Ricevimento studenti – martedì 4 febbraio 2025

Titolo nota 04/02/2025

$$A(t,0,0);\ B(0,\sqrt{3},0);\ C(0,0,-3)$$

$A, B, C \in \alpha$

$\vec{BA} = (t, -\sqrt{3}, 0)$

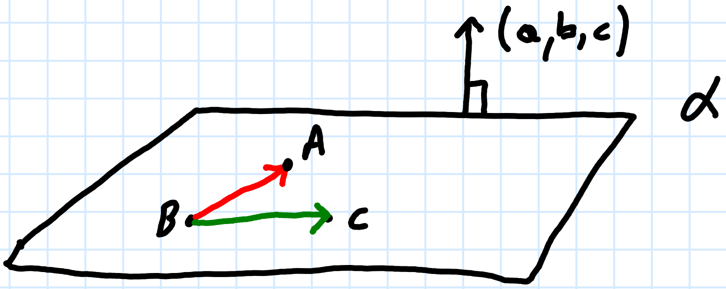
$\vec{BC} = (0, -\sqrt{3}, -3)$

$$\theta = \widehat{\alpha, \text{asse } X} = \frac{\pi}{6} \text{ radianti} = 30° \rightarrow \sin\theta = \frac{1}{2}$$

angolo RETTA – PIANO

$$\sin\theta = \frac{|al + bm + cn|}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{l^2+m^2+n^2}}$$

$$\text{RETTA} = \text{asse } X \Rightarrow (l, m, n) = (1, 0, 0)$$



$$\vec{u} = [\vec{BA}] \wedge [\vec{BC}] \perp \alpha$$

$$\begin{bmatrix} t & -\sqrt{3} & 0 \\ 0 & -\sqrt{3} & -3 \end{bmatrix} = \begin{array}{l} 3\sqrt{3} \\ 3t \\ -\sqrt{3}t \end{array}$$

$$\vec{u} \perp \alpha \rightarrow \vec{u} = (\underset{a}{3\sqrt{3}}, \underset{b}{3t}, \underset{c}{-\sqrt{3}t})$$

$(l, m, n) = (1, 0, 0)$

$$\frac{1}{2} = \frac{|3\sqrt{3}|}{\sqrt{27+9t^2+3t^2}\cdot\sqrt{1^2+0^2+0^2}} = \frac{3\sqrt{3}}{\sqrt{27+12t^2}}$$

$$\sqrt{27+12t^2} = 6\sqrt{3}$$

$$27+12t^2 = 108$$

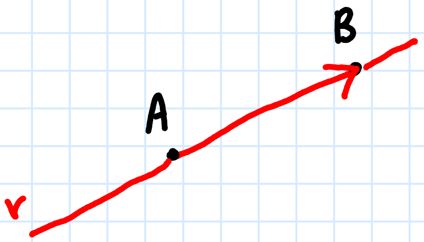
$$12t^2 = 81$$

$$4t^2 = 27$$

$$t^2 = \frac{27}{4} \Rightarrow \boxed{t = \pm\frac{3}{2}\sqrt{3}}$$

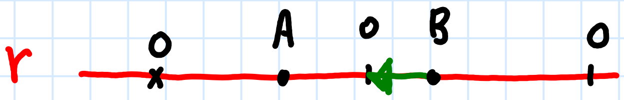
$A(1,-t,t-1)$; $B(t-1,t-1,1-t)$; $A,B \in r$

$t \in \mathbb{R}$; $O \in r$



$$[\overrightarrow{AB}] = (t-2,\ 2t-1,\ -2t+2)$$

$$[\overrightarrow{OB}] = (t-1,\ t-1,\ 1-t)$$



$$[\overrightarrow{OB}] \parallel [\overrightarrow{AB}]$$

$$(t-1,\ t-1,\ 1-t) \parallel (t-2,\ 2t-1,\ 2-2t)$$

$$(t-1,\ t-1,\ \underline{1-t}) \parallel (t-2,\ 2t-1,\ 2\cdot\underline{(1-t)})$$

se $\underline{t \neq 1}$ $\quad [\overrightarrow{OB}] = \alpha\,[\overrightarrow{AB}] \Rightarrow \alpha = 2$

$$\begin{cases} t-2 = 2(t-1) \\ 2t-1 = 2(t-1) \end{cases} \quad \begin{cases} t-2 = 2t-2 \\ 2t-1 = 2t-2 \end{cases} \quad \begin{cases} t = 0 \\ -1 = -2 \end{cases} \quad \underline{\text{No}}$$

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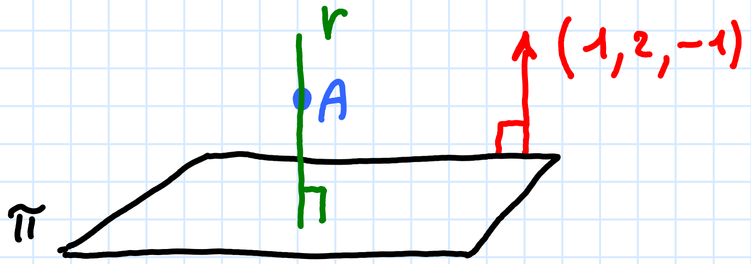
se $\underline{t=1}$, allora $B(0,0,0)$ coincide con l'origine O e, quindi, O, A e B sono allineati

$A(0,-3,1)$; $\pi : x+2y-z=0$;

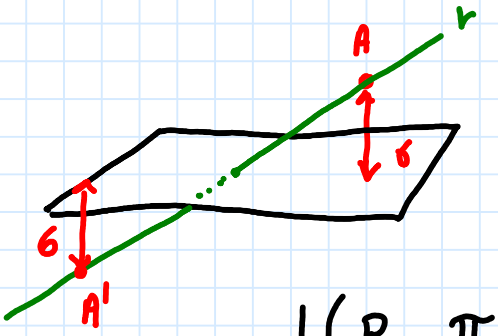
retta r tale che $A \in r$ et $r \perp \pi$

punti di r distanti 6 da $\pi' : z+1=0$



$$r : \begin{cases} x = 1 \cdot t + 0 \\ y = 2 \cdot t + (-3) \\ z = -1 \cdot t + 1 \end{cases}$$

$$P \in r \implies P(t, 2t-3, -t+1) \quad \forall t \in \mathbb{R}$$



$\pi' : z+1=0$

$$d(\underbrace{P}_{P \in r}, \pi') = 6$$

$$\frac{|(-t+1)+1|}{\sqrt{0^2+0^2+1^2}} = 6$$

$|-t+2| = 6$ $\qquad\qquad$ $P(t, 2t-3, -t+1)$

$|t-2| = 6$

$t-2 = \pm 6$

$t = 2 \pm 6$

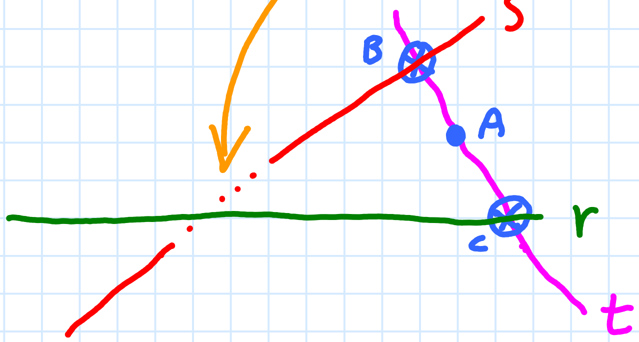
$t_1 = -4 \longrightarrow A(-4,-11,5)$

$t_2 = 8 \longrightarrow A'(8,13,-7)$

$A(3,0,-1)$; $\underbrace{\text{asse } X}_{s}$; $r: x = z+7 = 0$;

$\downarrow$ $r //$ asse Y

s e r sono $\boxed{\text{SGHEMBE}}$; $A \notin s$; $A \notin r$



$t = \alpha \cap \beta$

① $\alpha \in F(s)$ et $A \in \alpha$

② $\beta \in F(r)$ et $A \in \beta$

$s =$ asse X : $y = z = 0 \rightarrow F(s) : \lambda \cdot y + \mu \cdot z = 0$

$A(3,0,-1) \in \alpha : \lambda y + \mu z = 0 \rightarrow \lambda \cdot 0 + \mu(-1) = 0 \Rightarrow$

$\Rightarrow \mu = 0 \Rightarrow \lambda \neq 0 \Rightarrow$ scelgo $\lambda = 1$

$\boxed{\alpha : y = 0}$

$r : x = z+7 = 0 \longrightarrow F(r) : \lambda \cdot x + \mu(z+7) = 0$

$A(3,0,-1) \in \beta : \lambda x + \mu(z+7) = 0 \Rightarrow 3 \cdot \lambda + \mu(-1+7) = 0 \rightarrow$

$\rightarrow 3\lambda + 6\mu = 0 \rightarrow \lambda + 2\mu = 0 \longrightarrow$ scelgo $(\lambda, \mu) = (2, -1)$

$\beta : 2 \cdot x + (-1) \cdot (z+7) = 0$ $\boxed{\beta : 2x - z - 7 = 0}$

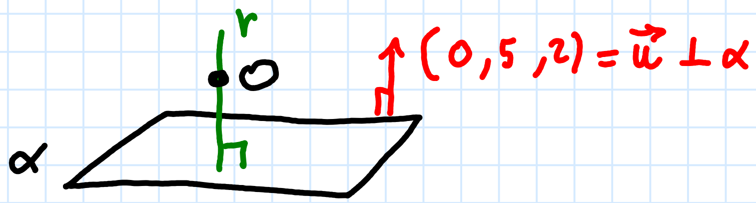
$$t : \begin{cases} y = 0 \\ 2x - z - 7 = 0 \end{cases}$$

$\pi : 3x + 17y + z - 9 = 0$

$\{A\} = \pi \cap \text{asse}X \qquad \text{asse}X : y = z = 0$

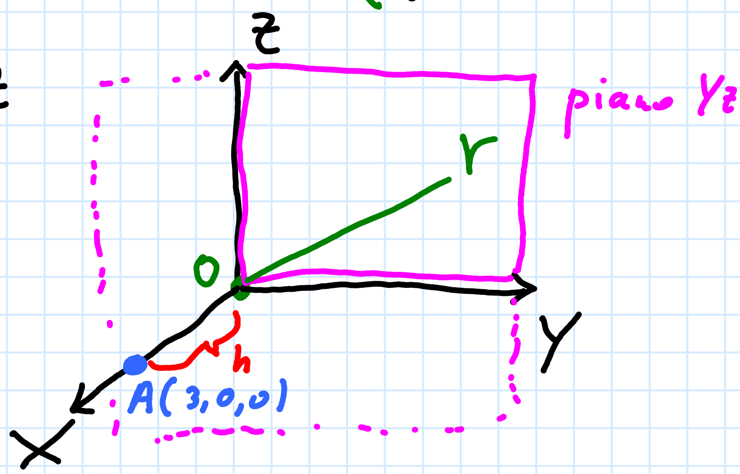
$A(3, 0, 0)$

$O \in r$ et $r \perp \alpha : 5y + 2z + 11 = 0$



$r : \begin{cases} x = 0 \cdot t + 0 \\ y = 5 \cdot t + 0 \\ z = 2 \cdot t + 0 \end{cases}$

$r : \begin{cases} x = 0 \\ y = 5t \\ z = 2t \end{cases} \longrightarrow$ piano Yz

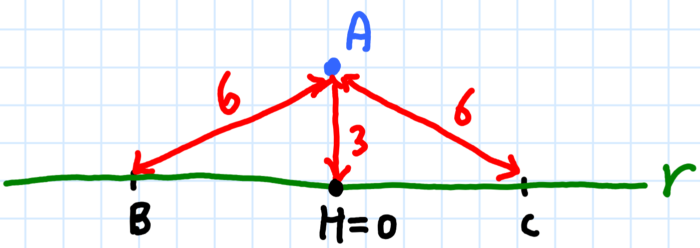


$h \overset{DEF}{=\!=} d(A, r)$

$\boxed{h = 3}$

$B, C \in r \quad : \quad d(B, A) = d(C, A) = 2 \cdot h = 6$

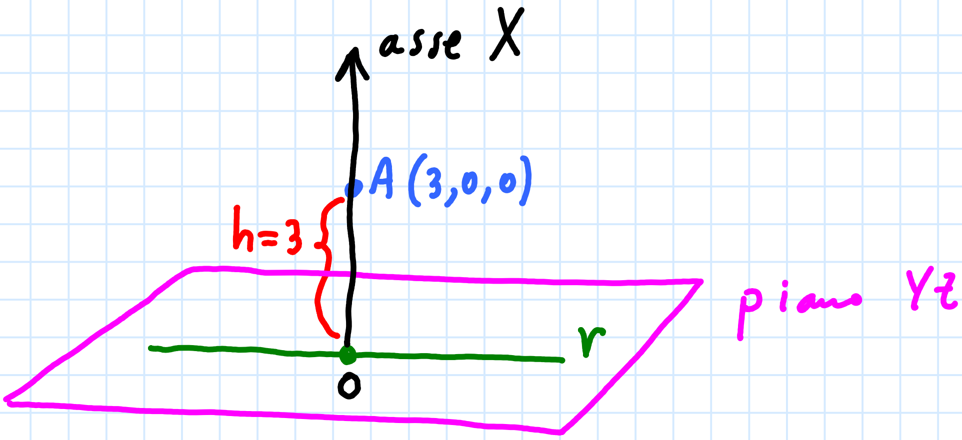
area $\overset{\Delta}{ABC}$

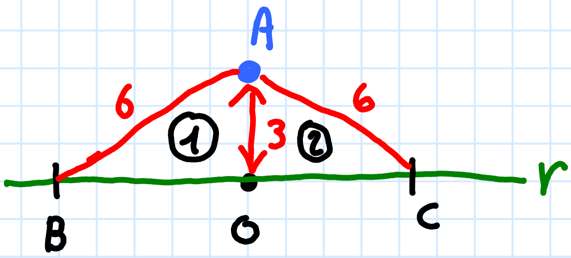


area $\overset{\Delta}{ABC} = 2 \cdot$ area $\overset{\Delta}{AHC} = 2 \cdot \frac{1}{2} \cdot HC \cdot AH = 3 \cdot HC$

$HC = \sqrt{6^2 - 3^2} = \sqrt{36 - 9} = \sqrt{27} = 3\sqrt{3}$

area $\overset{\Delta}{ABC} = 9\sqrt{3}$





$\text{area } A\overset{\triangle}{B}C =$

$= 2 \cdot \text{area } A\overset{\triangle}{O}C$

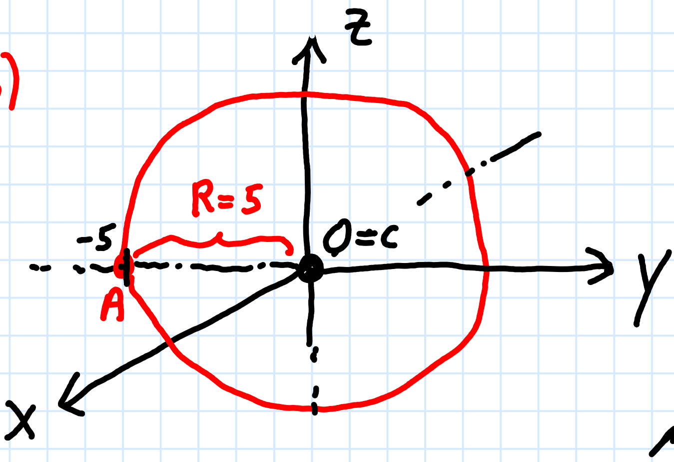
$C = O(0,0,0)$; $A(0,-5,0)$;

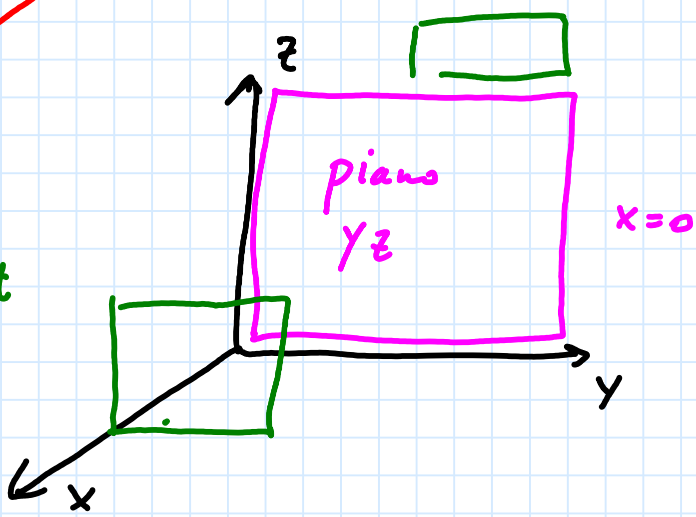
C centro sfera Σ passante per A

π // piano yz passante per $B(-4,-3,z)$.

$\mathcal{C} = \Sigma \cap \pi$ circonferenza $\nearrow$ CENTRO di $\mathcal{C}$; $\searrow$ RAGGIO di $\mathcal{C}$

$A(0,-5,0)$

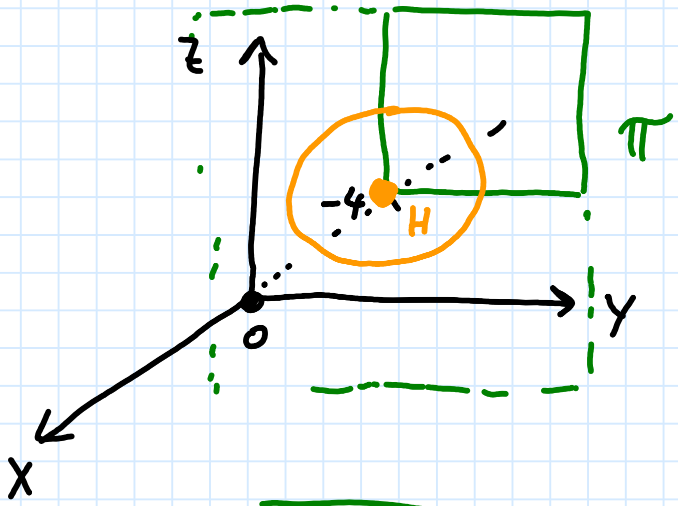


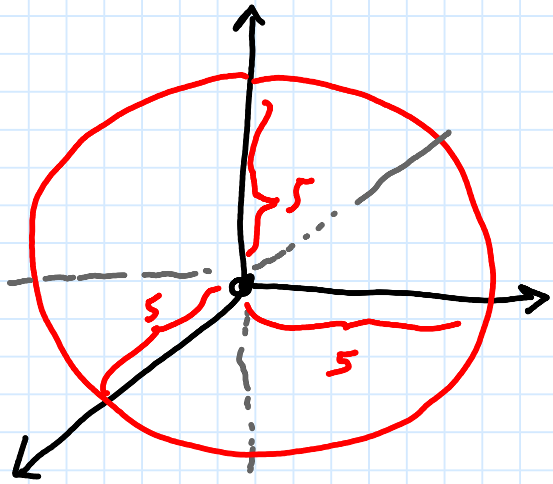


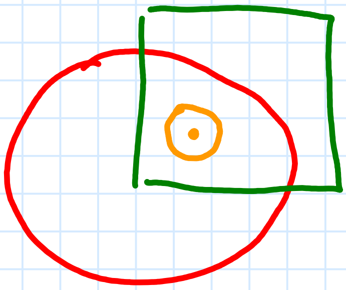
$B(-4,-3,z) \in \pi$: $x = \text{cost}$

π : $x = -4$

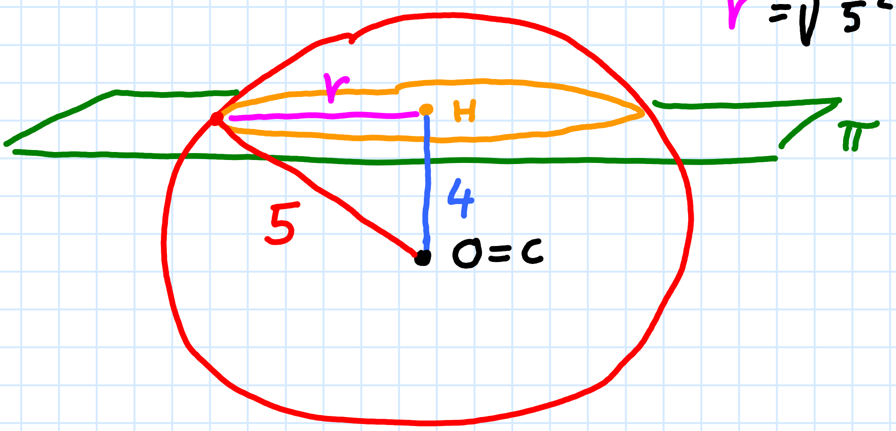
$x + 4 = 0$







$\boxed{H(-4,0,0)}$

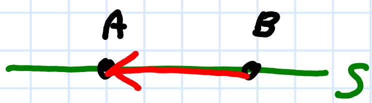


$$r = \sqrt{5^2 - 4^2} = 3$$

$$\boxed{r = 3}$$

$r: \quad 3x + (t+1)\cdot y + (t^2-10)z = x + 17 = 0$

$A(6, 5t+1, -6)$; $B(6, 2t+1, -t-7)$; $A, B \in s$



$r \parallel s$

$$r: \begin{bmatrix} 3 & (t+1) & (t^2-10) \\ 1 & 0 & 0 \end{bmatrix}$$

$l = 0$

$m = t^2 - 10$

$n = -(t+1)$

$[\overrightarrow{BA}] = (0, 3t, t+1) \Longrightarrow s:$

$l' = 0$

$m' = 3t$

$n' = t+1$

$$r \parallel s \iff rg\begin{bmatrix} 0 & (t^2-10) & -(t+1) \\ 0 & 3t & t+1 \end{bmatrix} = 1 \iff$$

$$\iff rg\begin{bmatrix} (t^2-10) & -(t+1) \\ 3t & (t+1) \end{bmatrix} = 1 \iff det\begin{bmatrix} (t^2-10) & -(t+1) \\ 3t & (t+1) \end{bmatrix} = 0$$

$$(t^2-10)\cdot(t+1) + 3t\cdot(t+1) = 0$$

$$(t+1)\cdot(t^2+3t-10) = 0$$

$$(t+1)\cdot(t+5)(t-2) = 0$$

$$t_1 = -1 \qquad t_2 = -5 \qquad t_3 = 2$$

$$\alpha: \quad x + (t+7)y = 0 \qquad (a,b,c,d) = (1, t+7, 0, 0)$$

$$\beta: \quad ty + z = 0 \qquad (a',b',c',d') = (0, t, 1, 0)$$

$$\gamma: \quad 3x - (t+7)z = 0 \qquad (a'',b'',c'',d'') = (3, 0, -(t+7), 0)$$

$$rg\begin{bmatrix} 1 & (t+7) & 0 & 0 \\ 0 & t & 1 & 0 \\ 3 & 0 & -(t+7) & 0 \end{bmatrix} = 2 \iff rg\begin{bmatrix} 1 & (t+7) & 0 \\ 0 & t & 1 \\ 3 & 0 & -(t+7) \end{bmatrix} = 2$$

$$\iff det\begin{bmatrix} 1 & (t+7) & 0 \\ 0 & t & 1 \\ 3 & 0 & -(t+7) \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & t+7 & 0 \\ 0 & t & 1 \\ 3 & 0 & -(t+7) \end{bmatrix} \begin{matrix} 1 & t+7 \\ 0 & t \\ 3 & 0 \end{matrix}$$

$$-t\cdot(t+7)+3(t+7)=0$$

$$(t+7)(3-t)=0 \qquad t_1=-7\ ; \quad t_2=3\ ;$$

$t_1=-7$

$\alpha: \ x=0$

$\beta: \ 7y-z=0$

$\gamma: \ x=0$

$\alpha=\gamma \Rightarrow$ NON SONO DISTINTI

NON è accettabile

$t_2=3$

$\alpha: \ x+10y=0$

$\beta: \ 3y+z=0$

$\gamma: \ 3x-10z=0$

Sono DISTINTI

UNICA soluzione del problema

$$A=\begin{bmatrix} 5 & 7 & 0 \\ 0 & h & 0 \\ -2 & K & 3 \end{bmatrix}$$

$(h,K)\in\mathbb{R}^2$ $\boxed{\text{NON}}$ ABBIA TRE AUTOVALORI REALI A 2 A DISTINTI E SIA DIAGONALIZZABILE

$$p_A(\lambda)=\det\begin{bmatrix} (5-\lambda) & 7 & 0 \\ 0 & (h-\lambda) & 0 \\ -2 & K & (3-\lambda) \end{bmatrix} = (h-\lambda)\cdot(5-\lambda)\cdot(3-\lambda)$$

$\lambda_1=h \qquad \lambda_2=5 \qquad \lambda_3=3$

$\boxed{h=5}$ oppure $\boxed{h=3}$

$\boxed{h=5}$

$$A = \begin{bmatrix} 5 & 7 & 0 \\ 0 & 5 & 0 \\ -2 & K & 3 \end{bmatrix} \qquad p_A(\lambda) = (5-\lambda)^2 \cdot (3-\lambda)^1$$

$m_a(3) = 1 \implies m_g(3) = 1$ OK

$m_a(5) = 2 \implies 1 \leq m_g(5) \leq 2$

A è DIAGON... $\iff m_g(5) = 2$

$$m_g(5) \overset{\text{DEF.}}{=\!=} \dim(E_5) = 3 - rg(A - 5 \cdot I_3) = 2 \implies$$

$$\implies rg \begin{bmatrix} 0 & 7 & 0 \\ 0 & 0 & 0 \\ -2 & K & -2 \end{bmatrix} = 1 \quad \text{MAI}$$

$h=5 \longrightarrow$ si scarta

$\boxed{h=3}$

$$A = \begin{bmatrix} 5 & 7 & 0 \\ 0 & 3 & 0 \\ -2 & K & 3 \end{bmatrix} \qquad p_A(\lambda) = (5-\lambda)^1 \cdot (3-\lambda)^2$$

$m_a(5) = 1 \longrightarrow m_g(5) = 1$ OK

$m_a(3) = 2 \longrightarrow 1 \leq m_g(3) \leq 2$

A è DIAGON..... $\iff m_g(3) = 2$

$$m_g(3) \overset{\text{DEF}}{=\!=} \dim(E_3) = 3 - rg(A - 3I_3) = 2$$

$$\operatorname{rg}(A - 3 \cdot I_3) = 1$$

$$\operatorname{rg}\begin{bmatrix} 2 & 7 & 0 \\ 0 & 0 & 0 \\ -2 & K & 0 \end{bmatrix} = 1 \iff K = -7$$

$$(h, K) = (3, -7) \in \mathbb{R}^2$$