \quad \begin{matrix} \nearrow = 0 \\ \rightarrow = -15 \\ \searrow = -6 \end{matrix}$$

$\vec{t} = (\underset{a}{0}, \underset{b}{5}, \underset{c}{2}) \perp \pi$ et $\underset{d=0}{O} \in \pi$

$$\boxed{\pi : 5y + 2z = 0}$$

φ di centro C e passante per O

$\varphi : x^2 + y^2 + z^2 + a'x + b'y + c'z + d' = 0$ $\hookrightarrow d' = 0$

$C\left(-\frac{a'}{2}, -\frac{b'}{2}, -\frac{c'}{2}\right) = (3, 0, 0)$

$-\frac{a'}{2} = 3 \quad ; \quad -\frac{b'}{2} = 0 \quad ; \quad -\frac{c'}{2} = 0$

$a' = -6 \quad ; \quad b' = 0 \quad ; \quad c' = 0$

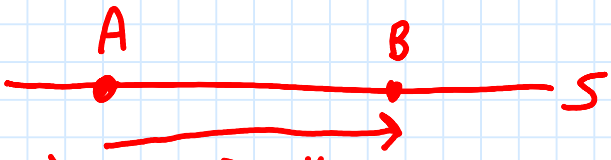
$\varphi : x^2 + y^2 + z^2 - 6x = 0$

$$\boxed{\mathcal{C} = \pi \wedge \varphi : \begin{cases} 5y + 2z = 0 \\ x^2 + y^2 + z^2 - 6x = 0 \end{cases}}$$

RISULTATO FINALE

$r: \ (t-1)x + 5y + (t^2-12)z = y+13 = 0$ $\qquad r // s$

$s: \ A(2-4t, 5, -9), \ B(2-3t, 5, -t-8)$



$[\overrightarrow{AB}] = (t, 0, 1-t)$

$\vec{u}_s = [\overrightarrow{AB}] // s$

$\vec{u}_r // r \qquad \begin{bmatrix} (t-1) & 5 & (t^2-12) \\ 0 & 1 & 0 \end{bmatrix} = \begin{cases} 12-t^2 \\ 0 \\ t-1 \end{cases}$

$$\vec{u}_r = (12-t^2, 0, t-1)$$

$$\vec{u}_s = (t, 0, 1-t)$$

$$r // s \iff \vec{u}_r // \vec{u}_s \iff rg \begin{bmatrix} (12-t^2) & 0 & (t-1) \\ t & 0 & (1-t) \end{bmatrix} = 1$$

$$\iff rg \begin{bmatrix} (12-t^2) & (t-1) \\ t & (1-t) \end{bmatrix} = 1 \iff \det \begin{bmatrix} (12-t^2) & (t-1) \\ t & (1-t) \end{bmatrix} = 0$$

$$(12-t^2)\cdot(1-t) - t(t-1) = 0$$

$$(12-t^2)\cdot(1-t) + t(1-t) = 0$$

$$(1-t)\cdot(12-t^2+t) = 0$$

$$(t-1)\cdot(t^2-t-12) = 0 \ ; \ (t-1)\cdot(t-4)\cdot(t+3) = 0$$

$$\boxed{t=-3 \ ; \ t=1 \ ; \ t=4}$$

RISULTATO FINALE