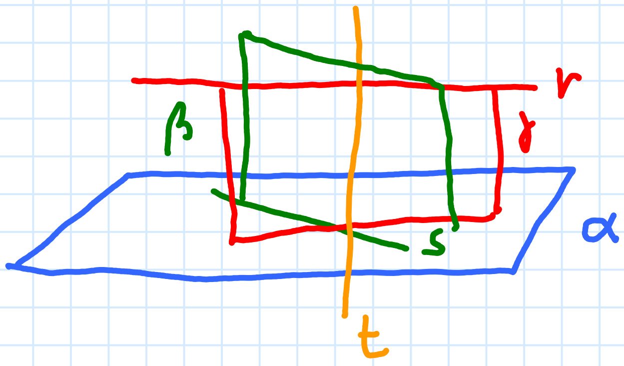
# Ricevimento studenti - lunedì 10/02/2025

Titolo nota 10/02/2025

$r: \ x+1 = 7x+y = 0$ $\qquad F(r)$ $\qquad \begin{bmatrix} 1 & 0 & 0 \\ 7 & 1 & 0 \end{bmatrix}$ $\to l = 0$, $\to m = 0$, $\to n = 1$

$s: \ x-2 = 2x+z = 0$ $\qquad F(s)$




$\alpha \in F(s) : \alpha \,//\, r$

$\beta \in F(s) : \beta \perp \alpha$

$\gamma \in F(r) : \gamma \perp \alpha$

$t = \beta \cap \gamma$

$\alpha : \lambda(x-2) + \mu(2x+z) = 0$

$\alpha : \underbrace{(\lambda+2\mu)}_{a} x + \underbrace{0}_{b} y + \underbrace{\mu}_{c} z - 2\lambda = 0 \qquad (l, m, n) = (0, 0, 1)$

$\alpha \,//\, r \iff al + bm + cn = 0 \iff \mu = 0 \implies \lambda \neq 0$

scelgo $\lambda = 1$

$\boxed{\alpha : x - 2 = 0} \longrightarrow (a', b', c') = (1, 0, 0)$

$\beta : (\lambda + 2\mu)x + 0y + \mu z - 2\lambda = 0 \to (a, b, c) = (\lambda + 2\mu, 0, \mu)$

$\beta \perp \alpha \iff aa' + bb' + cc' = 0 \iff \lambda + 2\mu = 0$

scelgo $(\lambda, \mu) = (2, -1)$

$\boxed{\beta : z + 4 = 0}$

$\gamma : \lambda(x+1) + \mu(7x+y) = 0$

$\gamma : (\lambda + 7\mu)x + \mu y + 0z + \lambda = 0 \quad (a, b, c) = (\lambda + 7\mu, \mu, 0)$

$\gamma \perp \alpha \iff aa' + bb' + cc' = 0 \iff \lambda + 7\mu = 0$

scelgo $(\lambda, \mu) = (7, -1)$

$$\boxed{\gamma : y - 7 = 0}$$

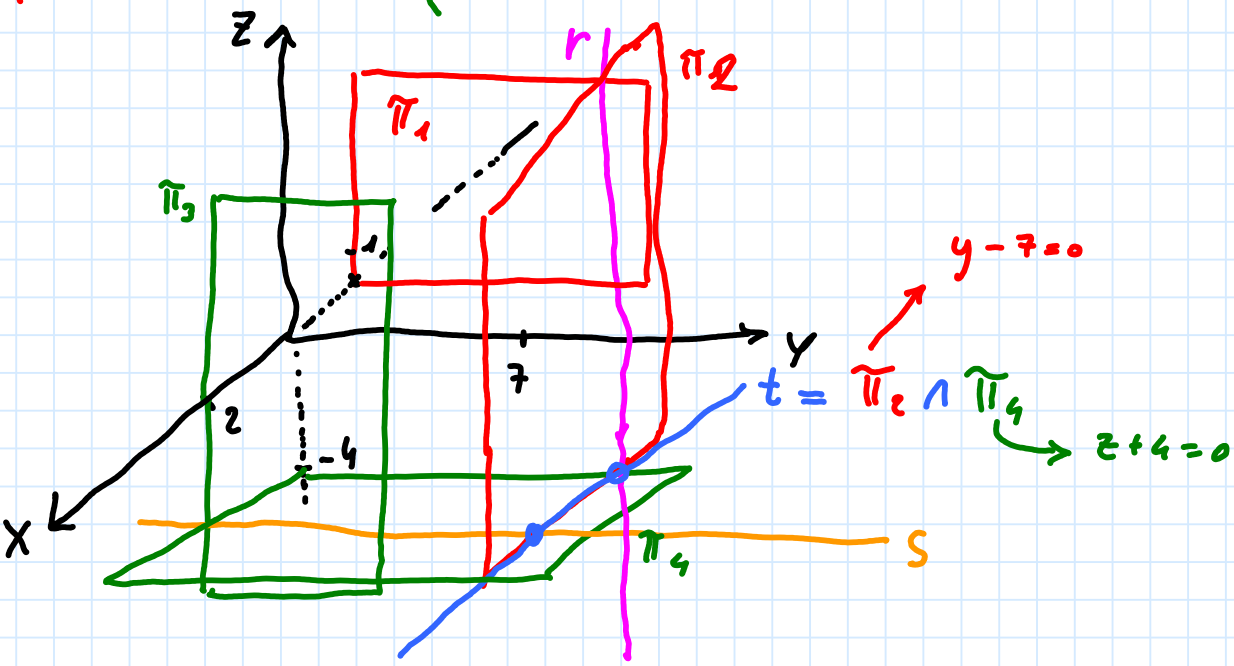
$$t = \beta \cap \gamma : \begin{cases} z + 4 = 0 \\ y - 7 = 0 \end{cases}$$

---

$r : x + 1 = 7x + y = 0$ $\qquad \boxed{x = -1}$ $\qquad r : x + 1 = y - 7 = 0$

$s : x - 2 = 2x + z = 0$ $\qquad x = +2$ $\qquad s : x - 2 = z + 4 = 0$

$$r : \begin{cases} x = -1 \leftarrow \pi_1 \\ y = +7 \leftarrow \pi_2 \end{cases} \qquad s : \begin{cases} x = 2 \leftarrow \pi_3 \\ z = -4 \leftarrow \pi_4 \end{cases}$$




---

$r \parallel$ asse $Y$ et $A(\sqrt{3}, -6, -1) \in r$

$\hookrightarrow (l, m, n) = (0, 1, 0)$

$$r : \begin{cases} x = 0 \cdot t + \sqrt{3} \\ y = 1 \cdot t - 6 \\ z = 0 \cdot t - 1 \end{cases} \qquad r : \begin{cases} x = \sqrt{3} \\ y = t - 6 \\ z = -1 \end{cases} \qquad r : \begin{cases} x - \sqrt{3} = 0 \\ z + 1 = 0 \end{cases}$$

piani $\in F(r)$ : $\lambda\cdot(x-\sqrt{3}) + \mu\cdot(z+1) = 0$

$$\boxed{\lambda\cdot x + 0\cdot y + \mu\cdot z + (\mu - \sqrt{3}\lambda) = 0}$$

$\theta = \frac{\pi}{3}$ rad   asse $z \rightarrow (l, m, n) = (0, 0, 1)$

$(a, b, c) = (\lambda, 0, \mu)$   $(l, m, n) = (0, 0, 1)$

ANGOLO RETTA - PIANO

$$\sin\theta = \frac{|al + bm + cn|}{\sqrt{a^2+b^2+c^2}\cdot\sqrt{l^2+m^2+n^2}}$$

$$\frac{\sqrt{3}}{2} = \frac{|\mu|}{\sqrt{\lambda^2+0^2+\mu^2}\cdot\sqrt{0^2+0^2+1^2}} = \frac{|\mu|}{\sqrt{\lambda^2+\mu^2}}$$

$3\cdot(\lambda^2+\mu^2) = 4\mu^2$   $\boxed{\mu^2 = 3\lambda^2}$

scelgo $\lambda = 1 \Rightarrow \mu = \pm\sqrt{3}$

$\pi_1$ $(\lambda, \mu) = (1, -\sqrt{3})$   $\pi_2$ : $(\lambda, \mu) = (1, \sqrt{3})$

$\pi_1$ : $x - \sqrt{3}\cdot z - 2\sqrt{3} = 0$   $\pi_2$ : $x + \sqrt{3}\cdot z = 0$

---

NEL PIANO   $r$ : $3x - 4y - 5 = 0$ direttrice

parabole per $O(0,0)$ con fuochi su asse $X$.

Trovare i fuochi

$F$ fuoco $\qquad r$ direttrice $\qquad O \in$ parabola

$$d(O,F) = d(O,r) \longleftarrow \text{per definizione}$$

$F(\alpha, 0) \in$ asse $X \qquad O(0,0)$

$$d(O,F) = \sqrt{(\alpha-0)^2+(0-0)^2} = \sqrt{\alpha^2} = |\alpha|$$

$$d(O,r) = \frac{|3\cdot 0 - 4\cdot 0 - 5|}{\sqrt{3^2+(-4)^2}} = \frac{|-5|}{\sqrt{25}} = 1$$

$$|\alpha| = 1 \Rightarrow \alpha = \pm 1 \begin{cases} F_1(1,0) \\ F_2(-1,0) \end{cases}$$

---

$$3x^2 + 2\sqrt{3}xy + x^2 + 2\sqrt{3}x + 2y + 1 = 0$$

$$A = \begin{bmatrix} 3 & \sqrt{3} \\ \sqrt{3} & 1 \end{bmatrix} ; \quad p_A(\lambda) = \dots = \lambda^2 - 4\lambda = \lambda \cdot (\lambda - 4)$$

$A_{2\times 2} \qquad p_A(\lambda) = \lambda^2 - (\mathrm{Tr}A)\lambda + \det A$

$\lambda_1 = 4 \qquad \lambda_2 = 0$

$\lambda_1 = 4 \longrightarrow$ autovettori
$$\begin{bmatrix} -1 & \sqrt{3} \\ \sqrt{3} & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{cases} -x + \sqrt{3}y = 0 \\ \cancel{\sqrt{3}x - 3y = 0} \end{cases} \longrightarrow x = \sqrt{3}y$$

$$(x,y) = (\sqrt{3}y, y) = y(\sqrt{3}, 1) \qquad \forall y \neq 0$$

scelgo $y=\frac{1}{2}$ auto VERSORE $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$$C=\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} \qquad \begin{bmatrix} 2\sqrt{3} & 2 \end{bmatrix}\cdot\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}=\begin{bmatrix} 4 & 0 \end{bmatrix}$$

dopo la rotazione

$$4\cdot(x')^2+0\cdot(y')^2+4x'+0\cdot y'+1=0$$

$$\boxed{4(x')^2+4x'}+1=0$$

$$\boxed{4\cdot\left(1\cdot x'+\frac{1}{2}\right)^2-1}+1=0$$

traslazione $x''=x'+\frac{1}{2}$

$$4\cdot(x'')^2-1+1=0$$

$$(x'')^2=0 \quad \begin{matrix} \nearrow x''=0 \text{ retta} \\ \searrow x''=0 \text{ retta} \end{matrix} \Big\} \text{ stessa}$$

Conica unione di 2 rette reali

COINCIDENTI

---

$r: \; 3x-2y=2z+5=0$ $\qquad d=1$

$\alpha: \; x-4=0$ $\qquad A, B \in r$

$$H\left(0,0,-\frac{5}{2}\right)\in r \qquad \begin{bmatrix} 3 & -2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{matrix} \nearrow l=-4 \\ \rightarrow m=-6 \\ \searrow n=0 \end{matrix}$$

$$(l,m,n)=(2,3,0)$$

$$r:\begin{cases} x=2\cdot t+0 \\ y=3\cdot t+0 \\ z=0\cdot t-\frac{5}{2} \end{cases} \qquad r:\begin{cases} x=2t \\ y=3t \\ z=-\frac{5}{2} \end{cases} \qquad \forall t\in r$$

$$P\left(2t,3t,-\frac{5}{2}\right)\in r \quad ; \quad \alpha: x-4=0$$

$$d=1$$

$$d(P,\alpha)\stackrel{\downarrow}{=}1$$

$$\frac{|2t-4|}{\sqrt{1^2+0^2+0^2}}=1 \quad ; \quad |2t-4|=1$$

$$2t-4=\pm 1 \quad ; \quad 2t=4\pm 1 \begin{matrix} \nearrow 2t=5 \rightarrow t_1=\frac{5}{2} \\ \searrow 2t=3 \rightarrow t_2=\frac{3}{2} \end{matrix}$$

$$t_1=\frac{5}{2} \longrightarrow A\left(5,\frac{15}{2},-\frac{5}{2}\right)$$

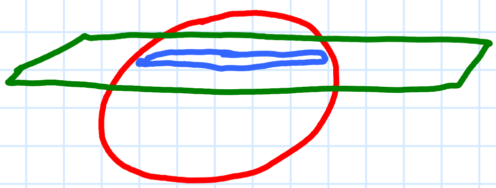
$$t_2=\frac{3}{2} \longrightarrow B\left(3,\frac{9}{2},-\frac{5}{2}\right)$$

---

$$\mathscr{C}: \underbrace{x^2+y^2+z^2-20x-2\sqrt{10}\,y}_{\text{sfera}} = \underbrace{x-3z=0}_{\text{piano}}$$

H r

centro e raggio di $\mathscr{C}$

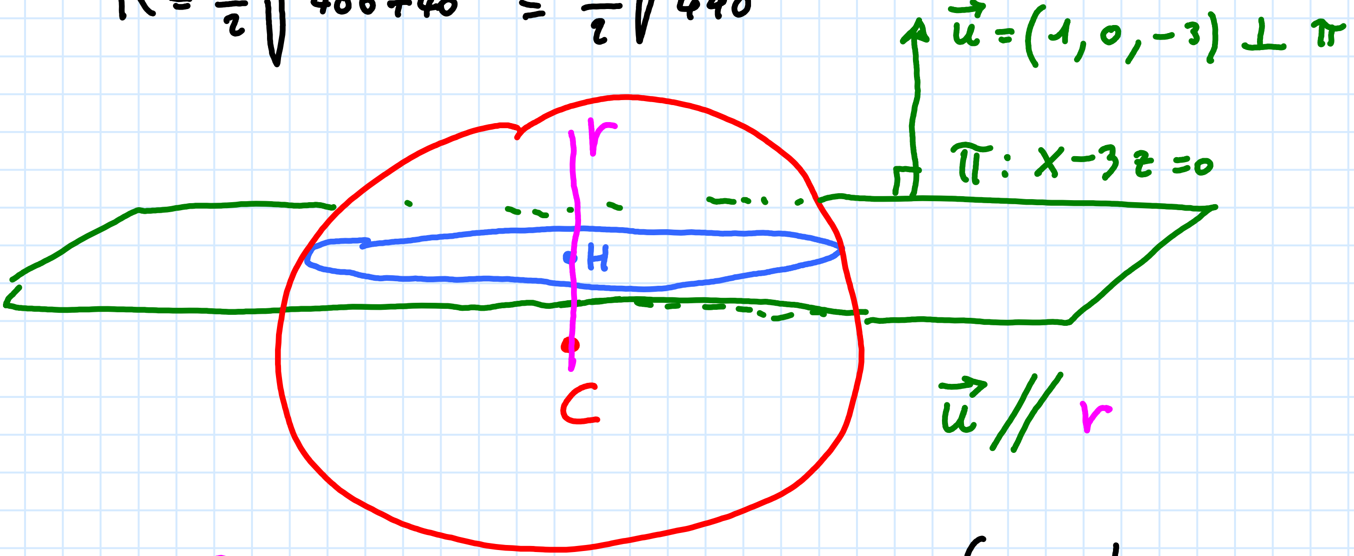


C centro sfera

R raggio sfera

$$C(10, \sqrt{10}, 0) \qquad R = \frac{1}{2}\sqrt{a^2+b^2+c^2-4d}$$

$$R = \frac{1}{2}\sqrt{400+40} = \frac{1}{2}\sqrt{440}$$



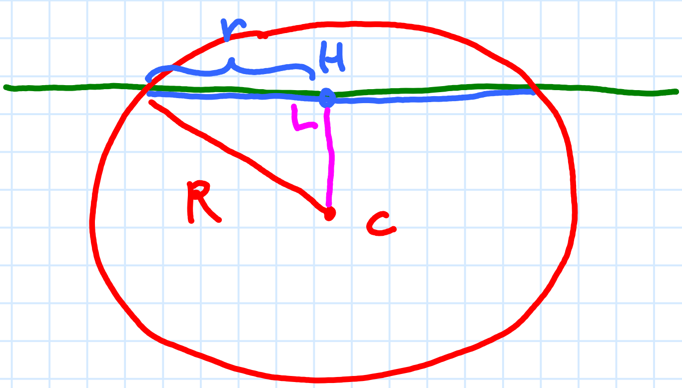

$\vec{u} \,/\!/\, r$

$$r : \begin{cases} x = 1 \cdot t + 10 \\ y = 0 \cdot t + \sqrt{10} \\ z = -3 \cdot t + 0 \end{cases} \qquad r : \begin{cases} x = t + 10 \\ y = \sqrt{10} \\ z = -3t \end{cases}$$

$$\{H\} = r \cap \pi : \begin{cases} x = t + 10 \\ y = \sqrt{10} \\ z = -3t \\ x - 3z = 0 \end{cases} \rightarrow (t+10) - 3(-3t) = 0$$

$$t + 1 = 0 \rightarrow t = -1 \rightarrow \boxed{H(9, \sqrt{10}, 3)}$$

$$R = \sqrt{110} \qquad C(10, \sqrt{10}, 0)$$




$$d(C, H) = \sqrt{1 + 9} = \sqrt{10}$$

$$r = \sqrt{R^2 - [d(C,H)]^2} = \sqrt{110 - 10} = \sqrt{100}$$

$$\boxed{r = 10}$$

$$A = \begin{bmatrix} t & t^2 & 0 \\ 0 & 0 & 3 \\ t & -t & 0 \end{bmatrix} ; \quad \det A = (-3)\cdot(-t^2 - t^3) = 3t^2(1+t)$$

$\forall t \in \mathbb{R} - \{0, -1\} \rightarrow \det A \neq 0 \rightarrow rg\, A = 3$

$t = 0$ $\quad A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow rg\, A = 1$

$\boxed{t = -1}$ $\quad A = \begin{bmatrix} -1 & +1 & 0 \\ 0 & 0 & 3 \\ -1 & +1 & 0 \end{bmatrix} \rightarrow rg\, A = 2$

---

$$A = \begin{bmatrix} 1 & -t & 2 \\ 0 & -5 & (t-4) \\ 0 & 0 & 1 \end{bmatrix} ; \quad p_A(\lambda) = \det \begin{bmatrix} (1-\lambda) & -t & 2 \\ 0 & (-5-\lambda) & (t-4) \\ 0 & 0 & (1-\lambda) \end{bmatrix}$$

$$p_A(\lambda) = (1-\lambda)^2 \cdot (-5-\lambda)^1$$

$\boxed{\lambda_1 = 1}$ $\quad m_a(\lambda_1) = 2 \longrightarrow 1 \leq m_g(\lambda_1) \leq 2$

$\boxed{\lambda_2 = -5}$ $\quad m_a(\lambda_1) = 1 \rightarrow m_g(\lambda_2) = 1$

A è DIAGON $\iff m_g(1) = 2$

$m_g(1) = 2 \iff \dim(E_1) = 2 \iff$

$$\Leftrightarrow 3 - \mathrm{rg}\begin{bmatrix} 0 & -t & 2 \\ 0 & -6 & (t-4) \\ 0 & 0 & 0 \end{bmatrix} = 2 \Leftrightarrow$$

$$\Leftrightarrow \mathrm{rg}\begin{bmatrix} 0 & -t & 2 \\ 0 & -6 & (t-4) \\ 0 & 0 & 0 \end{bmatrix} = 1 \Leftrightarrow \mathrm{rg}\begin{bmatrix} -t & 2 \\ -6 & (t-4) \end{bmatrix} = 1 \Leftrightarrow$$

$$\Leftrightarrow \det\begin{bmatrix} -t & 2 \\ -6 & (t-4) \end{bmatrix} = 0 \Leftrightarrow -t\cdot(t-4)+12=0 \Leftrightarrow$$

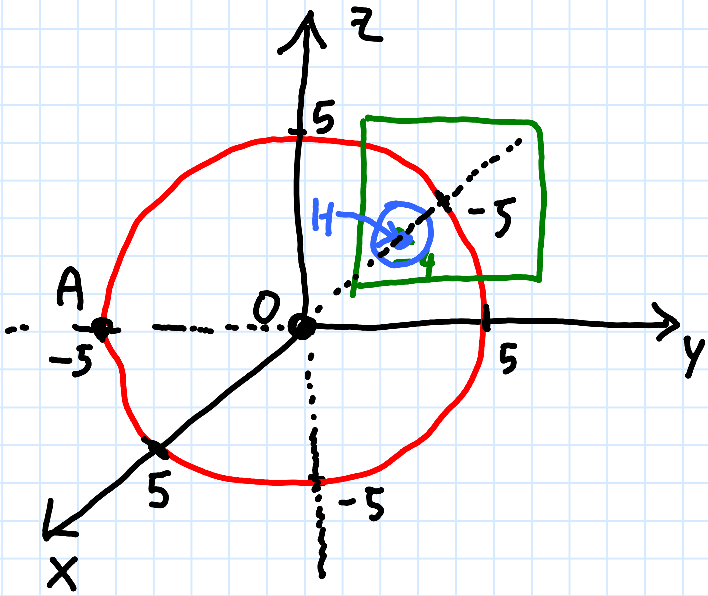
$$\Leftrightarrow t^2-4t-12=0 \Leftrightarrow (t-6)\cdot(t+2)=0 \Leftrightarrow$$

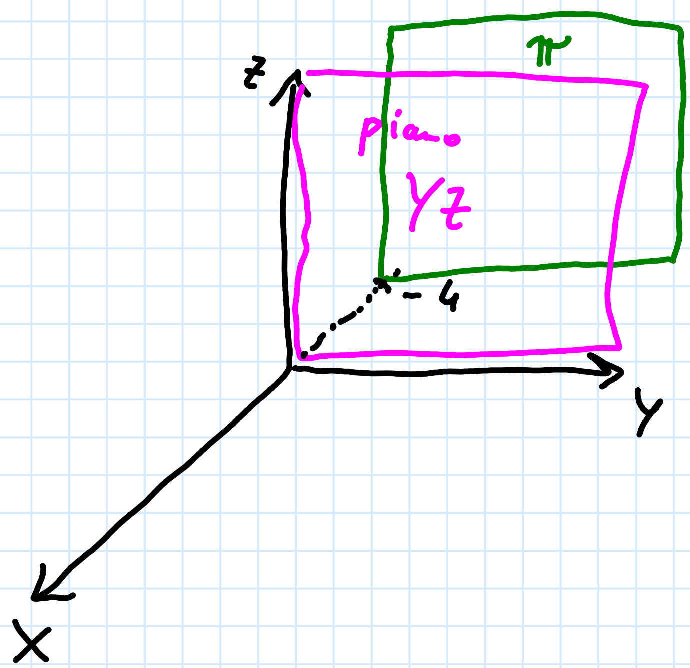
$$\Leftrightarrow t=+6 \text{ vel } t=-2$$

---

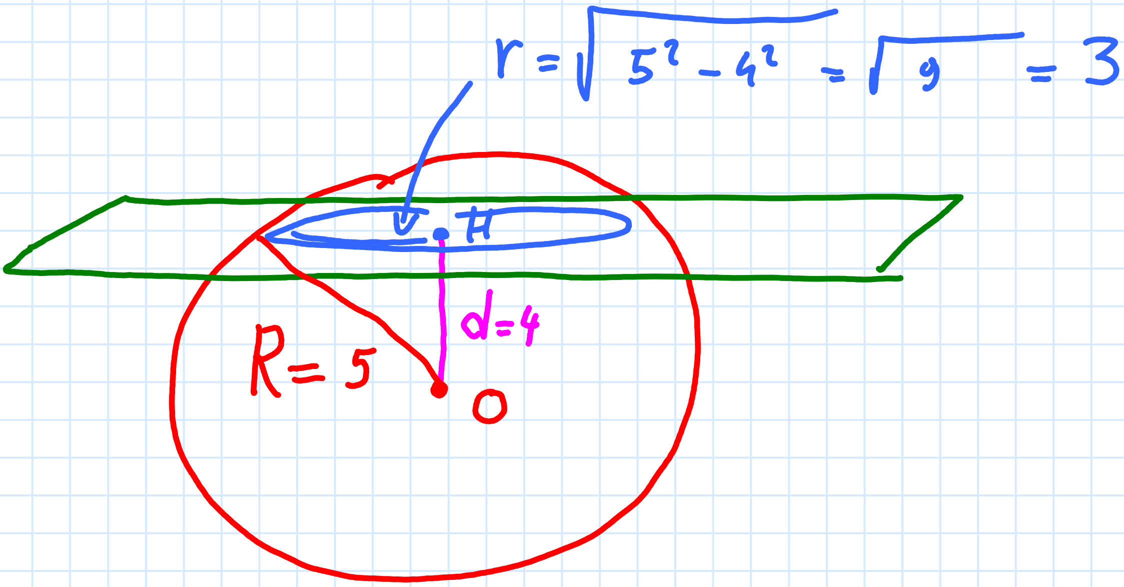
$O(0,0,0)$ centro sfera passante per $A(0,-5,0)$

$\pi \parallel yz$ passante per $B(-4,-3,2)$




$H(-4,0,0)$




$$A = \begin{bmatrix} 3t & 2t^2 \\ 6t & -t \end{bmatrix}; \quad p_A(\lambda) = \ldots\ldots = \lambda^2 - (2t)\cdot\lambda - 3t^2(1+4t)$$

$$\Delta = t^2 + 3t^2(1+4t) = 4t^2 + 12t^3 = $$
$$= 4t^2 \cdot (1+3t)$$

$A$ è diagon. $\Leftrightarrow \Delta > 0 \Leftrightarrow t \neq 0$ et $1+3t>0$

$\Leftrightarrow t \neq 0$ et $t > -\frac{1}{3}$

ma per $t=0$ si ha che $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ è già diagonale

$$\boxed{t > -\frac{1}{3}}$$