Ricevimento studenti – lunedì 27 gennaio 2025

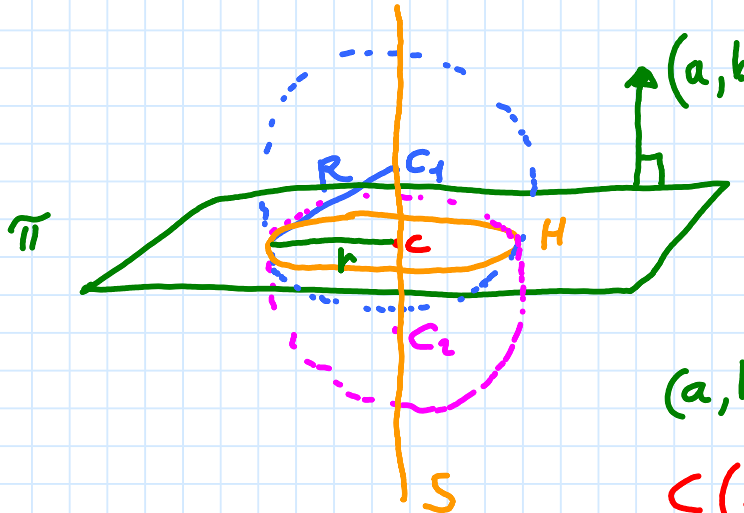
Titolo nota 27/01/2025

$\pi : 4x - 2y + z = 0 \qquad R = 22$

$C(1,0,-4) \in \pi \qquad r = 20 \qquad$ circonferenza H

C_1, C_2 sfere S_1, S_2 tali che $S_1 \cap \pi = S_2 \cap \pi = H$

$(a,b,c) = (4,-2,1)$



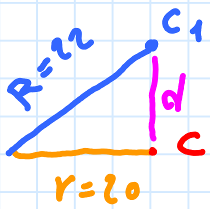
$s : s \perp \pi$ et $C \in s$

$(a,b,c) = (4,-2,1) \; // s$

$C(1,0,-4)$

$$s : \begin{cases} x = 4 \cdot t + 1 \\ y = -2 \cdot t + 0 \\ z = 1 \cdot t + (-4) \end{cases}$$

$P(4t+1, -2t, t-4) \in s$



$d = \sqrt{(22)^2 - (20)^2} = \sqrt{84} = 2\sqrt{21}$

$C(1,0,-4)$

$d(C_1, C) = 2\sqrt{21} \quad ; \quad [d(C_1,C)]^2 = 84$

$(4t+1-1)^2 + (-2t-0)^2 + (t-4+4)^2 = 84$

$16t^2 + 4t^2 + t^2 = 84 \quad ; \quad 21t^2 = 84 \; ; \quad t^2 = 4$

$t_1 = 2 \; ; \; t_2 = -2$

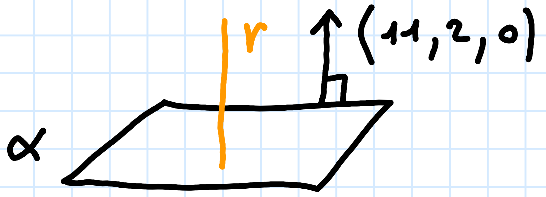
$t_1 = 2 \longrightarrow C_1(9,-4,-2)$

$t_2 = -2 \longrightarrow C_2(-7,4,-6)$

$\pi: 3x + 3y + 7z - 14 = 0$

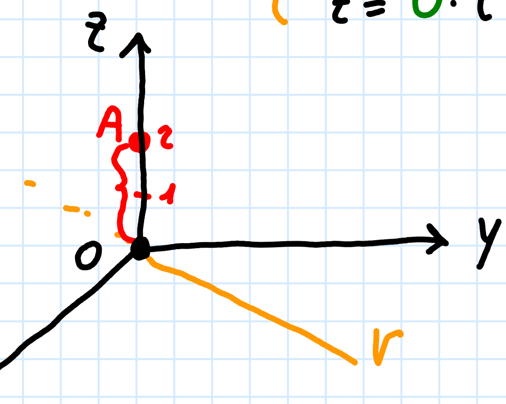
$\{A\} = \pi \cap$ asse z 　　 asse z: $x = y = 0$ 　　 $A(0,0,2)$

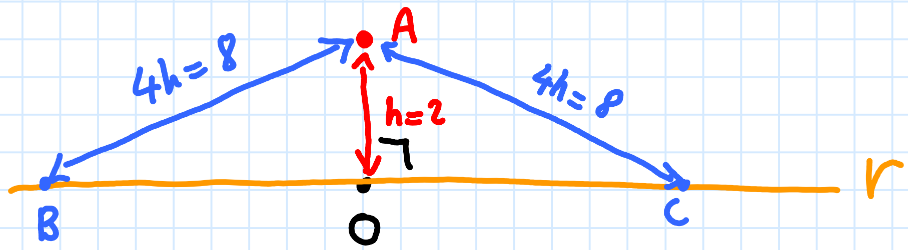
$O(0,0,0) \in r$ et $r \perp \alpha: 11x + 2y + 3 = 0$



$r: \begin{cases} x = 11 \cdot t + 0 \\ y = 2 \cdot t + 0 \\ z = 0 \cdot t + 0 \end{cases}$ 　　 $r: \begin{cases} x = 11t \\ y = 2t \\ z = 0 \end{cases}$

$h = d(A,r) = 2$





$\text{area } B\hat{A}C = ? = 2 \cdot \text{area } A\hat{O}C = 2 \cdot \left(\frac{1}{2} \cdot OC \cdot OA\right) =$

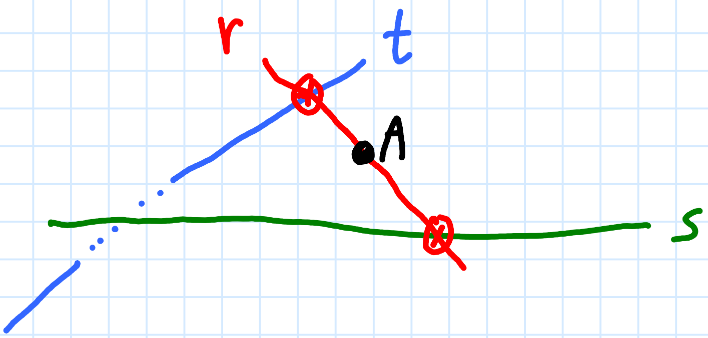
$= OC \cdot OA = 2 \cdot OC$

$OC = ? = \sqrt{8^2 - 2^2} = \sqrt{60}$; 　 $\text{area } B\hat{A}C = 2 \cdot \sqrt{60}$

$A(2,0,0)$ 　　 $s: z + 2 = y - 10x = 0$ 　　 $A \notin s$

$t: y - 5 = z + 20x = 0$ 　　 $A \notin t$

r : $A \in r$; si appoggia a s e t



$r = \alpha \cap \beta$

$\alpha \in F(t)$: $A \in \alpha$

$\beta \in F(s)$: $A \in \beta$

$\alpha \in F(t)$: $\lambda(y-5) + \mu(z+20x) = 0$

$A(2,0,0) \in \alpha \Rightarrow -5\lambda + 40\mu = 0 \Rightarrow \lambda - 8\mu = 0$

$(\lambda, \mu) = (8, 1) \Rightarrow \alpha : 8(y-5) + 1 \cdot (z+20x) = 0$

$$\boxed{\alpha : 20x + 8y + z - 40 = 0}$$

$\beta \in F(s) \Rightarrow \lambda(z+2) + \mu(y-10x) = 0$

$A(2,0,0) \in \beta \Rightarrow 2\lambda - 20\mu = 0 \Rightarrow \lambda - 10\mu = 0$

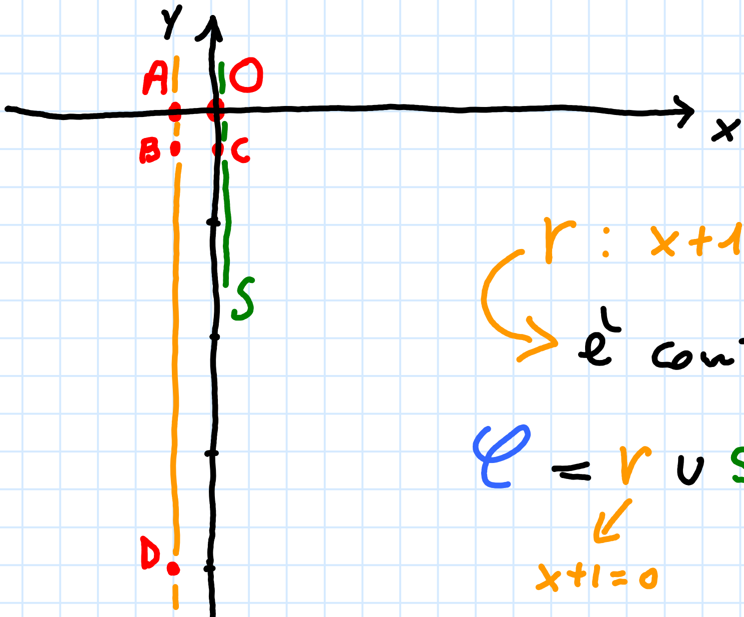
$(\lambda, \mu) = (10, 1) \Rightarrow \beta : 10(z+2) + 1 \cdot (y-10x) = 0$

$$\boxed{\beta : 10x - y - 10z - 20 = 0}$$

$$r = \alpha \cap \beta : \begin{cases} 20x + 8y + z - 40 = 0 \\ 10x - y - 10z - 20 = 0 \end{cases}$$

$$\begin{bmatrix} 20 & 8 & 1 \\ 10 & -1 & -10 \end{bmatrix} \begin{array}{l} \nearrow \ell = \\ \rightarrow m = \\ \searrow n = \end{array}$$

$O(0,0)$; $A(-1,0)$; $B(-1,-1)$; $C(0,-1)$; $D(-1,-12)$



$r: x+1=0$ $\quad A, B, C \in r$

è contenuta nella conica

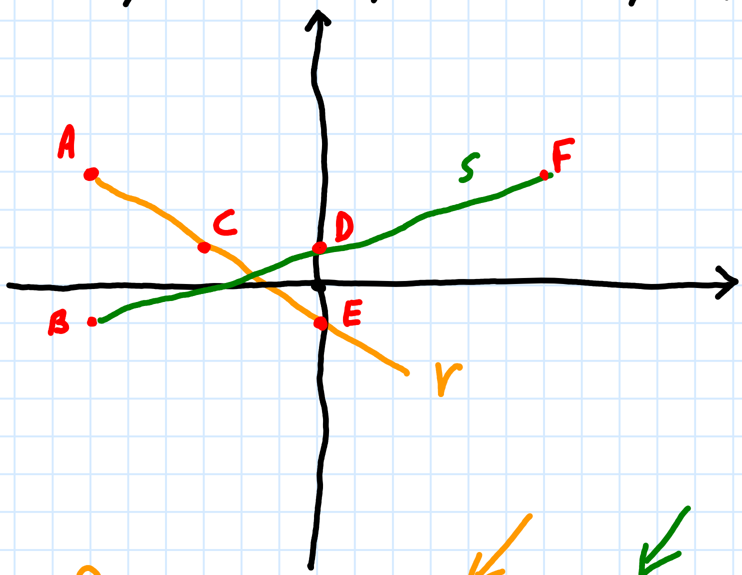
$\mathscr{C} = r \cup s$ $\quad s$ è l'asse Y: $x=0$

$x+1=0$ $\quad x=0$

$\mathscr{C}: (x+1)\cdot x = 0$

$$\boxed{\mathscr{C}: x^2+x=0}$$

$A(-6,3)$; $B(-6,-1)$; $C(-3,1)$; $D(0,+1)$; $E(0,-1)$; $F(6,3)$



$r: y = -\frac{2}{3}x - 1$

$A, C, E \in r$

$s: y = +\frac{1}{3}x + 1$

$B, D, F \in s$

$r: 2x+3y+3=0$ $\quad s: x-3y+3=0$

$\mathscr{C} = r \cup s = (2x+3y+3)\cdot(x-3y+3) = 0$

$5x^2+2xy+5y^2-10y+7=0$

$A = \begin{bmatrix} 5 & 1 \\ 1 & 5 \end{bmatrix}$; $\quad p_A(\lambda) = \ldots = \lambda^2 - 10\lambda + 24 = (\lambda-6)\cdot(\lambda-4)$

$\lambda_1 = 6$ $\quad \lambda_2 = 4$

$\boxed{\lambda_1 = 6}$ $\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$; $\begin{cases} -x+y=0 \\ x-y=0 \end{cases} \to y = x$

$(x,y) = (x,x) = x(1,1) \quad \forall x \neq 0$ (autovettori)

scelgo $x = \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}}{2}$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ auto VERSORE relativo a $\lambda_1 = 6$

$C = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ auto VERSORE relativa $\lambda_2 = 4$

$\det C = +1 \to$ matrice associata ad una ROTAZIONE

$$\begin{bmatrix} 0 & -10 \end{bmatrix} \cdot \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix} = \begin{bmatrix} -5\sqrt{2} & -5\sqrt{2} \end{bmatrix}$$

$$6 \cdot (x')^2 + 4 \cdot (y')^2 - 5\sqrt{2}\, x' - 5\sqrt{2}\, y' + 7 = 0$$

$$6 \cdot \left(x' - \frac{5\sqrt{2}}{12}\right)^2 - \frac{25}{12} + 4 \cdot \left(y' - \frac{5\sqrt{2}}{8}\right)^2 - \frac{25}{8} + 7 = 0$$

traslazione $\begin{cases} x'' = x' - \frac{5\sqrt{2}}{12} \\ y'' = y' - \frac{5\sqrt{2}}{8} \end{cases}$

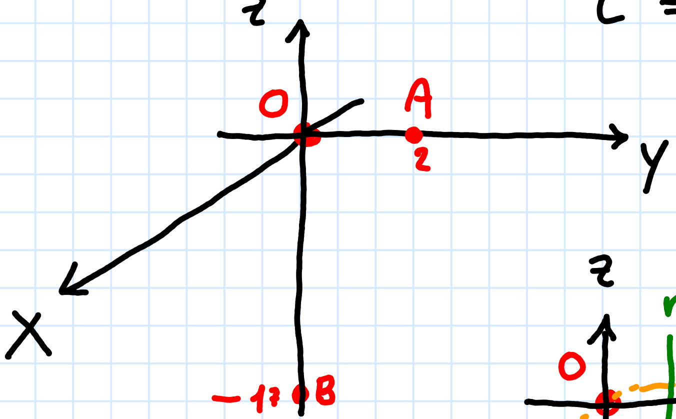
$$6 \cdot (x'')^2 + 4 \cdot (y'')^2 = \frac{25}{12} + \frac{25}{8} - 7 = d < 0$$

$$\frac{(x'')^2}{\left(\frac{-d}{6}\right)} + \frac{(y'')^2}{\left(-\frac{d}{4}\right)} = -1$$ ellisse immaginaria

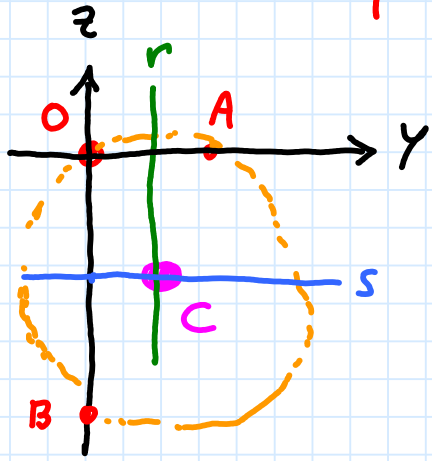
$O(\boxed{0},0,0)$; $A(\boxed{0},2,0)$; $B(\boxed{0},0,-17)$

$\mathscr{C}$ circonferenza passante per O, A e B nello SPAZIO

$\mathscr{C} = \mathscr{S} \cap \pi$



il piano π è il piano YZ. $\boxed{x=0}$



$C\left(0, 1, -\frac{17}{2}\right)$ ↳ centro sfera

$$C\left(\underset{x_0}{0}, \underset{y_0}{1}, \underset{z_0}{-\tfrac{17}{2}}\right)$$

$O(0,0,0) \in \mathscr{S} \Rightarrow \mathscr{S}: x^2+y^2+z^2+ax+by+cz=0$

$a = -2x_0 = -2\cdot 0 = 0$

$b = -2y_0 = -2\cdot 1 = -2$

$c = -2z_0 = -2\left(-\frac{17}{2}\right) = 17$

$\mathscr{S}: x^2+y^2+z^2-2y+17z=0$

$\pi: x=0$

$$\mathscr{C}: \begin{cases} x^2+y^2+z^2-2y+17z=0 \\ x=0 \end{cases}$$

$$A = \begin{bmatrix} 5 & 7 & 0 \\ 0 & h & 0 \\ -2 & K & 3 \end{bmatrix}$$

$(h,K) \in \mathbb{R}^2$ tale che

non 3 autovalori reali a 2 a 2 distinti

<u>et</u> A diagonalizzabile

$$P_A(\lambda) = \det \begin{bmatrix} (5-\lambda) & 7 & 0 \\ 0 & (h-\lambda) & 0 \\ -2 & K & (3-\lambda) \end{bmatrix} = (h-\lambda)\cdot(5-\lambda)\cdot(3-\lambda)$$

$h=5$ oppure $h=3$

$\boxed{h=5}$ $m_a(5)=2$; $m_a(3)=1$

devo avere che $m_g(5)=2$

impongo

$$m_g(5) = \dim E(5) = 3 - rg(A-5\cdot I_3) \overset{\downarrow}{=} 2 \Rightarrow$$

$$rg(A-5\cdot I_3) = 1 \quad ; \quad \begin{bmatrix} 0 & 7 & 0 \\ 0 & 0 & 0 \\ -2 & K & -2 \end{bmatrix} = (A-5I_3)$$

$rg(A-5I_3)=2$ quindi <u>NON</u> mai uguale a 1

$h=5$ DA SCARTARE

$\boxed{h=3}$ $m_a(3)=2$

deve essere anche $m_g(3)=2$

$$3 - rg(A-3I_3) = 2 \Rightarrow rg(A-3\cdot I_3)=1$$

$$rg\begin{bmatrix} 2 & 7 & 0 \\ 0 & 0 & 0 \\ -2 & K & 0 \end{bmatrix} = 1 \iff K=-7$$

SOLUZIONE FINALE $(h,K) = (3,-7) \in \mathbb{R}^2$

$r: 3x+2 = 3x+5z+2 = 0$ sghembe

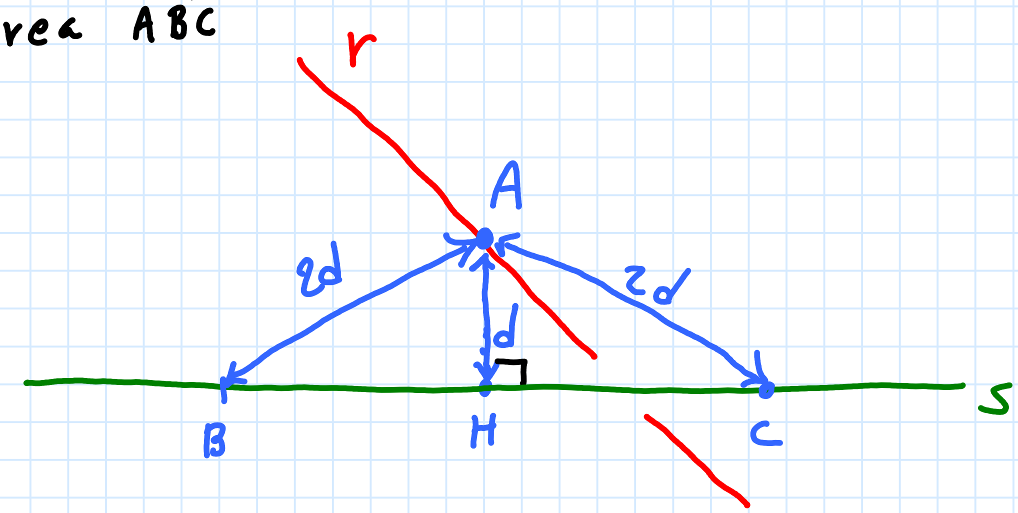
$s: 7y+20 = 7y+4z = 0$

t retta minima distanza $\{A\} = t \cap r$

$d = d(A,H)$ $\{H\} = t \cap s$

$B, C \in s$ tali che $d(A,B) = d(A,C) = 2d$

area $\overset{\Delta}{ABC}$



$$\text{area } \overset{\Delta}{ABC} = 2 \cdot \text{area } \overset{\Delta}{AHC} = 2 \cdot \left(\frac{1}{2} \cdot HC \cdot AH\right) = HC \cdot d$$

$$HC = \sqrt{(2d)^2 - d^2} = d \cdot \sqrt{3}$$

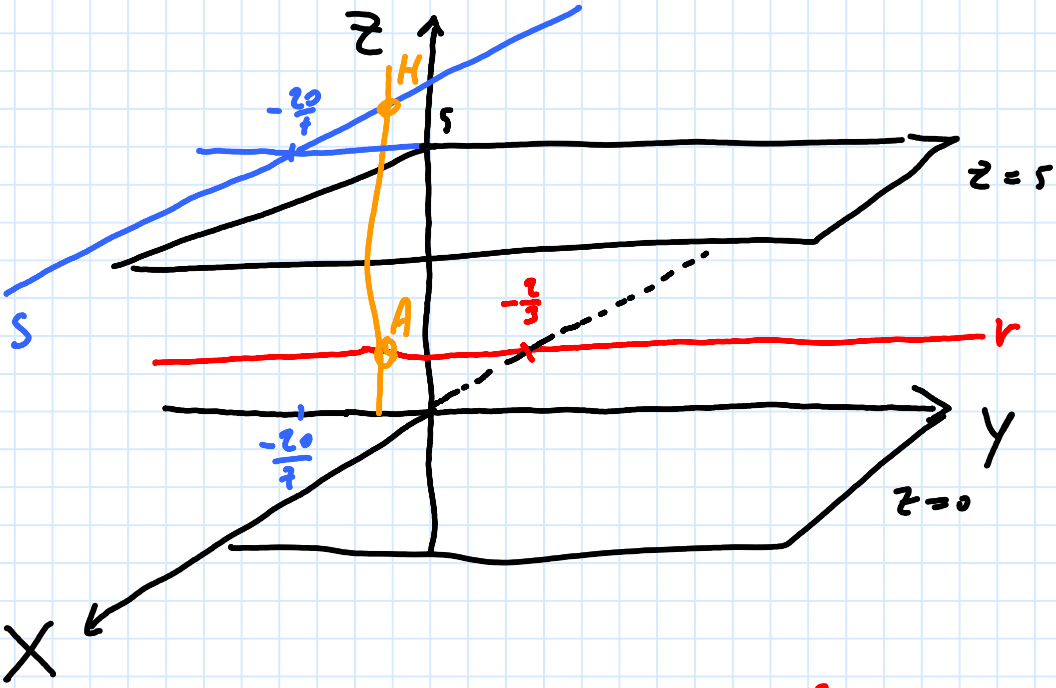
risultato finale $\overset{\Delta}{ABC} = \sqrt{3} \cdot d^2$

$r: 3x+2 = 3x+5z+2 = 0$

$s: 7y+20 = 7y+4z = 0$

$r: 3x+2 = z = 0$ $\quad r: \begin{cases} x = -\frac{2}{3} \\ z = 0 \end{cases}$

$s: 7y+20 = z-5 = 0$ $\quad s: \begin{cases} y = -\frac{20}{7} \\ z = 5 \end{cases}$



$d(r,s) = 5$

$d(A,H) = 5$

area richiesta $= \sqrt{3}\cdot d^2 = \sqrt{3}\cdot(5)^2 = 25\cdot\sqrt{3}$