Ricevimento studenti - martedì 21 gennaio 2025

Titolo nota 21/01/2025

$A(9, \sqrt{5}, 23)$; $r: x+3y-7 = 2y+z+9 = 0$;

$\pi : 3x+2z+13=0 \longrightarrow (\tilde{a}, \tilde{b}, \tilde{c}) = (3, 0, 2)$

$\pi' : A \in \pi'$; $r // \pi'$; $\pi \perp \pi'$

$A \in \pi' \Rightarrow \pi' \in S(A) \Rightarrow \pi' : a(x-9)+b(y-\sqrt{5})+c(z-23)=0$

$r : \begin{bmatrix} 1 & 3 & 0 \\ 0 & 2 & 1 \end{bmatrix} \rightarrow \begin{cases} l = 3 \\ m = -1 \\ n = 2 \end{cases}$

$r // \pi' \Leftrightarrow al+bm+cn=0 \Leftrightarrow 3a-b+2c=0$

$b = 3a+2c$

$\pi' \perp \pi \Leftrightarrow a\tilde{a}+b\tilde{b}+c\tilde{c}=0 \Leftrightarrow 3a+0b+2c=0$

$3a+2c=0$ $\qquad (a,c)=(2,-3)$

$\Downarrow$

$b=0$

$a(x-9)+b(y-\sqrt{5})+c(z-23)=0$

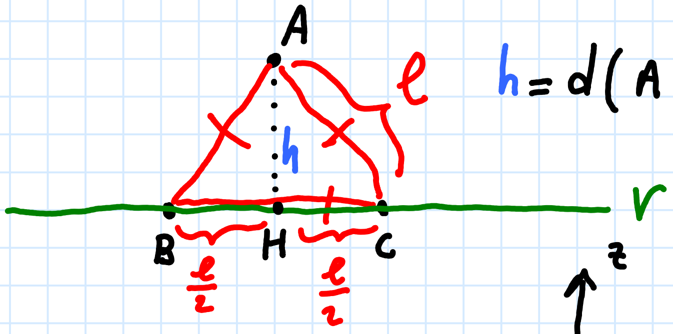
$2\cdot(x-9)+0\cdot(y-\sqrt{5})-3(z-23)=0$

$$\boxed{2\cdot x - 3z + 51 = 0}$$

$A(10, \sqrt{44}, 3)$; $r : y = x-8 = 0$; $B, C \in r$

tali che $\overset{\triangle}{ABC}$ EQUILATERO

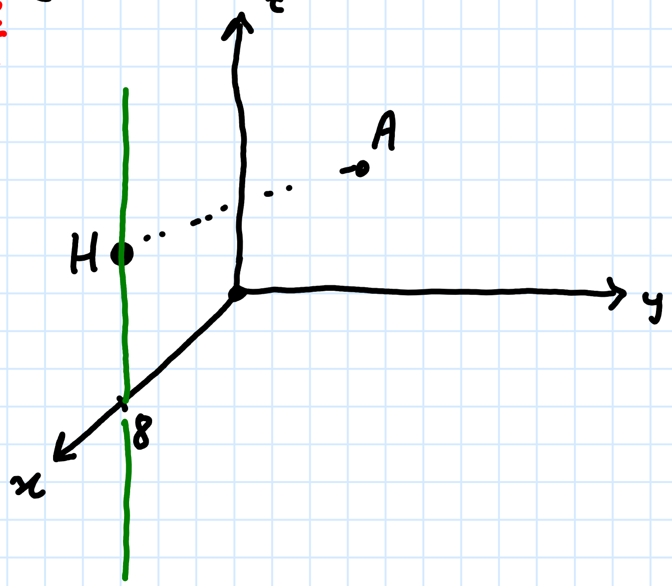
$A \notin r$



$h = d(A, r) = d(A, H)$

$l = h \cdot \dfrac{2}{\sqrt{3}}$

$r: \begin{cases} y = 0 \\ x = 8 \end{cases}$



$H \in r$

$H(8, 0, 3)$

$A(10, \sqrt{44}, 3)$

$h = d(A, H) = \sqrt{(10-8)^2 + (\sqrt{44} - 0)^2 + (3-3)^2} = \sqrt{48} = 4\sqrt{3}$

$l = h \cdot \dfrac{2}{\sqrt{3}} = 4 \cdot \sqrt{3} \cdot \dfrac{2}{\sqrt{3}} = 8 \implies \dfrac{l}{2} = 4$

$r: \begin{cases} y = 0 \\ x = 8 \end{cases}$

$B(8, 0, ?) \longrightarrow B(8, 0, 7)$

4

$H(8, 0, 3)$

4

$C(8, 0, ?) \longrightarrow C(8, 0, -1)$

$A(t, 0, 0)$; $B(0, -2, 0)$; $C(0, 0, \sqrt{23})$

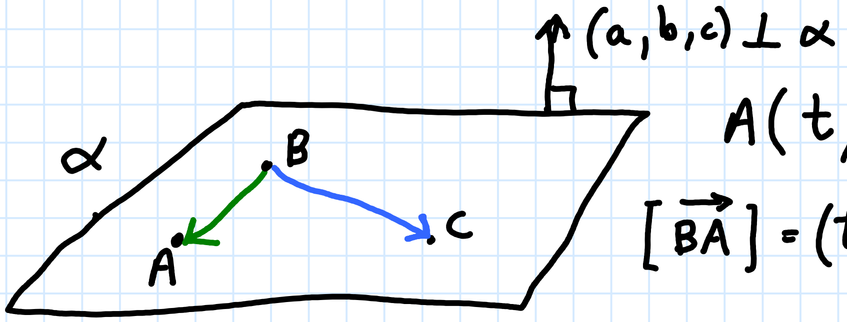
$t \in \mathbb{R}$; $A, B, C \in \alpha$; $\beta =$ plano YZ ;

$(\widehat{\alpha, \beta}) = \frac{2}{3}\pi$ RAD

$(a, b, c) \perp \alpha$ $\qquad$ $(a', b', c') \perp \beta$

$$\cos(\alpha \hat{,} \beta) = \pm \frac{aa' + bb' + cc'}{\sqrt{a^2+b^2+c^2} \cdot \sqrt{(a')^2+(b')^2+(c')^2}}$$

β = piano yz : $x=0 \longrightarrow (a', b', c') = (1, 0, 0)$



$A(t,0,0)$; $B(0,-2,0)$; $C(0,0,\sqrt{23})$

$[\vec{BA}] = (t,2,0)$; $[\vec{BC}] = (0,2,\sqrt{23})$

$$[\vec{BA}] \wedge [\vec{BC}] = \begin{bmatrix} t & 2 & 0 \\ 0 & 2 & \sqrt{23} \end{bmatrix} = (\underbrace{2\sqrt{23}}_{a}, \underbrace{-t\sqrt{23}}_{b}, \underbrace{2t}_{c})$$

$(\alpha \hat{,} \beta) = \frac{2}{3}\pi$ RAD $= 120 \rightarrow \cos(\alpha \hat{,} \beta) = -\frac{1}{2}$

$(a', b', c') = (1, 0, 0)$

$$-\frac{1}{2} = \pm \frac{2\sqrt{23}}{1 \cdot \sqrt{92 + 23t^2 + 4t^2}}$$

$$-\sqrt{92 + 27t^2} = \pm 4 \cdot \sqrt{23}$$

$$92 + 27t^2 = 16 \cdot 23 = 368$$

$$27t^2 = 368 - 92 = 276$$

$$9t^2 = 92 \qquad t^2 = \frac{92}{9} \qquad t = \pm \frac{\sqrt{92}}{3}$$

$$A = \begin{bmatrix} 1 & -t & 2 \\ 0 & -5 & (t-4) \\ 0 & 0 & 1 \end{bmatrix} \qquad t \in \mathbb{R}$$

$$p_A(\lambda) = \begin{bmatrix} (1-\lambda) & -t & 2 \\ 0 & (-5-\lambda) & (t-4) \\ 0 & 0 & (1-\lambda) \end{bmatrix} = (1-\lambda)^2 \cdot (-5-\lambda)^1$$

$\lambda_1 = 1$ $\qquad$ $\lambda_2 = -5$

$m_a(1) = 2$; $m_a(-5) = 1$;

$\Downarrow \qquad\qquad \Downarrow$

$m_g(1) \leq 2 \qquad m_g(-5) = 1$

A è diagonalizzabile $\iff m_g(1) = 2$

$$m_g(1) = 3 - rg(A - 1 \cdot I) = 2 \Rightarrow$$

$$rg(A - 1 \cdot I) = 1 \Rightarrow rg \begin{bmatrix} 0 & -t & 2 \\ 0 & -6 & (t-4) \\ 0 & 0 & 0 \end{bmatrix} = 1 \Rightarrow$$

$$\Rightarrow rg \begin{bmatrix} -t & 2 \\ -6 & (t-4) \end{bmatrix} = 1 \Longrightarrow \det \begin{bmatrix} -t & 2 \\ -6 & (t-4) \end{bmatrix} = 0 \Rightarrow$$

$$\Rightarrow -t(t-4) + 12 = 0 \Rightarrow -t^2 + 4t + 12 = 0 \Rightarrow$$

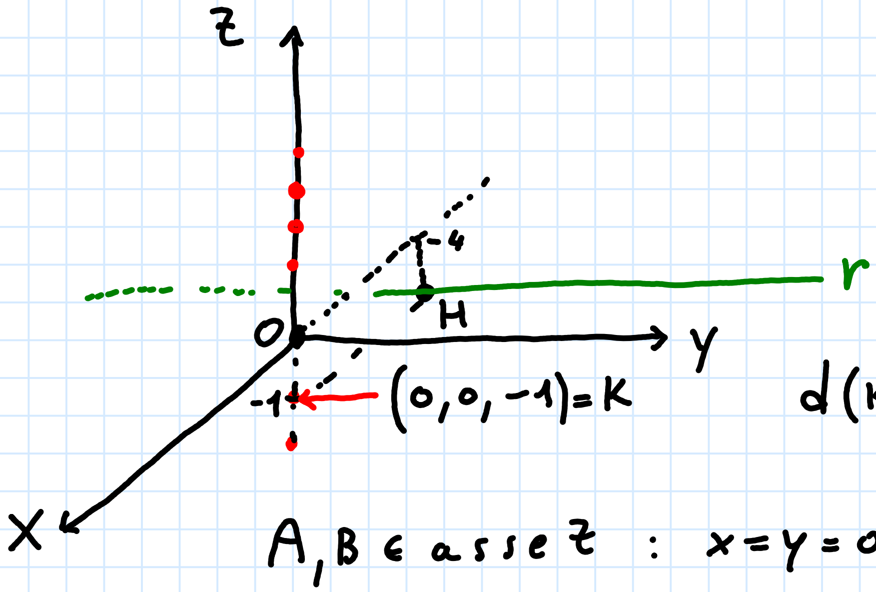
$$\Rightarrow t^2 - 4t - 12 = 0 \Rightarrow (t-6) \cdot (t+2) = 0 \Rightarrow$$

$$t = 6 \ ; \ t = -2 \ ;$$

$r: x+4 = z+1 = 0$; $A, B \in$ asse z :

$d(A, r) = d(B, r) = 5$

$$r: \begin{cases} x=-4 \\ z=-1 \end{cases}$$



$d(K,r) = 4 < 5$

$A, B \in$ asse z : $x = y = 0$

$A(0,0,\gamma)$ $\qquad H(-4,0,-1)$

$$d(A,H) = 5 \implies [d(A,H)]^2 = 25$$

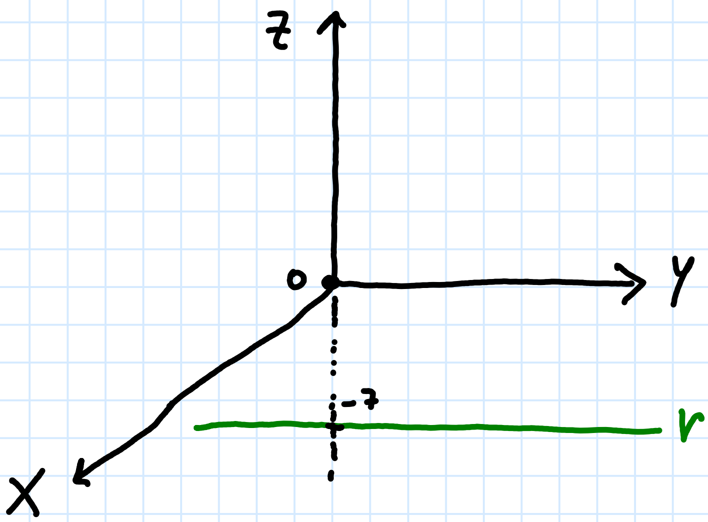
$$4^2 + 0^2 + (\gamma+1)^2 = 25$$
$$(\gamma+1)^2 = 9$$
$$\gamma + 1 = \pm 3 \qquad \gamma = -1 \pm 3 \begin{array}{l} \nearrow -4 \\ \searrow +2 \end{array}$$

$A(0,0,-4)$; $B(0,0,2)$;

$A(2\sqrt{26}, 2, -5)$; r: $x = z+7 = 0$

H proiezione di A su r $\implies H(0, 2, -7)$

$B \in r$ tale che area $A\hat{H}B = 9\sqrt{3}$

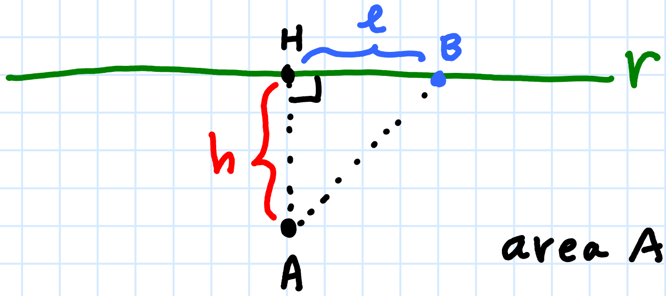


$B \in r \implies B(0, ?, -7)$

$A(2\sqrt{26}, 2, -5)$

$H(0, 2, -7)$

$B(0, \beta, -7)$



$\text{area } A\hat{H}B = 9\sqrt{3}$

$\text{area } A\hat{H}B = \frac{1}{2} \cdot \ell \cdot h$

$h = d(A, H) = \sqrt{104 + 4} = \sqrt{108} = 6\sqrt{3}$

$\ell = d(H, B) = \sqrt{(\beta - 2)^2} = |\beta - 2|$

$$9\sqrt{3} = \frac{1}{2} \cdot |\beta - 2| \cdot 6 \cdot \sqrt{3}$$

$$9 = 3 \cdot |\beta - 2| \;;\; |\beta - 2| = 3 \;;\; \beta - 2 = \pm 3$$

$\beta = 2 \pm 3 \begin{cases} \nearrow 5 \\ \searrow -1 \end{cases}$

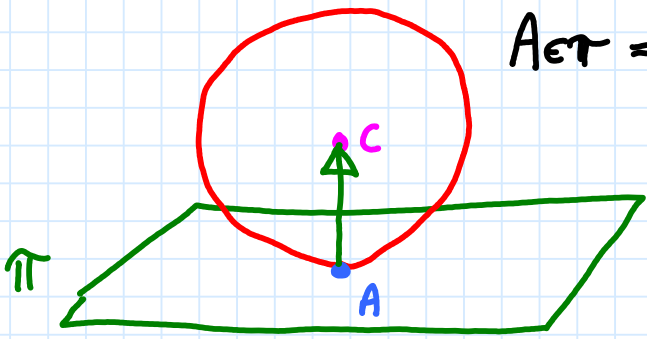
$B(0, 5, -7)$

$B'(0, -1, -7)$

$C(4, -7, -16)$; $A(4, -12, -10)$; sfera S

C centro sfera ; $A \in S$

π piano tangente a S nel punto A



$A \in \pi \Rightarrow a(x-4) + b(y+12) + c(z+10) = 0$

$[\vec{AC}] = (\underset{a}{0}, \underset{b}{5}, \underset{c}{-6})$

$5(y + 12) - 6(z + 10) = 0$

$$\boxed{\pi : 5 \cdot y - 6 \cdot z = 0}$$

$r: 3x - 4y - 5 = 0$ direttrice parabole passanti per $O(0,0)$ con fuochi su asse X.

Trovare coordinate dei FUOCHI.

$F(\alpha, 0)$ $\qquad d(O,F) = d(O,r)$

$$d(O,F) = \sqrt{(0-\alpha)^2 + (0-0)^2} = \sqrt{\alpha^2} = |\alpha|$$

$$d(O,r) = \frac{|3\cdot 0 - 4\cdot 0 - 5|}{\sqrt{3^2+(-4)^2}} = \frac{|-5|}{5} = \frac{5}{5} = 1$$

$$|\alpha| = 1 \implies \alpha = \pm 1 \quad \begin{cases} F_1(-1,0) \\ F_2(+1,0) \end{cases}$$

$\pi: 3x+3y+7z-14=0$ $\qquad A(0,0,2)$

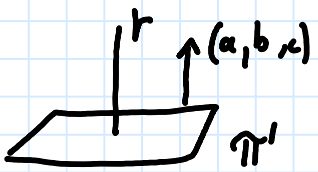
asse z: $x=y=0$

$(a,b,c) = (11,2,0)$

r: $O \in r$ et $r \perp \pi'$: $11x + 2y + 3 = 0$

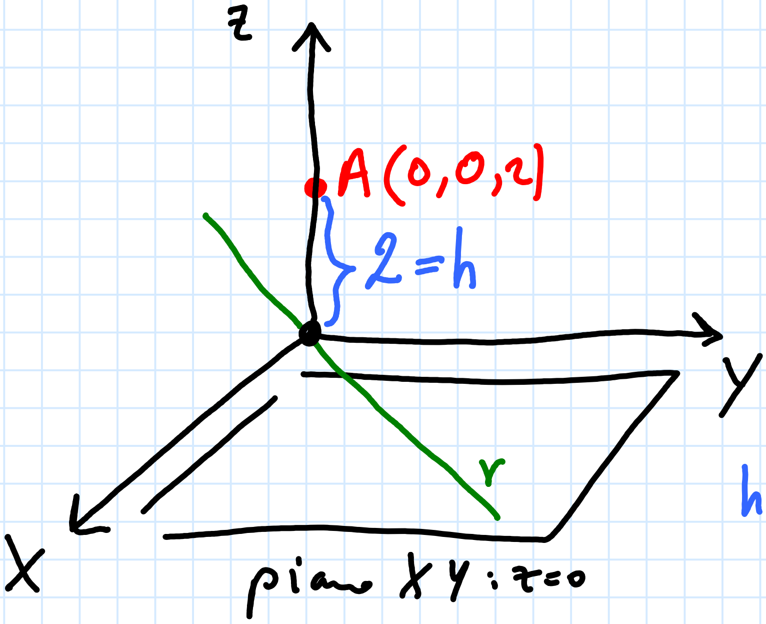
$h = d(A,r)$ $\qquad B, C \in r$: $d(B,A) = d(C,A) = 4h$

area $\overset{\triangle}{ABC} = ?$



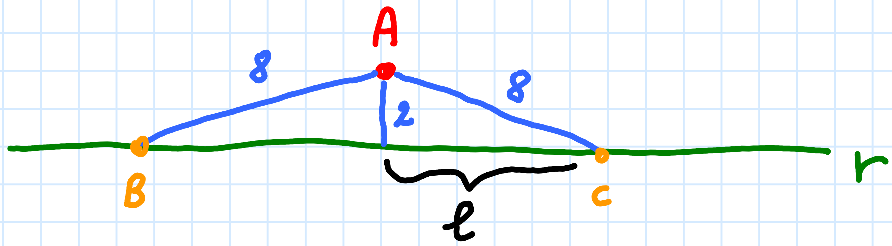
$$r: \begin{cases} x = 11t + 0 \\ y = 2\cdot t + 0 \\ z = 0\cdot t + 0 \end{cases} \qquad \boxed{r: \begin{cases} x = 11t \\ y = 2t \\ z = 0 \end{cases}}$$

r è una retta del piano XY: $z=0$
passante per l'origine O



$h=2 \Rightarrow 4h = 8$

$\ell = \sqrt{8^2-2^2} = \sqrt{60} = 2\cdot\sqrt{15}$

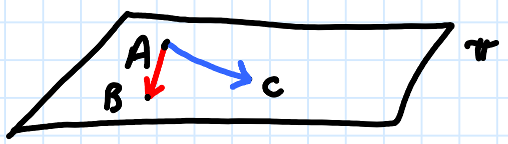


$\text{area } \widehat{ABC} = 2\cdot(2\cdot\sqrt{15}) = 4\cdot\sqrt{15}$

$A(\sqrt{2},0,0)\,;\ B(0,1,0)\,;\ C(0,0,-\sqrt{2})$

$A,B,C \in \pi\ ;\quad \pi' = \text{piano } yz\ ;\quad \theta = \widehat{\pi,\pi'}\ ;$

$\pi' = \text{piano } yz\ :\ x=0 \Rightarrow (a',b',c') = (1,0,0)$



$[\vec{AB}] = (-\sqrt{2}, 1, 0)$

$[\vec{AC}] = (-\sqrt{2}, 0, -\sqrt{2})$

$$[\vec{AB}]\wedge[\vec{AC}] = \begin{bmatrix} \vec{i} & \vec{j} & \vec{k} \\ -\sqrt{2} & 1 & 0 \\ -\sqrt{2} & 0 & -\sqrt{2} \end{bmatrix} = (-\sqrt{2}, -2, \sqrt{2})$$

$$\cos\vartheta = \pm \frac{a\cdot a' + b\cdot b' + c\cdot c'}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{(a')^2+(b')^2+(c')^2}}$$

$$\cos\vartheta = \pm \frac{-\sqrt{2}}{\sqrt{8}\cdot\sqrt{1}} = \mp \frac{\sqrt{2}}{2\sqrt{2}} = \mp \frac{1}{2}$$

$\vartheta = 60°$; $\vartheta = 120°$;
